IINITIAL PRESSURE DISTRIBUTION OVER A WAVEMAKER AFTER AN IMPULSIVE MOTION

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The hydrodynamic problem under consideration is shown in Figure 1. The wavemaker on the left will start moving suddenly with speed U . This is a classic problem and analytical solution based on the potential flow can be obtained easily at the early stage of the motion. Peregrine (1972), for example, solved the problem based on an expansion in terms of time t . His solution reveals that the vertical velocity at the intersection is infinite, which is due to the incompatibility of boundary conditions on the free surface and the body surface at that point. It was later found (Lin, Newman & Yue 1985) that the proper treatment of this singularity is crucial for obtaining accurate numerical solution. Here we will show that this incompatibility also has profound effect on the pressure distribution. Some of features seem not to have been observed before.

As argued by Wu (1998), for this kind of problem due to sudden motion, the initial impact can be divided into two stages: (1) the impulse stage between $0_- \le t < 0_+$ and (2) the post impulse stage $t = 0_+$. From Peregrine's solution (1972), we can write the velocity potential at $t = 0_+$, or the solution at the second stage, as

$$\phi = \frac{2U}{d} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{k_n^2} e^{-k_n x} \cos k_n (y + d) \tag{1}$$

where $k_n = (n\pi + \pi / 2) / d$. From this the pressure impulse (Batchelor, 1967, p471) at the first stage can be obtained from $\Pi = -\rho\phi$, where ρ is the density of the fluid. The solution for the pressure at the second stage is, however, less straightforward. In fact it should be obtained from the Bernoulli equation

$$p = -\rho\phi_t - \frac{1}{2}\rho\nabla\phi\nabla\phi \tag{2}$$

In the equation, ϕ is that given in equation (1) while ϕ_t is still unknown. To find the solution for the time derivative term, we notice that the term satisfies the Laplace equation, and the boundary conditions

$$\phi_t = -\frac{1}{2}\nabla\phi\nabla\phi \tag{3}$$

on $y = 0$ and

$$\frac{\partial \phi_t}{\partial x} = -U \frac{\partial^2 \phi}{\partial x^2} \tag{4}$$

on $x = 0$ (Wu 1998). It would then be possible to find ϕ_t using a Fourier series similar to that in equation (1). But here there is a short cut. In fact one can easily confirm that the solution for ϕ_t can be written as

$$\phi_t = -\frac{1}{2}(\phi_y^2 - \phi_x^2) - 2U\phi_x \tag{5}$$

despite the product terms. This leads to

$$p = 2\rho U\phi_x - \rho\phi_x^2 \tag{6}$$

On the wavemaker, because we have $\phi_x = U$, the above equation (6) becomes

$$p = \rho U^2 \tag{7}$$

Now at the intersection point A, the pressure is ρU^2 if it is approached along the wavemkaer, but it is zero if it is approached along the free surface. Thus the pressure is discontinuous, which is clearly due to the incompatibility of the free surface and the body surface boundary conditions imposed on the potential at the intersection point. This result does not seem to have been noticed before.

If the flow field is the main interest in the time marching analysis, the observed discontinuity in the pressure should not cause too much concern, because the pressure is a product obtained after the potential has been found. It has no feed back to the flow if the body is rigid and fixed. In other cases, such as an elastic plate or a non-fixed rigid body, correct prediction of pressure is an essential part of the analysis, as observed by Lu, He and Wu (2000). Error in pressure will lead to a false response of the body, which will in turn give a false feed back to the flow. It is quite possible that the error will accumulate in the time domain and the numerical solution will depart from the real one and even instability will occur. Thus proper understanding of the pressure behaviour is extremely important in the code development.

Finally, the discontinuity observed above may be confirmed by the numerical results. One can obtain the pressure distribution by solving the boundary values problems for ϕ and ϕ_t. Table 1 shows that the nondimensionalized pressure along the wavemkaer. The results are obtained from a boundary element method based on the complex potential with linear elements (Wu & Eatock Taylor 1995). Because the intention is not about computational efficiency, panels with constant length s are used over the boundary of the fluid domain which is truncated at $x = 10d$. The table gives results up to $y/d = -0.20$ to highlight the behaviour of the pressure near the intersection. It shows that $p/\rho U^2$ is near one almost everywhere apart from a small region very close to $y = 0$ where it behaves rather erratically. But the region in which the erratic behaviour can be observed is getting smaller and smaller when the elements are getting smaller and smaller, and at a given point below $y = 0$, the result will tend to one as s decreases. All these are consistent with what is found above.

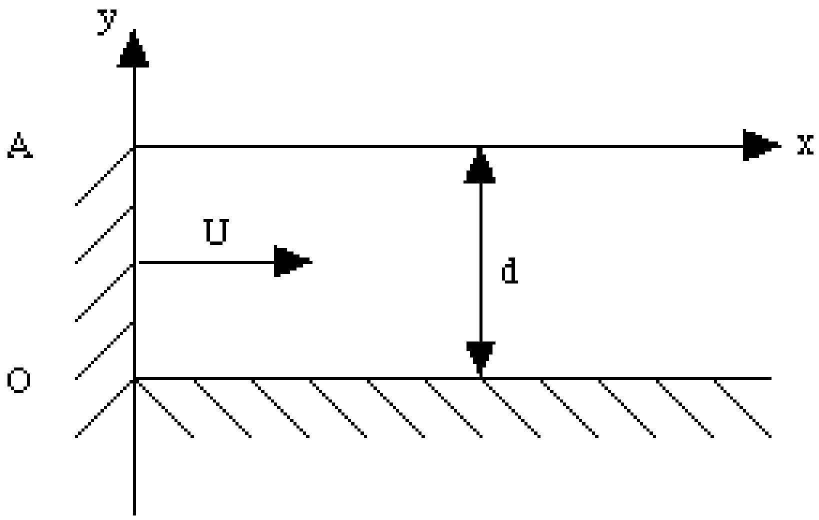

Figure 1. Computational model

Table 1. Pressure along the body surface

y/d	$p/\rho U^2$ $(s/d = 0.01)$	$p/\rho U^2$ $(s/d = 0.02)$	$p/\rho U^2$ $(s/d = 0.04)$
0.00	0.000	0.000	0.000
-0.01	-0.257		
-0.02	0.842	-0.114	
-0.03	0.920		
-0.04	0.954	0.868	0.029
-0.05	0.970		
-0.06	0.979	0.933	
-0.07	0.985		
-0.08	0.988	0.962	0.895
-0.09	0.990		
-0.10	0.992	0.975	
-0.11	0.993		
-0.12	0.994	0.983	0.946
-0.13	0.995		
-0.14	0.996	0.987	
-0.15	0.996		
-0.16	0.996	0.990	0.969
-0.17	0.997		
-0.18	0.997	0.992	
-0.19	0.997		
-0.20	0.998	0.993	0.980

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