NONLINEAR WAVES GENERATED BY A SURFACE-PIERCING BODY USING A UNIFIED SHALLOW-WATER THEORY

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Introduction

Modified Boussinesq's equations have been successfully applied in coastal engineering for simulating wave propagation from the deep sea to a shallow-water region, e.g. Nwogu (1993) and Schröter (1995), and in naval architecture for computing ship waves in shallow water, e.g. Jiang (1998) for a slender ship and Jiang & Sharma (1998) for a flat ship. But practical application to a general ship-form and to ship-form optimization was not possible because neither the slender-body theory nor the hydrostatic pressure assumption, used to approximate the ship's influence on the ambient flow, satisfies the tangential-flow condition on the wetted ship-surface. To overcome this fundamental difficulty in applying Boussinesq's equations to wave-body interactions, a new theory was recently derived by Jiang (1999) in which the tangential-flow condition on the wetted body-surface is explicitly included in a set of equations of Boussinesq type for the flow region under the body. After interfacial coupling of these equations for the flow inside the waterline with the associated Boussinesq's equations for the flow outside the waterline, a unified shallow-water theory emerges for many wave-related problems in shallow water.

A Unified Shallow-Water Theory for 2-D Wave-Body Interactions

For the sake of record we give here only the final formulation of the unified shallow-water theory for two-dimensional wave-body interactions. The mathematical derivation for general three-dimensional cases of varying water depth is fully documented in the work by Jiang (1999).

This unified shallow-water theory comprises a set of Boussinesq's equations

$$\left.\begin{aligned} &\zeta_t + [(\zeta + h)\overline{u}]_x = 0 \\ &\overline{u}_t + \overline{u}\,\overline{u}_x + g\zeta_x - \frac{h^2}{3}\overline{u}_{txx} = 0 \end{aligned}\right\}, \qquad (1)$$

for the flow outside the waterline with the unknown depth-averaged horizontal velocity $\overline{u}$ and wave elevation ζ, and a set of nonlinear partial-differential equations of Boussinesq type

$$\left.\begin{aligned} &-T_t + [(h - T)\overline{u}]_x = 0 \\ &\overline{u}_t + \overline{u}\,\overline{u}_x + \left(\frac{p_s}{\rho}\right)_x - \frac{(h-T)^2}{3}\overline{u}_{txx} - \frac{h-T}{3}T_t\overline{u}_{xx} + (h-T)T_x\overline{u}_{tx} = gT_x \end{aligned}\right\}, \qquad (2)$$

for the flow inside the waterline with the unkown depth-averaged horizontal velocity $\overline{u}$ and pressure p_s acting on the wetted body-surface. Herein g denotes the acceleration of gravity, ρ the water density, T the local instantaneous draft and h the constant water-depth; t is the independent time variable and x the horizontal coordinate.

[1] We thank Professor Sharma for his steady support and helpful discussions.

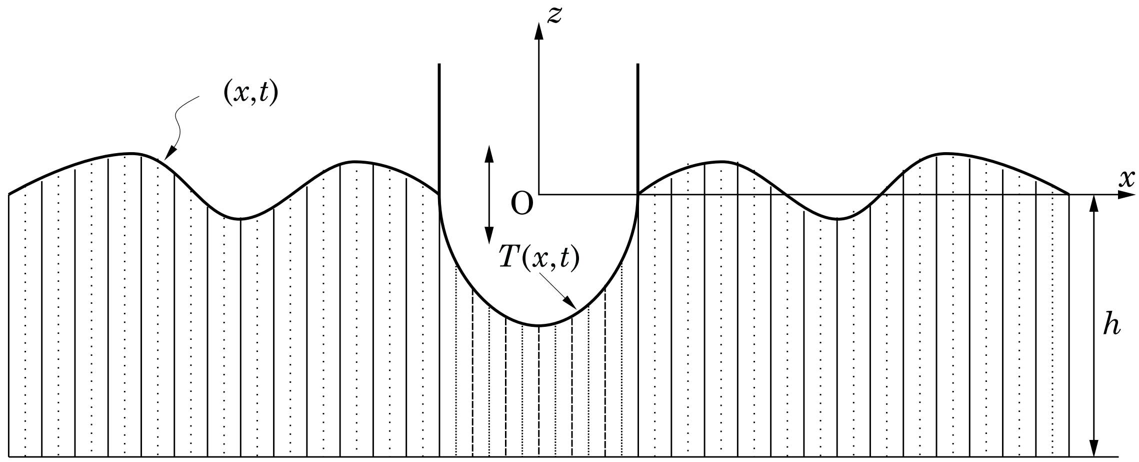

Figure 1: Schematic of space discretization using staggered grid

These two sets of equations are coupled by the interfacial condition at the waterline $x = x_{\mathrm{w}}$

$$\left.\begin{array}{l} \overline{u}_{\mathrm{I}}(x_{\mathrm{w}}, t) = \overline{u}_{\mathrm{O}}(x_{\mathrm{w}}, t) \\ \\ p_{\mathrm{s}}(x_{\mathrm{w}}, t) = 0 \end{array}\right\}, \tag{3}$$

without a vertical sidewall, i.e. $T_{\mathrm{w}} = T(x_{\mathrm{w}}, t) = -\zeta(x_{\mathrm{w}}, t)$, and

$$\left.\begin{array}{l} (h - T_{\mathrm{w}})\overline{u}_{\mathrm{I}}(x_{\mathrm{w}}, t) = (h + \zeta)\overline{u}_{\mathrm{O}}(x_{\mathrm{w}}, t) \\ \\ p_{\mathrm{s}}(x_{\mathrm{w}}, -T_{\mathrm{w}}, t) = p_{\mathrm{O}}(x_{\mathrm{w}}, -T_{\mathrm{w}}, t) \end{array}\right\}, \tag{4}$$

with a vertical sidewall, i.e. $T_{\mathrm{w}} = T(x_{\mathrm{w}}, t)$, where the subscripts I and O denote the flow fields inside and outside the waterline, respectively. The pressure at $z = -T_{\mathrm{w}}$ can be approximated by

$$p_{\mathrm{O}}(x_{\mathrm{w}}, -T_{\mathrm{w}}, t) = \rho g(\zeta + T_{\mathrm{w}}) + \rho(\frac{T_{\mathrm{w}}^2}{2} - hT_{\mathrm{w}})\overline{u}_{tx}, \tag{5}$$

which is obtainable from the outside flow.

Nonlinear Waves Generated by a Vertically Oscillating Body

Considering the analogy of the decoupling of the pressure p_{s} on the wetted body-surface from the depth-averaged continuity equation to that familar from the Euler's equations, the well-established concept of a staggered grid, see Figure 1, is implemented in our computer code based on a full Crank-Nicolson scheme.

In our preliminary study we apply this unified theory to simulate the waves generated by two vertically oscillating bodies. One has the shape

$$T_{\mathrm{S}} = T_{\mathrm{o}} \cos^2 \pi \frac{x}{L} \quad \text{for} \quad |x| \leq \frac{L}{2}, \tag{6}$$

where T_{o} and L denotes the maximum draft and the body length, respectively. The second has a rectangular shape with a constant draft $T_{\mathrm{S}} = T_{\mathrm{o}}$ and with the same length as the first one. These two bodies, both having a wall-sided freeboard over the still waterline, are forced to

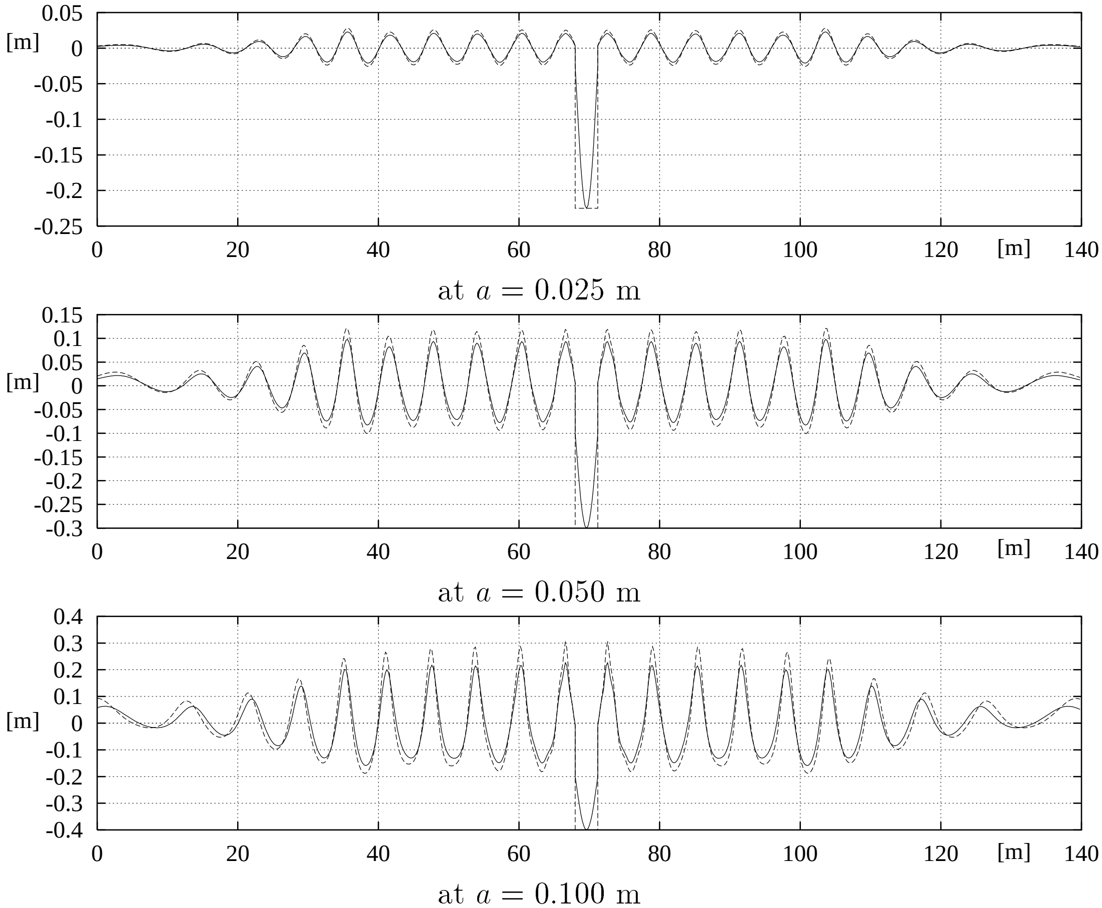

Figure 2: Comparison of waves generated by a rectangular body (dashed lines) with those by a trigonometric one (solid lines) oscillating in shallow water of depth $h = 1$ m

oscillate harmonically in shallow water of depth $h = 1$ m. The associated instantaneous local draft is then defined by

$$T = T_{\mathrm{S}} + a(1 - \cos \omega t), \tag{7}$$

where ω is the forcing frequency and a the forcing amplitude.

For a combination of parameter values $L = 3.14$ m, $T_{\mathrm{o}} = 0.2$ m, and $\omega = 2.73$ s^{-1}, Figure 2 compars the waves generated by a rectangular body (dashed lines) with those by a trigonometric one (solid lines) for three forcing amplitudes a =0.025, 0.05, and 0.10 m. As expected, for the same forcing amplitude, the rectangular body with larger displacement generates also waves with higher amplitudes. For the small forcing amplitude in graph (a), the waves behave more harmonically. For the large forcing amplitude in graph (c), the waves are characterized by steeper wave crests and by flatter hollows. This is typical for nonlinear shallow-water waves.

More fundamentally, Figure 3 shows the total pressure, corresponding to eight representative phases within a full period for the forcing amplitude $a = 0.10$ m. This reasonable looking pressure distribution, acting on the wetted-body surface, could be calculated for the first time by a shallow-water wave theory, namely the unified one.

Based on these preliminary results, which need to be theoretically verified and experimentally validated, we do believe in a large potential for the application of the new unified theory, consisting of two sets of Boussinesq type equations interfacially coupled at the waterline, for many wave-related 3-D problems in shallow water, reduced to the 2-D horizontal plane.

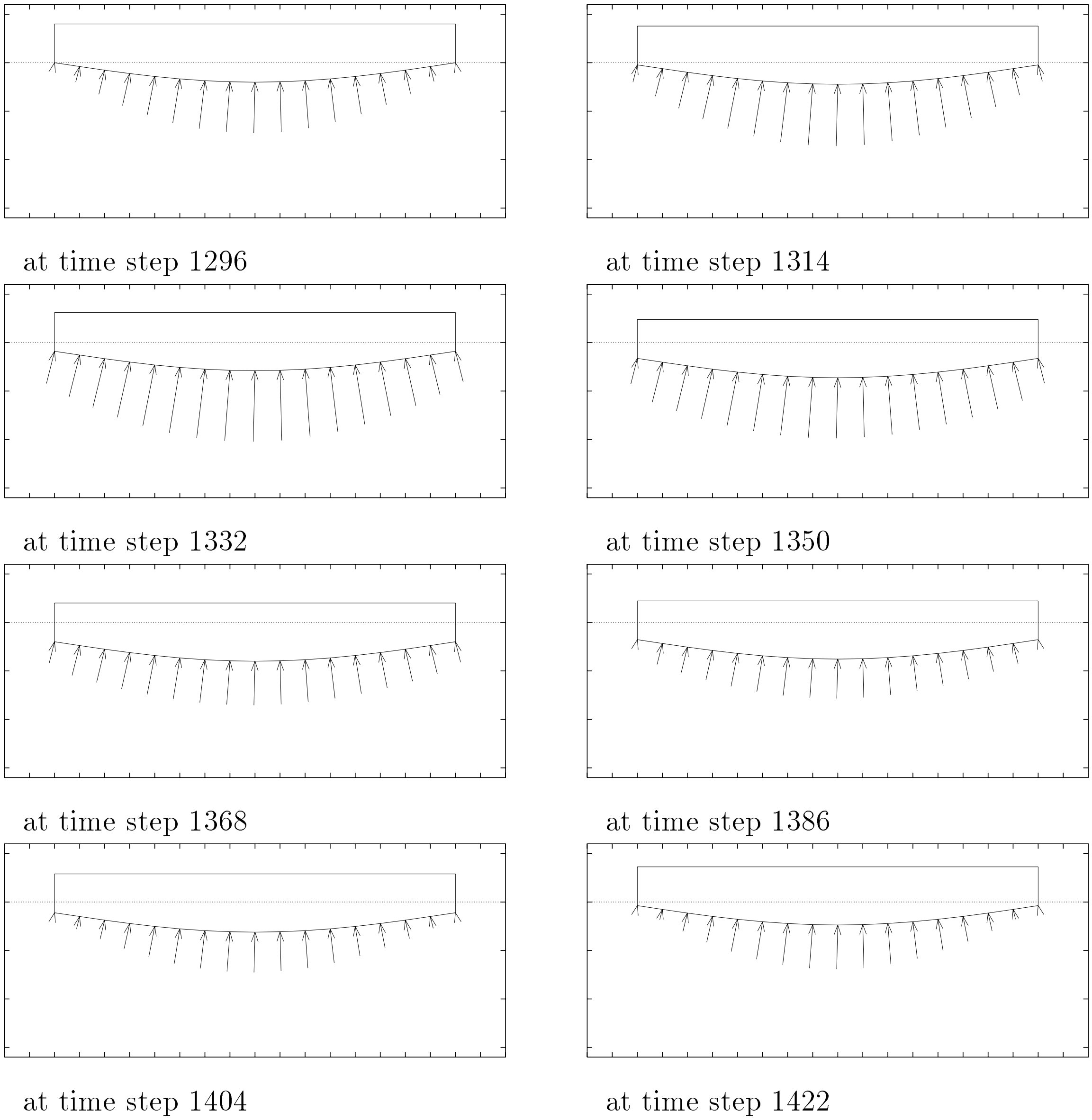

Figure 3: Evolution of the pressure distribution acting on the wetted-surface of the subject trigonometric rigid-body oscillating in shallow water of depth $h = 1$ m at eight representative phases within a full period

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