DIFFRACTION OF WATER WAVES BY AN AIR CHAMBER

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The wave-induced loads on a very large floating structure can be reduced by using an air cushion to provide part of the static support. This idea has been advanced by Pinkster *et al* (1997, 1998), with computations and experimental measurements of the motions in waves and the air-cushion pressure. They consider a long rectangular barge with side walls and ends enclosing the air cushion. Further computations have been performed by Lee & Newman (1999) using a more complete description of the acoustic disturbance, with particular attention to the resonant modes of the air cushion which are analogous to the 'cobblestone effect' suffered by air-cushion vehicles (Ulstein & Faltinsen, 1996).

Here we consider a simpler two-dimensional diffraction problem, which provides qualitative insight into the coupling between the acoustic and water waves. In the plane (x, y), water occupies the semi-infinite domain $-\infty < x < \infty$, $-\infty < y < 0$. Air is enclosed within a chamber bounded by vertical walls at $x = \pm a$, a horizontal lid at $y = b$, and by the free surface $-a < x < a$, $y = 0$, as shown in Figure 1. The walls and lid are fixed. Plane waves of amplitude A, frequency ω and wavenumber $k = \omega^2/g$ are incident from $x = -\infty$. The motions of the water and air are assumed sufficiently small to justify a linearized analysis.

With the time-dependence represented by the factor $e^{i\omega t}$, the velocities of the air and water are equal to the gradients of the complex potentials $\Phi(x, y)$ and $\phi(x, y)$. (Upper/lower case symbols or the subscripts a/w are used where appropriate to distinguish the air/water domains, respectively.) The governing equations are the Helmholtz and Laplace equations

$$\nabla^2\Phi + K^2\Phi = 0, \qquad \nabla^2\phi = 0, \tag{1}$$

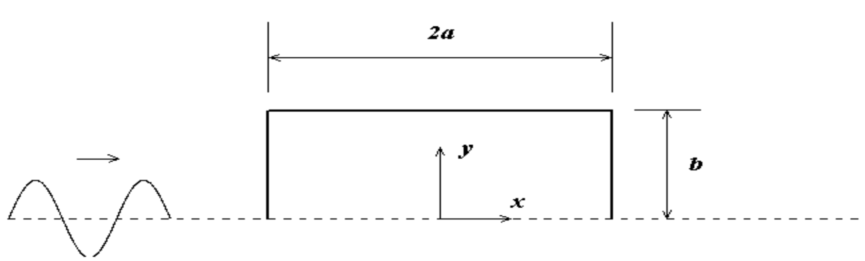

Figure 1: Sketch showing the air chamber and incident wave.

where $K = \omega/c$ is the acoustic wavenumber and c is the sound velocity. The corresponding pressures are given by the linearized Bernoulli equation:

$$P(x,y) = -\,\mathrm{i}\rho_a\omega\Phi(x,y), \qquad p(x,y) = -\,\mathrm{i}\rho_w\omega\phi(x,y) - \rho_w g y, \tag{2}$$

where ρ is the density. The aerostatic pressure $-\rho_a g y$ is neglected on the assumption that $c >> g/\omega$.

Zero normal velocity is prescribed on the ends and lid of the chamber. On the air-water interface the kinematic and dynamic conditions are combined in the usual manner to give the linear free-surface condition

$$\rho_w(\omega^2\phi - g\phi_y) = \rho_a\omega^2\Phi. \tag{3}$$

The boundary-value problem for ϕ is completed by imposing the conventional conditions, including the homogeneous form of (3) on the the exterior free surface, the requirement that ϕ vanishes as $y \to -\infty$, and an appropriate radiation condition in the far field.

The potential in the air chamber can be expanded in the form

$$\Phi = \,\mathrm{i}\omega A \sum_{m=0}^{\infty} \xi_m \Phi_m(x,y), \tag{4}$$

where

$$\Phi_m(x,y) = f_m(x)\frac{\cosh v_m(b-y)}{v_m \sinh v_m b} \tag{5}$$

and

$$f_m(x) = \cos\left(u_m(x-a)\right). \tag{6}$$

Here

$$u_m = \frac{m\pi}{2a}. \tag{7}$$

The normalizing factors are chosen such that $\Phi_{my}(x,0) = f_m(x)$. The elevation of the interface is

$$\eta(x) = A\sum_{m=0}^{\infty} \xi_m f_m(x). \tag{8}$$

The eigenfunctions $\Phi_m(x,y)$ satisfy homogeneous Neumann conditions on the ends and top of the chamber. The coefficients v_m follow from the Helmholtz equation:

$$v_m^2 = u_m^2 - K^2. \tag{9}$$

These coefficients are either real or imaginary, and the eigenfunctions Φ_m are real.

The potential in the water can be derived by superposition of the incident-wave potential

$$\phi_I = \frac{\mathrm{i}gA}{\omega}\exp(ky - \,\mathrm{i}kx) \tag{10}$$

with the solution for an oscillatory pressure imposed on the free surface (Wehausen & Laitone, 1960, equation 21.17). After adapting to the present notation it follows that

$$\phi = \phi_I - \frac{i\omega}{\pi\rho_w g}\int_{-a}^{a} P(\xi,0)d\xi \int_0^\infty \cos\kappa(x-\xi)\,\mathrm{e}^{\kappa y}\frac{d\kappa}{\kappa - k}. \tag{11}$$

In the integral with respect to κ the contour of integration passes below the pole $\kappa = k$, in accordance with the radiation condition. After substituting (2) and (4-5),

$$\phi = \phi_I + \mathrm{i}\omega A \sum_{m=0}^{\infty} \xi_m \phi_m, \tag{12}$$

where

$$\phi_m = -\frac{k\rho_a}{\pi \rho_w v_m \tanh v_m b} \int_{-a}^{a} f_m(\xi) d\xi \int_0^\infty \cos\kappa(x-\xi)\, \mathrm{e}^{\kappa y} \frac{d\kappa}{\kappa - k}. \tag{13}$$

After imposing the kinematic boundary condition $\phi_y = \mathrm{i}\omega\eta$, multiplying by $f_n(x)/a$, and integrating over $(-a, a)$, a linear system of equations is derived for the unknown coefficients ξ_m in the form

$$\sum_{m=0}^{\infty} \xi_m C_{mn} = D_n, \tag{14}$$

where

$$C_{mn} = \frac{1}{a}\int_{-a}^{a} f_n(x)\left[f_m(x) - \frac{k\rho_a}{\pi\rho_w v_m \tanh v_m b}\int_{-a}^{a} f_m(\xi)d\xi \int_0^\infty \cos\kappa(x-\xi)\frac{\kappa d\kappa}{\kappa - k}\right]dx \tag{15}$$

and

$$D_n = \frac{1}{a}\int_{-a}^{a} f_n(x)\,\mathrm{e}^{-\mathrm{i}kx} dx = -\frac{2\,\mathrm{i}^n ka}{(u_n^2 - k^2)a^2}\sin\left(ka + \frac{n\pi}{2}\right). \tag{16}$$

The integrals in (15) with respect to x and ξ are elementary, and the remaining integral with respect to κ can be evaluated in terms of sine and cosine integrals.

The simplest physical parameters to consider are the vertical exciting force and pitch moment, due to the acoustic pressure P acting on the ends and lid of the air chamber. Neglecting the contribution to the pitch moment from the ends, the force and moment are given by

$$\begin{pmatrix} X_3 \\ X_5 \end{pmatrix} = \int_{-a}^{a} \begin{pmatrix} 1 \\ -x \end{pmatrix} P(x,0)dx. \tag{17}$$

After normalizing by the long-wavelength (hydrostatic) limits of the force and moment for a flat plate of the same width, it follows that

$$\frac{X_3}{2\rho_w g a A} = \frac{\rho_a k \xi_0}{\rho_w v_0 \sinh v_0 b} \tag{18}$$

and

$$\frac{X_5}{\frac{2}{3}\rho_w g k a^3 A} = -\frac{12\rho_a}{\pi^2 \rho_w} \sum_{\substack{n=1 \\ (n\ \text{odd})}}^{\infty} \frac{\xi_n}{n^2 v_n \sinh v_n b}. \tag{19}$$

For asymptotically large wave periods the normalized heave force tends to a value less than the usual hydrostatic limit of 1.0, due to the compressibility of the air. The pitch moment tends to zero more quickly than the corresponding hydrostatic moment for a conventional floating body, due to equalization of the pressure throughout the chamber when the period is very large.

Preliminary computations indicate a very large pitch exciting moment in relatively short waves, at the resonant frequency of the first anti-symmetric acoustic mode ($Ka = \pi/2$). The maximum heave force occurs in longer waves. More complete computations of the heave force, pitch moment, and modal amplitudes ξ_n will be presented at the Workshop.

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