# A Non-linear Wave Prediction Methodology for Surface Vessels

Donald C. Wyatt, Science Applications International Corporation, San Diego, CA

## INTRODUCTION

The inviscid potential flow problem is formulated for a vessel with steady forward speed, subject to the fully non-linear free-surface boundary conditions and a Neumann boundary condition on the hull-surface. The method is formulated as a boundary value problem to be solved by the method of Green's functions. The method employs both Rankine and Havelock singularities. Rankine singularities are distributed on uniform density hull-panels, and Rankine point sources are positioned on a de-singularized free surface. Havelock point singularities are distributed along the down stream edge of the free-surface domain to enforce a downstream radiation condition.

## FORMULATION

Consider a vessel with steady forward speed, U, in the presence of a free-surface, $z=\eta(x,y)$. Our coordinate system is fixed with the ship, with $x$ positive forward, $y$ positive to port , and $z$ positive upward. The origin is located on the center plane of the vessel and $z$=0 corresponds to the position of the undisturbed free-surface.

We write the velocity potential as the sum of the free-stream potential, -U$x$, and a perturbation potential, $\phi$,

$$\Phi = -\mathrm{U}x + \phi. \qquad (1)$$

The hull boundary condition is

$$\nabla\phi \bullet \mathbf{n} = \mathrm{U}\mathbf{n} \bullet \mathbf{i} \quad \text{on } B. \qquad (2)$$

The kinematic free-surface boundary condition is

$$\eta_t - \mathrm{U}\eta_x - \phi_z = -\phi_x\eta_x - \phi_y\eta_y \quad \text{on } z = \eta(x,y). \qquad (3)$$

The dynamic free-surface boundary condition is

$$\phi_t - \mathrm{U}\phi_x + g\eta = -\frac{p}{\rho} - \tfrac{1}{2}\nabla\phi \bullet \nabla\phi \quad \text{on } z = \eta(x,y). \qquad (4)$$

We proceed by formulating an implicit time-stepping scheme for the kinematic and dynamic boundary conditions. By forward differencing the kinematic condition, in time, we obtain,

$$\frac{\eta^{\mathrm{n+1}} - \eta^{\mathrm{n}}}{\Delta t} - \mathrm{U}\eta_x^{\mathrm{n+1}} - \phi_z^{\mathrm{n+1}} = K^{\mathrm{n}} \quad \text{where } K = -\phi_x\eta_x - \phi_y\eta_y. \qquad (5)$$

The superscripts n and n+1 indicate time iterations. We collect terms of n+1 on the left-hand side to obtain an implicit equation,

$$\frac{1}{\Delta t}\eta^{n+1} - U\eta_x^{n+1} - \phi_z^{n+1} = \frac{1}{\Delta t}\eta^n + K^n. \qquad (6)$$

Note that we have used the $n^{th}$ iteration of the nonlinear terms, which makes the equation first order accurate in time, with implicit treatment of the linear terms and explicit treatment of the non-linear terms. This treatment is useful given that we will be interested in only the steady state solution at this time.

Following similar lines for the dynamic condition, (9), we write

$$\frac{1}{\Delta t}\phi^{n+1} - U\phi_x^{n+1} + g\eta^{n+1} = \frac{1}{\Delta t}\phi^n + D^n - \frac{p}{\rho} \quad \text{where}, \ D = -\frac{1}{2}\nabla\phi \bullet \nabla\phi. \qquad (7)$$

Taking the limit of $\Delta t \to \infty$ in equations drives the time dependent terms to zero. Using the same iteration construct as in the time-dependent equations yields for the kinematic and dynamic boundary conditions:

$$-U\eta_x^{n+1} - \phi_z^{n+1} = K^n \qquad (8)$$

$$-U\phi_x^{n+1} + g\eta^{n+1} = D^n - \frac{p}{\rho} \qquad (9)$$

The steady solution is achieved by an iterative solution of equations (2), (8), and (9). Potentials are approximated by a combination of Rankine and Havelock singularities. Initially the non-linear terms are set to zero, and a solution to the linear set of equations is achieved. The non-linear terms are evaluated for this iteration, and matrix formulation is re-solved with the updated right hand side. The process is repeated until a steady-state non-linear solution is achieved. Once an elevation field is calculated new influence matricies are re-evaluated and the process is repeated. Convergence is evaluated by determining that the boundary conditions are satisfied to within a specified tolerance.

## VALIDATIONS

A Fortran code, named Das Boot, has been constructed to solve the steady formulation. The new code can is applied to hulls whose wave fields have been measured at model-scale to assess the proposed methodology.

### Case 1: Series 60, Cb=0.6, Fn = 0.316

The Series 60, Cb-0.6 hull form is a good validation case for the application of the method to the predictions of steady non-linear wave generated by a realistic hull form. The data to which we compare was collected at the Iowa Institute of Hydrodynamic Research (IIHR) for the Series 60 at a Froude number of 0.316.

Figure 1 shows a comparison of prediction and model-scale data. The correlation of prediction and data is high, $\rho$=0.974. Along the hull it is difficult to discern the elevation differences between prediction and data until just forward of the stern.

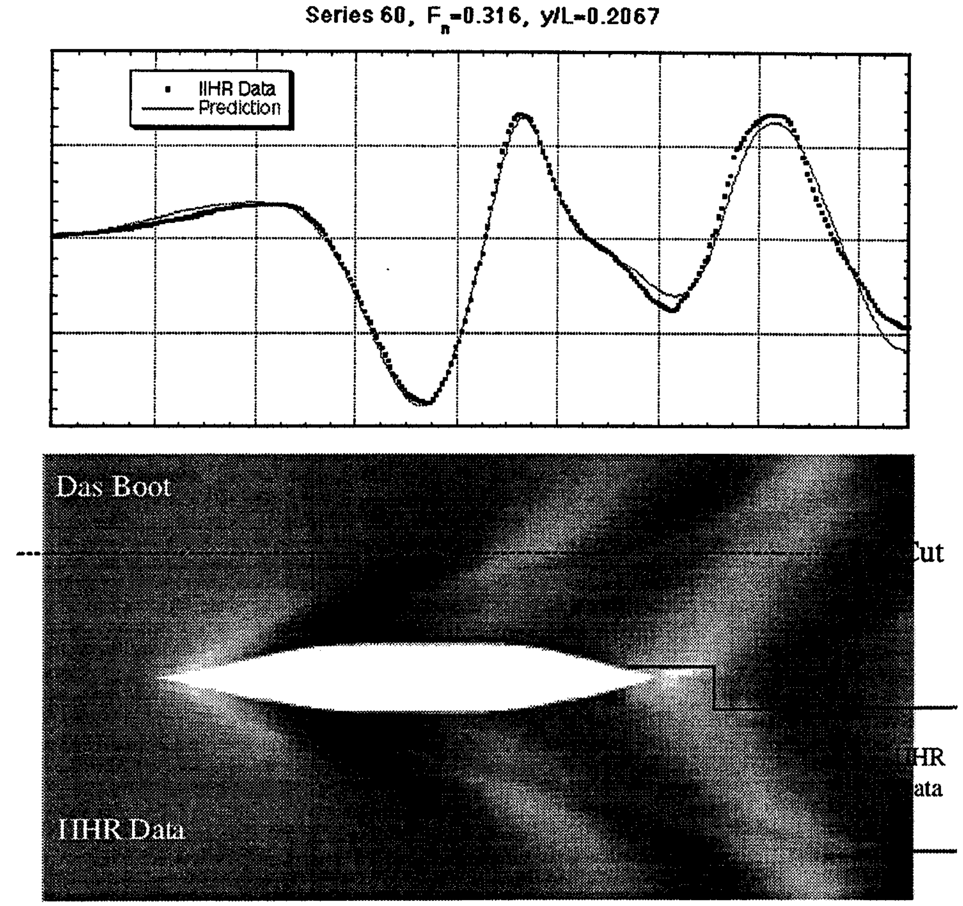


Figure 1. Comparison of predictions and measurement for a wave elevation data for Series 60, Cb=0.6, Fn=0.316.

<u>Case 2. DTMB Model 5415 Fn=0.28</u>

Model 5415 was an early design variant for the DDG51 currently deployed in the U.S. Naval fleet. Extensive model-scale data has been collected to which we can compare and assess the Das Boot predictions. The data to which we will compare was collected at the David Taylor Model Basin (DTMB) for the model 5415 at a Froude number of 0.28.

Figure 2 displays a top-down view of the predicted (5 surface re-grids) and measured data. The upper half of the figure is the Das Boot prediction for elevation on the entire computational domain at a panel density of 35 panels per transverse wavelength. On the lower half of the image the DTMB tank data has been plotted. This includes a longitudinal swath of model-scale wave cut data as indicated by the red lines, a patch of whisker probe data immediately adjacent to the stern, and a patch of whisker probe data near the port bow. The correlation of prediction and data is encouraging, ρ=0.81.

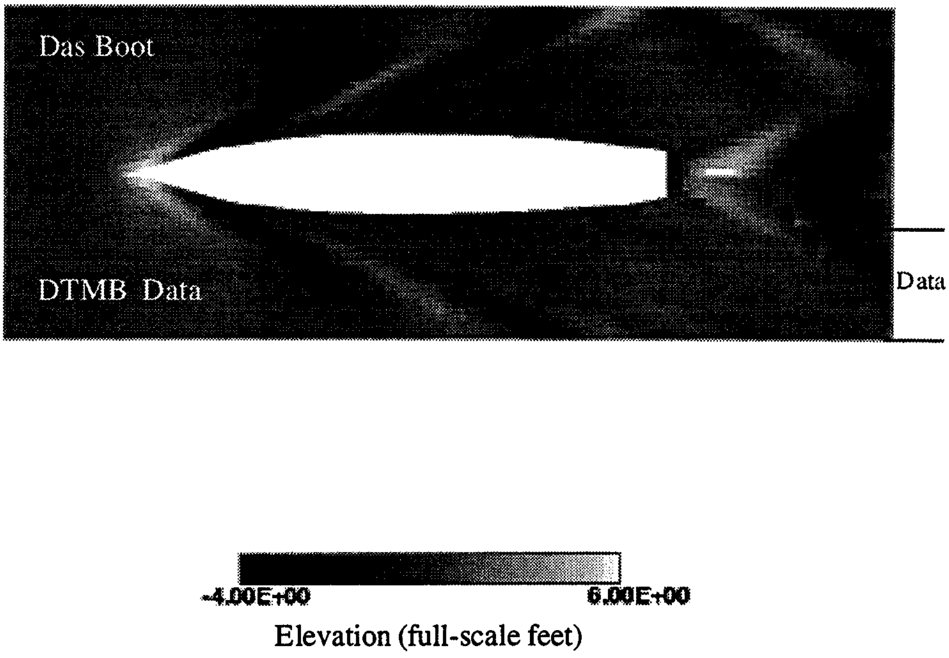


Figure 2. Comparison of non-linear wave elevation fields from Das Boot (upper half) and model scale data (lower half) for DTMB Model 5415 Fn = 0.28

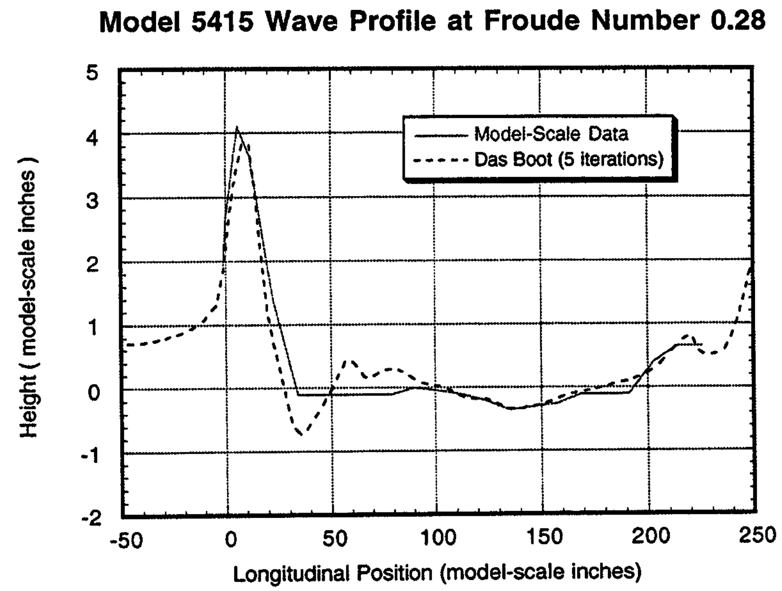


Figure 3. Comparison of prediction and measurement for wave profile on the hull for DTMB Model 5415 Fn = 0.28.