A 3D Potential Flow Computing Method Based on NURBS

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1. Introduction

Rankine panel method is an effective tool for computing potential flow about arbitrary three-dimensional bodies. It has been developed from first order panel method to high order panel method. Concerning the high order panel method, Hess (1979), Johnson (1980) and Kehr (1994) used quadratic curve panel and linear source density distribution, Hsin (1994) used the so-called hyperboloidal panel and constant source density. Although all the above methods can increase accuracy, the body surface need to be discretized into a large number of panels, and the derivative continuity at the boundaries between panels can not be ensured. Hsin (1993) and Maniar (1995) presented a B-spline based panel method. The body surface is divided into a number of patches, where the potential and geometry on each patch is a parametric representation of B-spline, and some degree of continuity is retained everywhere on the patch. But it doesn't ensure derivative continuity at the boundary between patches, and is unable to deal with conic and quadric shapes in an exact way.

Non-uniform rational B-spline (NURBS) retain not only all B-spline properties, but also can represent conic and quadric shapes exactly. It has been established as an industrial standard in CAD system. This paper adopts the NURBS to represent body surface and source density for potential computation. It doesn't need to divide the body surface into different panels or patches and can ensure some degree of continuity. Moreover, it can become a bridge linking CFD and CAD systems.

2. Basic Formulation

Let $\vec{p}(u,v)=[x(u,v),y(u,v),z(u,v)]$ be the parametric equation of a closed body surface, x, y and z denote the Cartesian coordinates, u and v denote the two parameters for surface definition, $\sigma(u,v)$ denote the source density of a point on the body. The $\vec{p}(u,v)$ and $\sigma(u,v)$ can be written as:

$$\bar{p}(u,v)=\frac{\sum_{i=0}^{m_b}\sum_{j=0}^{n_b}W_{ij}\bar{B}_{ij}N_{i,k}(u)N_{j,l}(v)}{\sum_{i=0}^{m_b}\sum_{j=0}^{n_b}W_{ij}N_{i,k}(u)N_{j,l}(v)} \quad (1)$$

$$\sigma(u,v)=\frac{\sum_{i=0}^{m_s}\sum_{j=0}^{n_s}W_{sij}S_{ij}N_{i,k}(u)N_{j,l}(v)}{\sum_{i=0}^{m_s}\sum_{j=0}^{n_s}W_{sij}N_{i,k}(u)N_{j,l}(v)} \quad (2)$$

Where $\bar{B}_{ij}$ and S_{ij} are the control net, W_{ij} and W_{sij} are the weights, $N_{i,k}$ and $N_{j,l}$ are the B-spline basis functions of order k and l, defined by the Cox-de Boor recursive expressions.

The negative x-axis is in the direction of uniform flow $\vec{U}$. The velocity potential at a point $\vec{q}$, represented by $\phi(\bar{q})$, due to a source distribution over point $\bar{p}(u,v)$ on the surface with a density $\sigma(\bar{p})$, can be written as:

$$\phi(\bar{q}) = \iint \frac{\sigma(\bar{p})}{4\pi r(\bar{p},\bar{q})} ds \qquad (3)$$

Where $r(\bar{p},\bar{q}) = |\bar{p} - \bar{q}|$ denote the distance between $\bar{q}$ and $\bar{p}$.

2.1 Non-lifting Potential Flow Problem

For this problem, the velocity potential on the body surface satisfies the integral equation [1]:

$$-0.5\sigma(\bar{q}) + \iint \frac{\partial}{\partial n}\left(\frac{1}{4\pi r(\bar{p},\bar{q})}\right)\sigma(\bar{p})\, ds = \bar{U}\cdot\bar{n}(\bar{q}) \qquad (4)$$

where $\bar{n}$ denotes the surface unit normal directed out of the body. Since $\bar{q}$ lies also on the body surface, $\bar{p}$ and $\bar{q}$ can be represented by different u, v, $r(\bar{p},\bar{q})$ can be written as:

$$r(\bar{p},\bar{q}) = \frac{1}{|\bar{p}(u,v) - \bar{p}(u_0,v_0)|} \qquad (5)$$

where $\bar{p}(u,v)$ denotes $\bar{q}$, $\bar{p}(u_0,v_0)$ denotes $\bar{p}$. Inserting formulation (1) and (2) into equation (4), equation (4) becomes:

$$\sum_{i=0}^{m_s}\sum_{j=0}^{n_s} S_{ij} f_{ij}(u,v) = \bar{U}\cdot\bar{n}[\bar{p}(u,v)] \qquad (6)$$

$$f_{ij}(u,v) = -\frac{W_{sij}N_{i,k}(u)N_{j,l}(v)}{2\sum_{e=0}^{m_s}\sum_{h=0}^{n_s} W_{seh}N_{ei,k}(u)N_{j,l}(v)}$$
$$+ W_{sij}\iint \frac{\nabla\left(\frac{1}{4\pi|\bar{p}(u,v) - \bar{p}(u_0,v_0)|}\right)\cdot\bar{n}[\bar{p}(u,v)]N_{i,k}(u_0)N_{j,l}(v_0)}{\sum_{e=0}^{m_s}\sum_{h=0}^{n_s} W_{seh}N_{ei,k}(u_0)N_{f,l}(v_0)}\cdot\sqrt{EG - F^2}\, du_0 dv_0$$

(7)

$$\bar{n}[\bar{p}(u,v)] = \frac{\bar{p}_u \times \bar{p}_v}{|\bar{p}_u \times \bar{p}_v|} \qquad (8)$$

(6) is a linear equation system for S_{ij} control net of source density. S_{ij} are to be solved by given $(m+1)\times(n+1)$ collocation points at the direction u and v.

2.2 Wave-making problem

The body surface and the source density on body surface are represented by NURBS in this work. In order to satisfy the radiation condition, the collocation point shift method is adopted. Furthermore, the free-surface boundary conditions are linearized in a first order Taylor Series expansion [3]. Thus the source density control net of body and free surface can be obtained by solving a linear equation system. It can be written as:

On the body

$$\sum_{i=0}^{m_b}\sum_{j=0}^{n_b} S_{bij} f_{bij}^{bb}(u,v) + \sum_{i=0}^{m_f}\sum_{j=0}^{n_f} S_{fij} f_{ij}^{bf}(u,v) = C_b(u,v) \tag{9}$$

On the free surface

$$\sum_{i=0}^{m_b}\sum_{j=0}^{n_b} S_{bij} g_{ij}^{fb}(u,v) + \sum_{i=0}^{m_f}\sum_{j=0}^{n_f} S_{fij} f_{ij}^{ff}(u,v) = C_f(u,v) \tag{10}$$

3. Numerical Results

3.1 Potential Flow around a Sphere without Free Surface

Fig.1 shows a spherical surface generated by NURBS, where $m_b = 4, n_b = 4, k = 2, l = 2$. The collocation point normal vectors are also showed. Tab.1 shows the results for different number of collocation points on $\frac{1}{4}$ spherical surface. It can be seen that the results are very accurate.

3.2 Potential Flow around an Ellipsoid with Free Surface

Fig.2 and Fig.3 show the calculated wave patterns for an ellipsoid with axis-lengths $l_x : l_y : l_z = 2:1:1$ at $F_n = 0.4$ and $F_n = 0.7$.

3.3 Potential Flow around a Wigley Hull with Free Surface

Fig.4 shows the wave pattern for a Wigley hull at F_n=0.348

4. Concluding Remarks

A numerical method for calculating potential flow around arbitrary three-dimensional bodies with or without free surface is proposed. Non-uniform rational B-spline (NURBS) is used to represented the boundary surface and the Rankine source density on the boundary surface. The boundary surface does not need to be divided into different panels or patches, and some degree of continuity can be ensured. Numerical results show that the potential flow can be calculated accurately by using this method.

References

[1] Hess, J.L and Smith, A.M.O., Calculation of Nonlifting Potential Flow about Arbitrary Three-Dimensional Bodies, J. of Ship Research, Sep.1964, PP22-44.

[2] Maniar, H., A B-spline Based Higher Order Method in 3D, Proc. of the 10th International Workshop on Water Waves and Floating Bodies, April 1995, Oxford, UK, PP153-157.

[3] Jensen, G., Berechnung der stationaeren Potentialstroemung um ein Schiff unter Beruecksichtigung der nichtlinearen Randbedingung an der freien Wasseroberflaeche, 1988, IFS-Bericht Nr.484, Uni. Hamburg.

[4] Janson, C.E., Potential Flow Methods for the Calculation of Free-surface Flows with Lift, Ph.D. Thesis, Chalmers University of Technology, Goteborg, April 1997.

Table 1

X/Radius	ANALYTIC	4X6	4X6	6X16	6X16
X0	V0	V	V/V0	V	V/V0
0.0000	1.5000	1.4763	98.42%	1.4920	99.46%
0.3846	1.3846	1.3307	96.11%	1.3523	97.67%
0.6897	1.0862	1.1973	110.23%	1.1087	102.07%
0.8824	0.7059	0.8648	122.51%	0.7381	104.56%
0.9756	0.3293	0.4569	138.71%	0.3401	103.20%
1.0000	0.0000	0.0000	100.00%	0.0000	100.00%

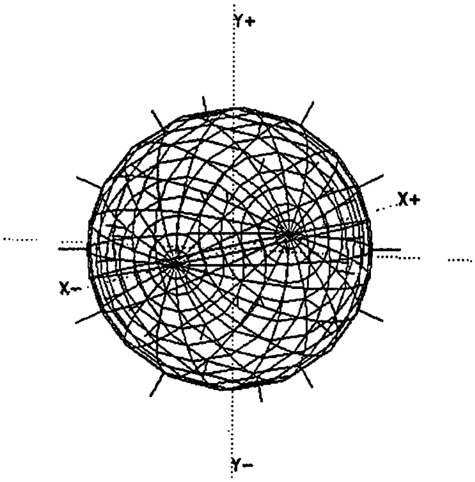

Fig 1

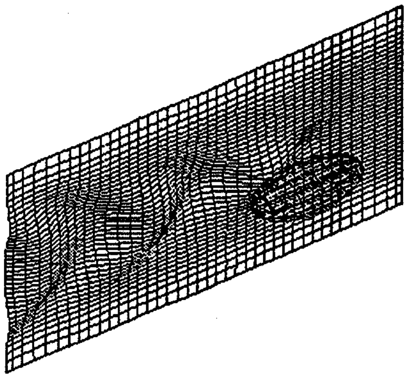
Fig 2

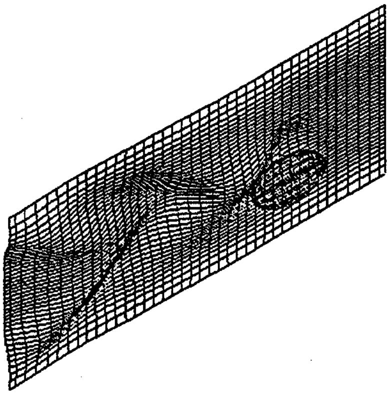
Fig 3

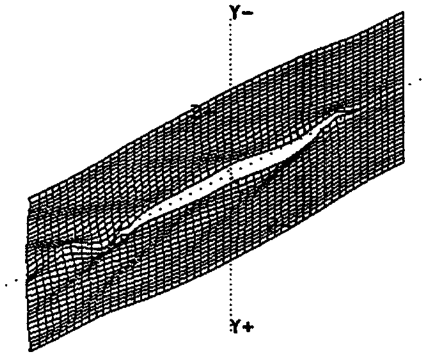

Fig 4