Suppression of Wave Breaking in Nonlinear Water Wave Computations Including Forward Speed

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INTRODUCTION

Much progress has been made over the years in the development of numerical methods for the prediction of nonlinear water waves and wave loads on structures (e.g., Beck, 1998). However, due to inherent limitations in the mathematical model in most of the prevalent methods, a major and persistent difficulty has existed in the simulation of highly nonlinear free-surface flows: the characteristic occurrence of spray and wave breaking causes the computations to terminate abruptly.

Motivated by the desire to prevent even a fleeting generation of spray and breaking waves from curtailing the numerical simulations, a variety of artifices have been formulated in order to overcome the difficulty. Examples include the works of Haussling and Coleman (1979); Wang et al. (1993); and Subramani et al. (1998). The present work describes recent advances in the suppressing of wave breaking.

The approaches above consist essentially of two parts: (1) detecting the likely incidence of wave breaking and then (2) controlling the wave breaking locally. In an improvement over the previous techniques, Subramani et al. (1998) devised a general yet simple numerical criterion, based on the local free-surface curvature, for the likely incidence of wave breaking. The wave breaking was then suppressed in a manner similar to Haussling and Coleman's (1979)—by exerting an additional pressure distribution on the wave, in the vicinity of the breaking wave.

However, the approach proved not to be robust: it was difficult to determine the appropriate amount of damping and there were also problems with extending the technique to calculations involving forward speed, especially in three dimensions. These limitations are overcome through a new, improved technique, which is used in conjunction with the existing, curvature-based criterion for wave breaking. In this paper, we will provide background and describe the technique briefly. Sample calculations will also be presented, including gravity waves in a two-dimensional numerical wave tank and two-dimensional transom stern calculations at forward speed.

BACKGROUND

On the basis of empirical studies, Subramani et al. (1998) proposed the following criterion as a "trigger" for the activation of localized wave damping in their fully nonlinear computations. A wave may go on to break if it satisfies the relation, $|ka| > 0.35$, where a is the wave amplitude and k is the free-surface curvature. Using this criterion and an additional free-surface pressure distribution (as in Haussling and Coleman, 1979), they demonstrated the successful suppression of the breaking wave observed in the physical experiment of Grue et al. (1993). This is reproduced in figure 1.

The curvature-based criterion for breaking is simple and easy to implement, even when waves of different frequencies are present. The curvature may be computed easily—in two dimensions at least—using a local three-point formula arising from the fitting of a circle through three consecutive free surface nodes. This formula gives agreeable results with the analytically obtained curvatures for wave-like profiles.

The new, localized-fairing technique continues to use the exceeding of the threshold $|ka|$ to flag "problem" free-surface nodes. The difference lies in how a breaking wave (the growth of the flagged nodes) is subsequently suppressed. The working of the technique is depicted in figure 2 (a detailed snapshot of the wave surface at the end of a time-step at time, t=11.30) and is described below.

METHODOLOGY

The technique lends itself well to Euler-Lagrange methods for solving water wave problems, but its "post-processing nature" also renders it adaptable to other kinds of methods. For illustrative purposes, consider an instant within the UM-DELTA method (Beck, 1998) when the boundary-value problem has been solved and the free surface has been updated by the growths, $\partial h / \partial t$ and $\partial f / \partial t$ (h being the wave elevation and f, the velocity potential). The updated free surface, before fairing, is given by the dashed line in figure 2, with the free-surface nodes marked by hollow circles. Wave breaking, when it occurs, is manifested as a large growth in h at certain nodes.

The use of the curvature-based criterion enables us to flag such nodes (which satisfy $|ka| > 0.35$) and simply to fair through them. As sketched in figure 2, we fit a cubic spline through the non-flagged nodes and re-position the problem nodes on this faired wave surface. In essence, we "bridge" through the flagged, problem free-surface nodes. Extensive tests have shown that a cubic spline gives reliable results.

This technique is essentially a variation of Wang et al.'s (1993) "peeling" technique; however, "bridging" may be more apt a description of the present treatment. In their studies on wave group evolution and deformation, Wang et al. had performed an ad hoc reduction—based on a limiting value of hk, where k is the wave number—of the wave crest at the leading wave front. Another important difference between the two techniques is that we do an identical fairing for f – in order to ensure that there is no mismatch between h and f for nodes on the new free surface. Wang et al. prescribed an exponential decay in elevation when faced with evaluating f for nodes on a free surface that had been lowered because of a peeling-away.

Note that implied in our technique is the discarding of the computed values of $\partial h / \partial t$ and $\partial f / \partial t$ for the nodes that seem poised—on the basis of the local curvature—to break. The flagged nodes by themselves are not discarded, it is important to add. Following the fairing and the setting of updated Dirichlet boundary conditions for the free surface, all the free-surface nodes are carried into the boundary value problem at the next time step. Also, the fairing process is carried out at the end of each intermediate time-step in the 4^{th}-order Runge-Kutta time-stepping algorithm.

The post-fairing calculation, in which the tendency to break has been suppressed at a particular instant of time, is also shown in figure 2. Note also that the difference seen between the "post-fairing wave surface" and the "without damping" calculation is a cumulative result of the damper having been activated over the previous time-steps.

Given the fairing-through of the problem free-surface nodes that occurs, there is an inevitable but slight loss of mass in our calculations. Figure 3 shows the extent to which mass conservation is controlled in the calculation discussed above. In the figure are plotted the percentage change of fluid volume in the tank (obtained by integrating the wave heights throughout the tank) for both, a typical "no breaking involved" calculation and one in which the breaking-wave damper is activated. For the calculation in question, the mass loss (about 0.05%) is much less than the uncertainty that creeps into the calculations (about 0.20%) due to an in-between-sources non-satisfaction of the body boundary condition at the ends of the tank.

FORWARD SPEED CALCULATIONS

One of the advantages of the present technique is its ease of application and robustness for instances of wave breaking in calculations involving forward speed. Consider, for example, the breaking wave behind a two-dimensional transom stern (in inviscid flow). This is a problem that has been tackled by several researchers, including—recently—Scorpio and Beck (1997). The first wave crest behind the transom stern has been observed to break in the case of dry-transom flow at F_H (Froude number based on transom depth) of 2.3.

In his doctoral work, Scorpio (1997) used a variant of Haussling and Coleman's artificial surface-pressure distribution to damp this breaking wave successfully. However, this approach is difficult to rely on because the appropriate amount of damping is not easily determined. Furthermore, questions also exist as to the optimal form of the damping term for forward speed calculations.

On the other hand, no such difficulties exist in the present fairing technique. Its ability to suppress the transom stern breaking wave is shown in figure 4. Convergence of the damped solutions is demonstrated in figure 5.

The technique holds promise for three-dimensional ship wave calculations as well. The wave-breaking criterion may be used to detect the likely occurrence of wave breaking along the predominant wave-propagation direction. The fairing technique may then be used to suppress any wave breaking. We have been using this technique to overcome the numerical instabilities otherwise experienced in the calculation of the bow wave. Some of the discrete free-surface nodes in the vicinity of the ship's bow are flagged and faired through, as previously described.

CONCLUSIONS AND FUTURE WORK

A new, improved technique has been developed for suppressing wave breaking in the computations of nonlinear water waves. The technique may be applied, in conjunction with Subramani et al.'s (1998) curvature-based criterion for the detection of likely wave breaking, within a variety of numerical methods for simulating free-surface flows.

The technique has been successfully tested for a wide range of water wave computations—gravity waves in a two-dimensional tank; waves behind a two-dimensional transom stern; and the spray sheet at the bow of several three-dimensional surface ships. Future work will involve the continued testing of the technique for extreme cases of wave breaking in three-dimensional ship-wave computations.

ACKNOWLEDGMENTS

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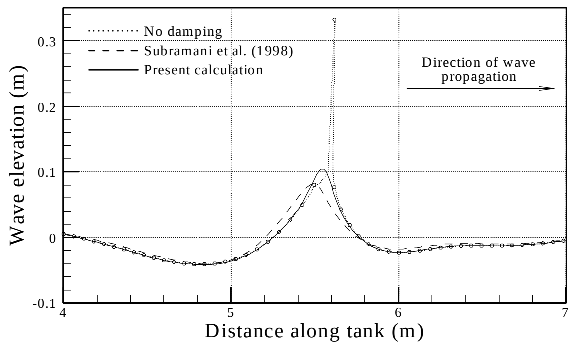

Figure 1. Snapshot of the wave surface at t=11.35 in a simulation of the physical experiment of Grue et al. (1993).

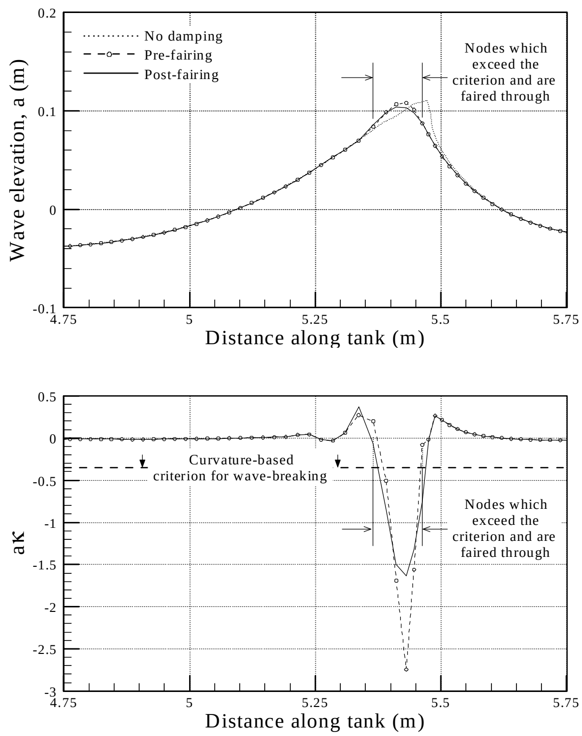

Figure 2. Damping of a breaking wave at t=11.30, using a new fairing technique in conjunction with the curvature-based criterion for wave-breaking.

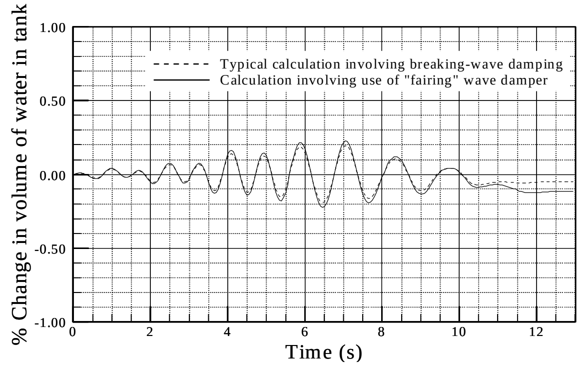

Figure 3. Estimation of mass lost in the process of suppressing the breaking wave in figures 1 and 2.

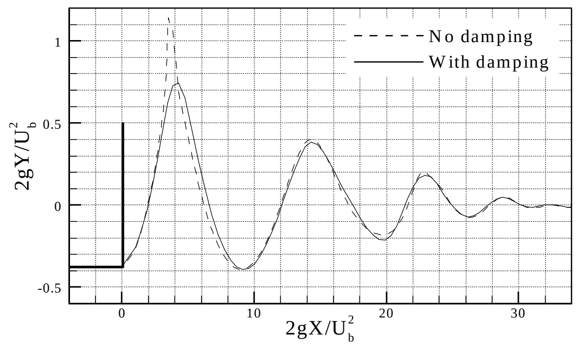

Figure 4. Application of the fairing technique to the breaking wave behind a two-dimensional transom stern.

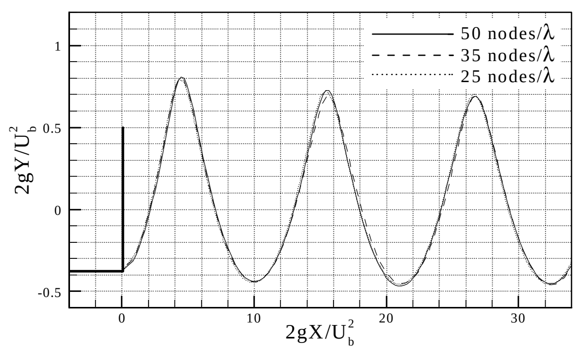

Figure 5. Convergence of solutions obtained using the fairing technique.