Application of the Fourier-Kochin theory to the farfield extension of nonlinear nearfield steady ship waves

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The Fourier-Kochin theory of steady and time-harmonic ship waves is based on two fundamental theoretical results. These important theoretical results are now summarized. One result, given in *Noblesse et al. (1997)*, is a boundary-integral representation of steady and time-harmonic free-surface potential flows with forward speed. This new boundary-integral representation defines the velocity field $\nabla\phi$ in a potential flow *explicitly* in terms of the velocity distribution $(u\,,v\,,w)$ at a boundary surface Σ. Thus, the new flow representation does not involve the potential ϕ at Σ — unlike the classical Green identity which expresses ϕ within a flow domain in terms of boundary values of the potential ϕ and its normal derivative $\partial\phi/\partial n$ — and defines the velocity field $\nabla\phi$ directly, instead of via numerical differentiation of ϕ. The new flow representation can be used to extend a given nearfield flow (determined using any nearfield flow solver) into the far field, and to couple a viscous nearfield flow — for which a velocity potential cannot be defined — and a farfield linear potential-flow representation.

Another result, given in *Noblesse et al. (1999a)*, is a new mathematical representation of steady and time-harmonic free-surface flows with forward speed generated by *arbitrary* distributions of singularities (e.g., sources and dipoles) over (flat or curved) hull-panels or waterline-segments. Such flows are called *super Green functions* because of the similarity and difference with ordinary Green functions, which are associated with a *point* source instead of a *distribution* of singularities. The mathematical representation of super Green functions given in *Noblesse et al. (1999a)* is valid for a broad class of waves in *generic* dispersive media. This representation of generic super Green functions defines the velocity field $\vec{u}$ as the sum of a *simple-singularity component* $\vec{u}^{\,S}$ given by distributions of simple Rankine singularities, and a *free-surface component* $\vec{u}^{\,F}$ given by a double Fourier superposition of elementary waves. The Fourier representation of free-surface effects $\vec{u}^{\,F}$ can be further decomposed into a *wave component* $\vec{u}^{\,W}$ and a *local component* $\vec{u}^{\,L}$. Thus, the velocity field $\vec{u}$ can be expressed in terms of the flow decomposition

$$\vec{u} = \vec{u}^{\,W} + \vec{u}^{\,L} + \vec{u}^{\,S} \tag{1}$$

This representation of generic super Green functions provides a useful formal decomposition of nearfield free-surface flows (and other dispersive waves) into nonoscillatory local components, which decay rapidly and are significant only in the near field, and a wave component which fully accounts for the waves in the near field (as well as in the far field where the local components are negligible). The expression for the wave component $\vec{u}^{\,W}$, given by single Fourier integrals along the curves (called dispersion curves) defined by the dispersion relation, is especially simple and is well suited for accurate and efficient numerical evaluation.

The Fourier representation of the wave component in the mathematical representation of generic super Green functions and the boundary-integral flow representation given in *Noblesse et al. (1997)* are used here to determine the farfield *steady* ship waves generated by a prescribed velocity distribution at a boundary surface, and to extend nonlinear nearfield steady ship waves into the far field.

For the purpose of verifying the Fourier-Kochin representation of farfield steady ship waves, the linear free-surface potential flow due to a submerged point source of unit strength, i.e. a Green function, is considered in *Noblesse et al. (1999b)*. Briefly, the disturbance velocity generated by the point source is evaluated at a boundary surface Σ that encloses the source. This boundary velocity distribution is then extended outside Σ using the Fourier-Kochin flow representation.

The *outer* flow fields — outside Σ — determined *directly*, by using the expressions for the gradient of the Green function summarized in *Ponizy et al. (1994)*, and *reconstructed* via the Fourier-Kochin representation are identical as expected.

The Fourier-Kochin representation of farfield steady ship waves can be used to extend nonlinear nearfield waves — predicted by any nearfield flow calculation method — into the far field. An illustrative application to the Wigley hull is presented here. The Wigley hull is defined by $y = \pm b\,(1-4x^2)\,[1-(z/d)^2]$ with $b = 0.05$ and $d = 0.0625$. The nearfield flow is evaluated at a boundary surface Σ defined by $x^2/a^2 + y^8/b^8 + z^8/c^8 = 1$ with $a = 0.6\,, b = 0.055\,, c = 0.1$. The nearfield flow in this example is determined using the fully nonlinear calculation method — based on the Euler flow equations — of *Yang and Löhner (1998)*.

The solution domains in the Fourier-Kochin flow representation and the nonlinear nearfield flow calculation method are respectively bounded by the mean free-surface plane $z = 0$ and the actual free surface $z = e$, where e stands for the computed free-surface elevation. The nearfield flow computed at the *Euler* matching boundary surface (with $z \le e$) is therefore mapped onto the *Fourier-Kochin* boundary surface (with $z \le 0$) required for the farfield flow extension. A continuous flow mapping based on linear interpolation is used here. The disturbance velocity distribution, predicted by the Euler nearfield flow solver and used in the Fourier-Kochin flow extension, at the previously-defined matching boundary surface is shown in Fig. 1 for $F = 0.316$.

The nonlinear Euler nearfield wave patterns and their linear Fourier-Kochin farfield extensions are shown in Fig. 2 for $F = 0.25\,, 0.316$, and 0.408. The nearfield and farfield wave patterns appear to be in fairly good agreement, especially in view of the limitations inherent to both the farfield and the nearfield flows. In particular, the simple-singularity component $\vec{u}^S$ and the local component $\vec{u}^L$ in the flow decomposition (1) are ignored in the present implementation of the Fourier-Kochin flow representation. In addition, numerical damping attenuates the nearfield waves relatively quickly.

The *nearfield drag* predicted by the nonlinear Euler nearfield flow solver (via integration of the hull pressure) and the *farfield drag* obtained in the Fourier-Kochin extension (via the Havelock formula for the wave energy radiated by the farfield waves) are listed below, together with the corresponding experimental values :

F	Near	Far	Exp
0.250	0.97	0.90	0.82
0.316	1.58	1.55	1.525
0.408	2.33	2.27	2.31

These theoretical and experimental values of the wave drag coefficient (multiplied by 1000) are in relatively fair agreement.

References

Noblesse F.; Yang C.; Chen X.B. (1997) *Boundary-integral representation of linear free-surface potential flows*, J. Ship Research 41 pp.10-16

Noblesse F.; Chen X.B.; Yang C. (1999a) *Generic Super Green functions*, Ship Technology Research 46

Noblesse F.; Yang C.; Löhner R.; Hendrix D. (1999b) *Fourier-Kochin representation of farfield steady ship waves*, Journées de l'Hydrodynamique, Marseille, France

Ponizy B.; Noblesse F.; Ba M.; Guilbaud M. (1994) *Numerical evaluation of free-surface Green functions*, J. Ship Research 38 pp.193-202

Yang C.; Löhner R. (1998) *Fully nonlinear ship wave calculation using unstructured grid and parallel computing*, Proc. 3rd Osaka Colloquium on Advanced CFD Applications to Ship Flow and Hull Form Design, Osaka, Japan, pp.125-150

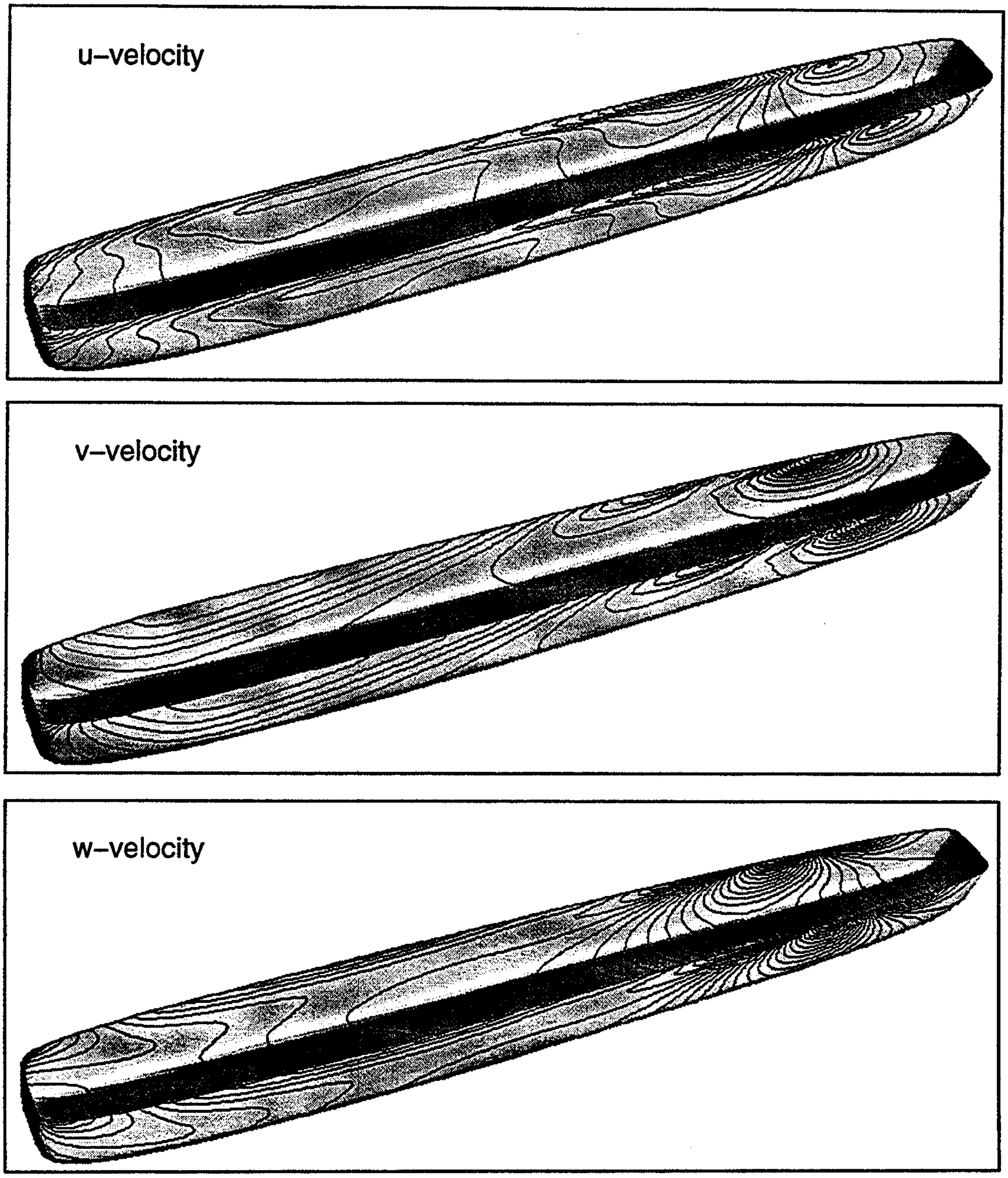

Fig.1 - Velocity distribution on matching boundary surface (Wigley hull, $F=0.316$)

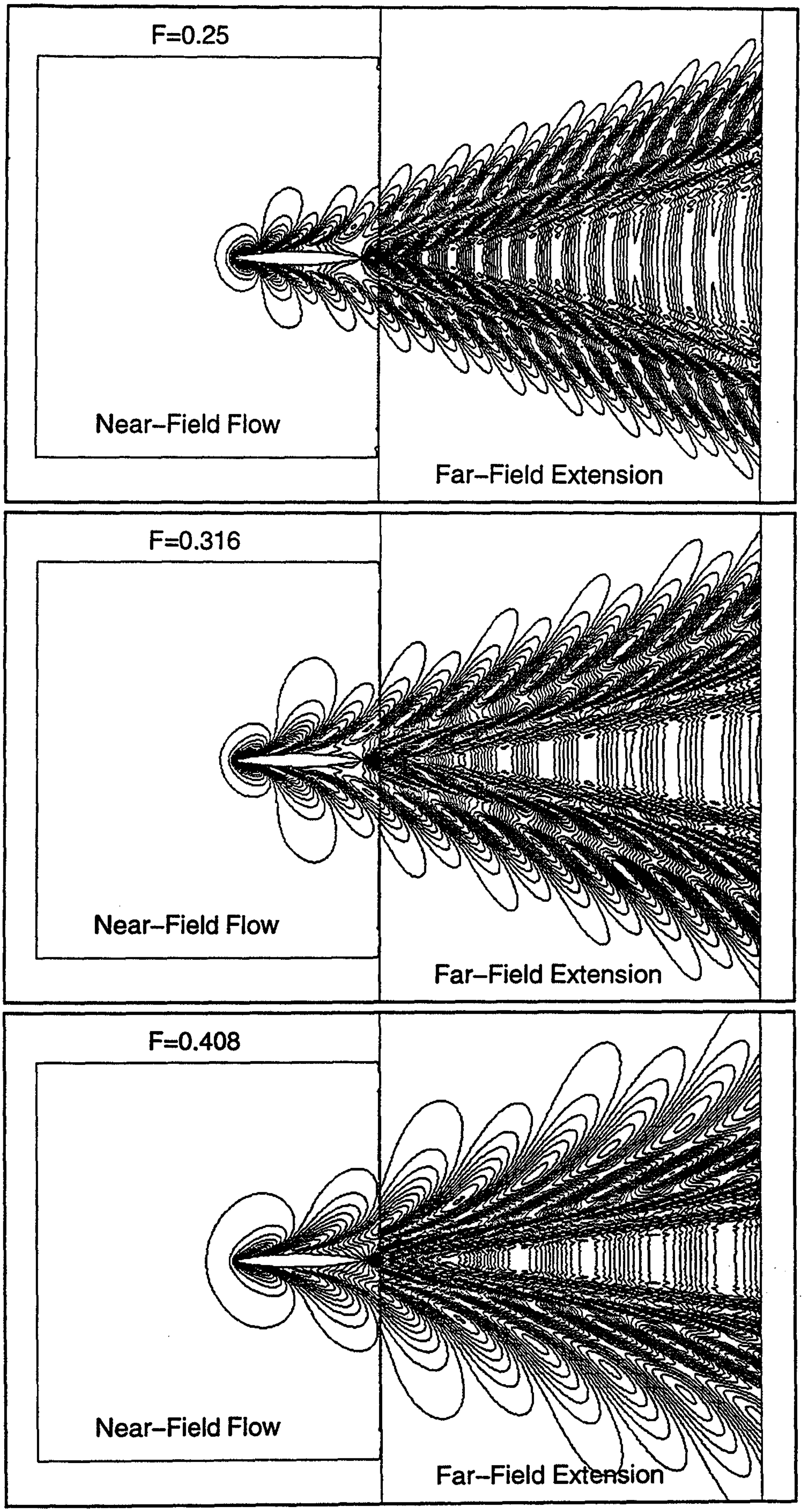

Fig.2 - Wave patterns generated by the Wigley hull