Numerical Simulation of the Linear and Second-Order Surface Flows around Circular Cylinders in Random Waves

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Introduction

Fluid flow simulation near offshore structures in waves has been studied many times. Most works were for monochromatic or bichromatic waves. In the present study, multi-frequency wave is considered, aiming the prediction of the multiple linear and quadratic transfer functions with a single run and finally the application to realistic ocean spectrum. The method of solution is a Rankine Panel Method, which has been developed at MIT. In order to extract the components of arbitrary frequencies, the Fourier transform was carried out using not FFT but an integral method. The computation was concentrated on the diffraction problem and the linear and quadratic transfer functions were compared with other benchmark-test results.

Boundary Value Problems

The linear and second-order diffraction problems are quite well known. The velocity potential of random or multi-component incident wave can be written as

$$\Phi_{1,I} = \mathrm{Re}\{ \sum_j \phi_{1,I}(x,y,z)e^{i\omega_j t} \} \tag{1}$$

$$\Phi_{2,I} = \mathrm{Re}\{ \sum_l \sum_m [\phi^+_{2,I}(x,y,z)e^{i(\omega_l+\omega_m)t} + \phi^-_{2,I}(x,y,z)e^{i(\omega_l-\omega_m)t}]\} \tag{2}$$

where the subscript 1,2 means the order of problem. In the second-order wave, there are sum- and difference-frequency components, being applied to numerical computation as an analytic form or being generated during the numerical simulation of the second-order wave flow. The free-surface boundary conditions can be written as follows:

$$\begin{aligned} \frac{\partial \Phi_k}{\partial t} + g\eta_k &= \delta_{2,k}(-\frac{1}{2}\nabla\Phi_1 \cdot \nabla\Phi_1 - \eta_1 \frac{\partial^2 \Phi_1}{\partial z \partial t}) \\ \frac{\partial \eta_k}{\partial t} - \frac{\partial \Phi_k}{\partial z} &= \delta_{2,k}(-\nabla\Phi_1 \cdot \nabla\eta_1 + \eta_1 \frac{\partial^2 \Phi_1}{\partial z^2}) \end{aligned} \qquad \text{on } z=0 \tag{3}$$

where $\delta_{2,k}$ means the delta function which becomes unit when the second-order problem is considered, i.e. k=2. Therefore, at a certain time, the linear solution should be known in order to solve the second-order problem. A zero-flux condition must be imposed on the body surface and other rigid boundary. Furthermore, we need the radiation condition for the uniqueness of solution.

Numerical Method

The method of solution is the Rankine panel method, which has been developed by Sclavounos(1988) for steady forward-speed problem and Nakos(1993) for the unsteady ship motion, and in particular by Kim(1997) for the second-order diffraction problem. Panels are flat, but the representation of the physical quantities is approximated using a B-spline basis function, which takes the form as below:

$$B(\bar{x}_i;\bar{x}_j) \equiv B_{ij} = b_j^{(p)}(\bar{\xi}_i) \times b_j^{(q)}(\bar{\varsigma}_i) \tag{4}$$

where (p) and (q) are the orders of B-spline function on two coordinates along panel surface. Time integration scheme is a modified Euler method, so that the free-surface boundary conditions are written as follows:

$$\frac{\eta_k^{n+1} - \eta_k^n}{\Delta t} - \Phi^n_{z,k} = \delta_{2,k} P_{kin}(\Phi_1^{n+1}, \eta_1^{n+1})$$

$$\frac{\Phi_k^{n+1}-\Phi_k^n}{\Delta t}+g\eta_k^{n+1}=\delta_{2,k}P_{dyn}(\Phi_1^{n+1},\eta_1^{n+1}) \tag{5}$$

where P_{kin} and P_{dyn} mean the forcing terms of equation (3). In addition, the velocity potential on solid boundary and normal flux on free surface can be obtained from Green's identity. The radiation condition is imposed using the artificial damping zone, in which the kinematic free-surface boundary condition is modified.

Stability Issue

The same idea with Nakos(1993) and Kim(1997) can be extended to the stability analysis for an arbitrary order of B-spline function. For a certain disturbance on the descretized free surface with constant-spacing panels, Δx and Δy, the dispersion relation in the discrete domain can be written as follows:

$$W=e^{2i\omega\Delta t}-(2-g\Delta t^2 S)e^{i\omega\Delta t}+1=0 \tag{6}$$

where Δt indicates time segment and $S(u,v)$ is the function of the wave number u and v, which is related with the Fourier transformation of basis function and Rankine source. For the arbitrary orders of basis function defined in equation (4), $S(u,v)$ becomes

$$\begin{aligned}S(u,v)=&\frac{1}{\sqrt{u^2+v^2}}+\frac{(-1)^{p+1}\Delta s(v\Delta s)^{q+1}}{(v\Delta s+2\pi)^{q+1}\sqrt{(u\Delta s)^2+(v\Delta s+2\pi)^2}}+\frac{(-1)^{q+1}\Delta s(u\Delta s)^{p+1}}{(u\Delta s+2\pi)^{p+1}\sqrt{(u\Delta s+2\pi)^2+(v\Delta s)^2}}\\&+\frac{(-1)^{p+q+2}\Delta s(u\Delta s)^{p+1}(v\Delta s)^{q+1}}{(u\Delta s+2\pi)^{p+1}(v\Delta s+2\pi)^{q+1}\sqrt{(u\Delta s+2\pi)^2+(v\Delta s+2\pi)^2}}+O(\Delta s^{p+q+3})\end{aligned} \tag{7}$$

when $\Delta s=O(\Delta x,\Delta y)\to 0$, and it approaches the inverse of continuous wave number. Therefore, equation (6) recovers the continuous dispersion relation as $\Delta t\to 0$. Figure 1 shows the comparison of discrete and continuous dispersion relations for different p and q, i.e. the order of basis function. Here β means $\sqrt{\Delta x/g\Delta t^2}$. It is interesting that the discrepancy for bi-quadratic and bi-4th order functions is not significant. A condition for temporally neutral stability can be derived as a simple form,

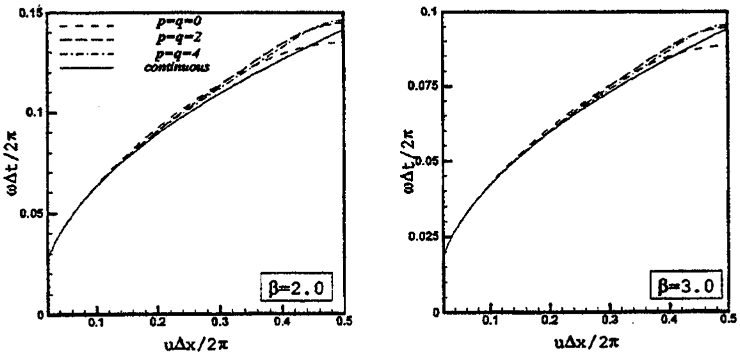

Figure 1 Discrete and continuous dispersion relations

$$\beta\ge\frac{\sqrt{S\Delta x}}{2}\ . \tag{8}$$

Figure 2 shows the contour plots of $S\Delta x/4\beta^2$ when the aspect ratio of panel is unit and $u=v$. Computation will be stable when $|S|\Delta x/4\beta^2\le 1$. In this figure, the basis functions are the bi-quadratic and bi-3rd order functions, showing almost identical borders. Therefore, we may conclude that the order of basis function higher than bi-quadratic doesn't provide significant improvement in the viewpoint of the consistency and stability of numerical scheme.

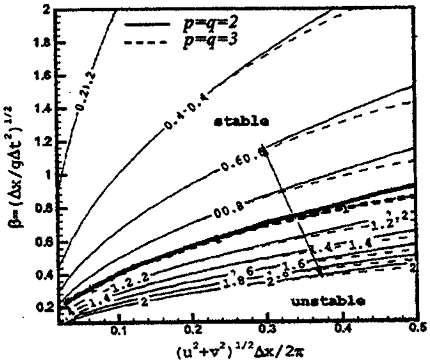

Fig.2 Contour of $|S|\Delta x/4\beta^2$

Fourier Transform

FFT is widely used in the Fourier transform of time signal. However, it may be not suitable when the frequencies of interest are not equally spaced. In the present study, a Fourier-transformation program based on an integral concept was developed in order to extract the arbitrary frequency components, which a user wants to select. The idea is simple. For a certain function, which is written as the sum of a constant and the series of exponential function, the following integral is satisfied.

$$f(t) = C_o + \sum_{k=1}^{N} C_k e^{i\omega_n t} \;\; => \;\; \int_{T_1}^{T_2} f(t)\, e^{i\omega_m t}\, dt = C_o \int_{T_1}^{T_2} e^{i\omega_m t} + \sum_{k=1}^{N} C_k \int_{T_1}^{T_2} e^{i(\omega_n+\omega_m)t}\, dt \qquad (9)$$

Here the weight function is an exponential function with frequency, ϖ_m. The integrals in right side are trivial and we know the analytic solution, while the left integral must be obtained using numerical integration. When we apply N+1 frequencies, which are equal to the basis frequencies of $f(t)$, a matrix equation for unknown C_k can be assembled. In the second-order problem, the sum- and difference-frequencies can be applied in equation (9). In the real numerical computation, there are some unexpected components, for example saw-tooth wave or slowly-decaying transient mode. Therefore, in order to minimize the numerical error, some dummy frequencies are recommended to be included. These dummy frequencies must cover a certain bandwidth, minimizing an aliasing error.

Numerical Computation & Results

The computation was carried out for single cylinders, being bottom-mounted and truncated. Polar grid system with proper stretching near the body was applied. In the simulation of multi-component wave, the solution grid must be fine enough to resolve the shortest wave and the computational domain should be large enough to contain the longest wave. Usually, in the second-order problem, the former comes from the sum-frequency components, while the latter from the difference-frequency. Fig. 3 shows the time signals of the linear and second-order surge force on a bottom-mounted circular cylinder. Four components are mixed in the linear signal, so that the 16(4X4) sum-frequency and 16(4X4) difference-frequency components are in the second-order signal. Quadratic transfer functions (QTFs) can be extracted from the second-order signal, using the Fourier transform

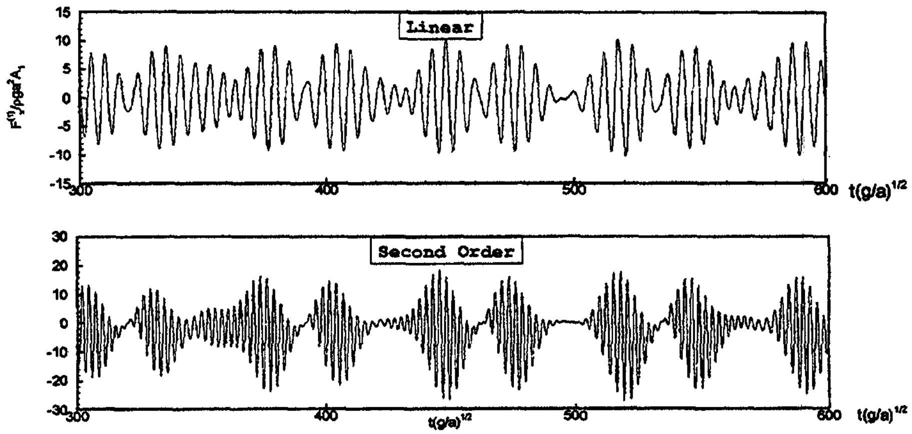

Figure 3 The linear and 2nd order surge forces on a bottom-mounted cylinder radius(a)/depth(d)=0.4, ka=1.0,1.2,1.4,1.6

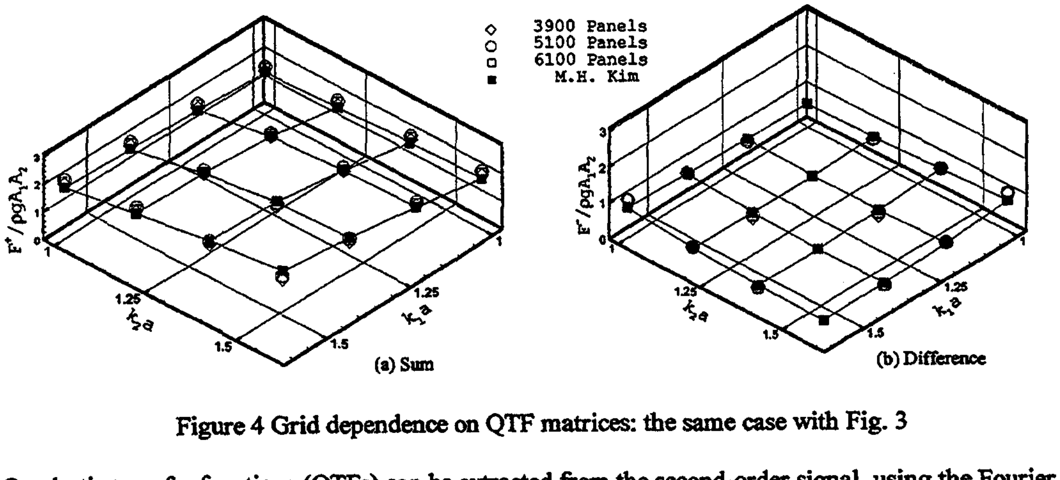

Figure 4 Grid dependence on QTF matrices: the same case with Fig. 3

described above. Fig.4 shows QTF matrices obtained from the time signals with different grid numbers, and the grid dependency in this case is not significant in this case.

The same computational method can be extended to random ocean spectra. Fig. 5 shows the instantaneous linear and second-order wave profiles near a truncated circular cylinder with 10m radius. In this case, ITTC spectrum of Sea State 5 was adopted. The significant wave height is 3.25 m and the modal wave period is 9.7 sec. In the second-order profile, local waves are significant near the body. Fig. 6 shows the force signals in random wave, and Sea State 6 was applied. In Sea State 6, the significant wave height is 5.0m and the modal wave period is 12.4m. The second-order surge force is not significant compared with the linear force, while the second-order heave force cannot be ignored. Microseim effect may be a major source of the second-order heave force. The analysis of the second-order force is not simple since a lot of sum- and difference-frequency components are mixed. The Fourier transform described above is not enough in this case. Therefore, signal-processing techniques will be very useful to analyze the nonlinear statistical characteristics of the second-order random signal.

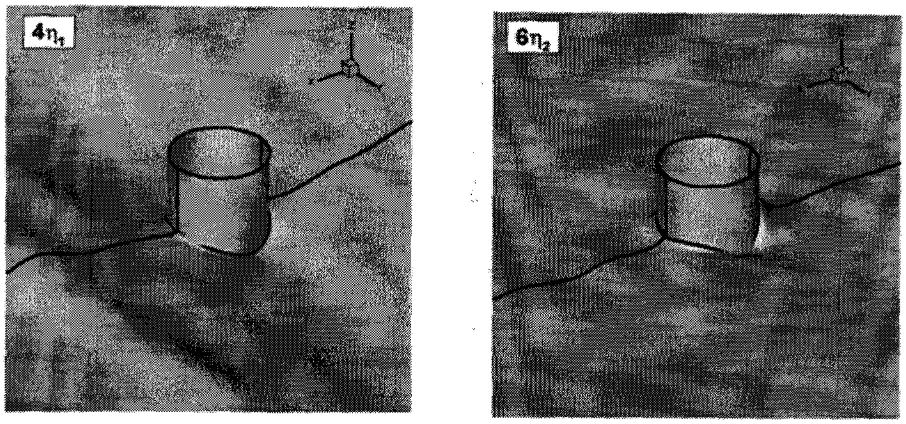

Figure 5 Instantaneous wave profile near a truncated cylinder
ITTC spectrum, sea state 5, d/a=4, Elevations are magnified.

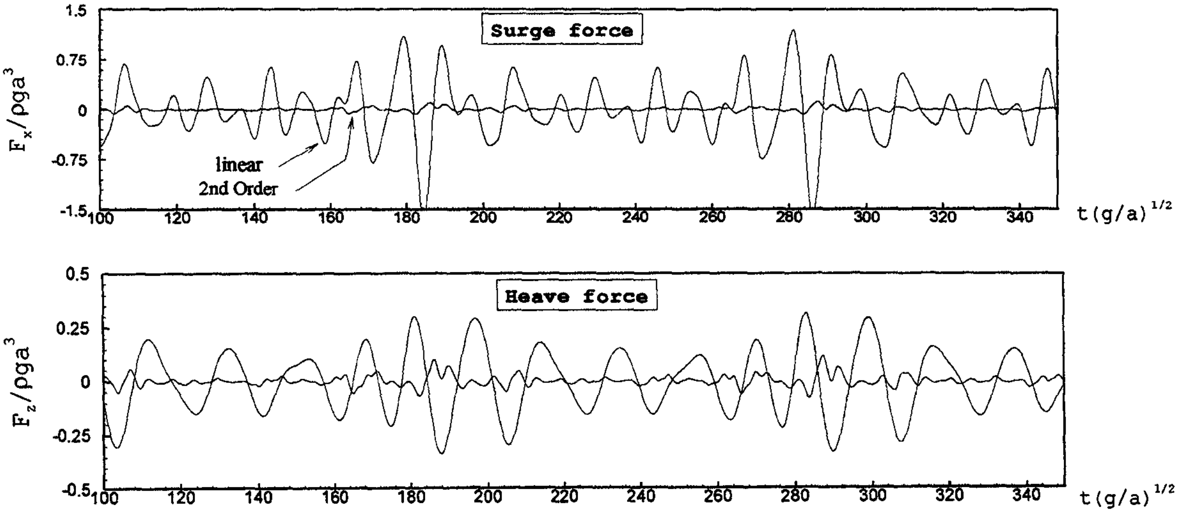

Figure 6 The linear and 2nd order forces on the cylinder in Figure 5, ITTC spectrum, Sea State 6

References

[1] Sclavounos, P.D. and Nakos, D.E., Stability analysis of panel methods for free surface flows with forward speed, *Proc. of 17th ONR*, hague, 1988

[2] Nakso, D.E., Stability of transient gravity waves on a discrete free surface, MIT report, 1993

[3] Kim, Y. and et al., Linear and nonlinear interactions surface waves bodies by a three-dimensional Rankine panel methods, *Applied Ocean Research*, Vol.19, 1997