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Numerical Reconstruction of Nonlinear Irregular Wave-Field Using Single or Multiple Probe Data

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1 Introduction

The objective of this work is to develop an effective algorithm for the reconstruction of irregular nonlinear wave-fields using wave records of limited duration at one or more fixed points. With wave reconstruction, we are able to obtain: (i) complete kinematics of entire nonlinear wave-fields; (ii) proper initial conditions for numerical simulations and wave-basin experiments of nonlinear wave-field evolutions; (iii) forecast of nonlinear wave-field evolution dynamics beyond specified wave records; and (iv) data assimilation for quantitative comparisons between numerical simulations and field and wave-basin measurements.

For small to mild waves, the kinematics of the irregular wave-field can be reconstructed based on the linear and second-order Stokes wave theory (e.g. Zhang *et al.* 1999). For moderate to steep waves or long-time wave-field evolutions, higher-order effects must be considered and efficient reconstruction schemes need to be used.

In this work, we develop a multiple-level iterative optimization scheme for the reconstruction of three-dimensional nonlinear wave-fields using single- or multiple-point measurements. For initial (low-level) optimizations, we apply the analytic low-order Stokes wave theory. For high-level optimizations, We employ the Rosenbrock optimization scheme (Bazaraa *et al.* 1993) with an extremely efficient high-order spectral method (HOS) (Dommermuth & Yue 1987) for the computation of nonlinear wave-field evolutions. Since the HOS can follow the evolution of a large number of wave modes and account for their interactions up to an arbitrary order in wave steepness, the present wave reconstruction scheme is capable of regenerating the kinematics of large nonlinear wave-fields for long-time evolutions. As an illustration, the scheme is applied to construct steep two- and three-dimensional irregular waves using wave-basin data. Agreement between the reconstructed wave-field and the measurements is excellent.

2 Problem definition

The mathematic problem of wave reconstruction can be described as follows: given probe data of wave elevations at multiple points in a limited duration ($\bar{\eta}(\mathbf{x}_p, t)$, p=1, ..., N_p, $t \in [0, T]$), find proper initial conditions of the free-surface elevation and velocity potential at t=0 ($\eta_0(\mathbf{x}) = \eta(\mathbf{x}, t=0)$ and $\phi_0(\mathbf{x}) = \phi(\mathbf{x}, z=\eta_0, t=0)$) for construction of the nonlinear wave-field such that the difference ($\mathcal{E}$) between the wave elevations of the reconstructed wave-field at the probes

($\eta(\mathbf{x}_p, t)$, $p = 1, \ldots, N_p$) and the specified probe data is minimized, i.e.

$$\mathcal{E} = \min \left\{ \frac{1}{N_p} \sum_{p=1}^{N_p} \int_0^T w(\mathbf{x}_p, t) \left[\eta(\mathbf{x}_p, t) - \bar{\eta}(\mathbf{x}_p, t) \right]^2 \, \mathrm{d}t \right\} \tag{1}$$

where $w(\mathbf{x}_p, t)$ is a weighting function. This is a nonlinear optimization problem. The main difficulties involved are:

- Evaluation of $\mathcal{E}$ is expensive since it requires to resolve the nonlinear wave-field evolution problem for specified initial conditions.
- The degree of freedom in optimization increases linearly with the duration of the records T and the size of the wave-field.
- There exist multiple solutions satisfying nonlinear equation (1).

Because of these, the wave reconstruction algorithm needs to employ (1) a highly efficient optimization scheme; (2) an efficient and accurate simulation scheme for nonlinear wave-field evolutions; and (3) an effective technique for proper initial solutions of the optimization.

3 Multiple-level optimization scheme

We develop a multiple-level iterative optimization scheme which employs the analytic Stokes wave solutions for low-level optimization and the high-resolution simulation results for high-level optimization. The low-level optimization is conducted analytically while the high-level optimization is performed numerically based on the Rosenbrock method (Bazaraa *et al.* 1993). For the simulation of nonlinear wave-field evolutions, a powerful high-order spectral method (HOS) developed for nonlinear wave-wave interactions by Dommermuth & Yue (1987) is employed. The high efficiency of optimization and HOS simulations allows the reconstruction of large-space/long-time wave-field evolutions including nonlinear wave effects.

The procedure of the multiple-level optimization scheme can be outlined as follows:

- *Level 1*: we use the linear wave solution with N_L free wave modes for $\eta(\mathbf{x}_p, t)$ in (1) and determine the unknown mode amplitudes and phases by analytic optimization.
- *Level 2*: we use the nonlinear (second-order) Stokes wave solution with N_L free wave modes for $\eta(\mathbf{x}_p, t)$ in (1) and determine the unknown mode amplitudes and phases by analytic optimization.
- *Level 3*: using the result of *Lever 2* as initial conditions, we conduct HOS simulations with N_H spectral modes and arbitrary order M to determine $\eta(\mathbf{x}_p, t)$. Through an iterative optimization, we obtain a complete solution of $\eta_0(\mathbf{x})$ and $\phi_0(\mathbf{x})$ with N_H spectral wave modes.

Note that with *Level 1* and *Level 2*, we are able to obtain a proper initial guess which speeds up the convergence of the optimization in *Level 3*. In practice, N_H can be as large as $O(10^4)$ and M can be $O(5 \sim 10)$. Thus, we can obtain an accurate reconstruction of nonlinear wave-fields in large domains. Moreover, with the reconstructed wave-field as initial conditions for HOS simulations, we can forecast the evolution of the nonlinear wave-field.

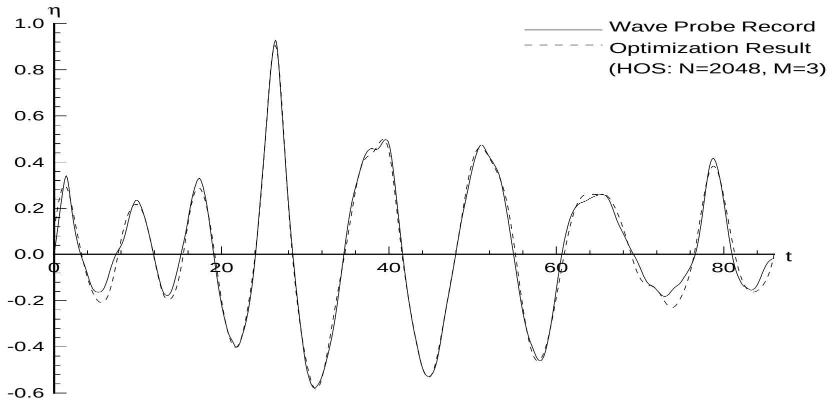

Figure 1: Comparison of computed versus experimentally measured free-surface elevation (*10 meters) at a given point of a two-dimensional wave-field as a function of time (seconds): ——, experiments of Stansberg *et al.* (1995); - - - -, HOS simulation with N_H=2048 spectral modes and M=3 order.

4 Results

The present multiple-level optimization scheme is applied to reconstruct both two- and three-dimensional steep irregular wave-fields using single or multiple-points measurements. Figure 1 shows the comparison between the HOS simulation result of the reconstructed wave-field and the experimental measurements of a steep two-dimensional wave-field (Stansberg *et al.* 1995) for about 10 dominant wave periods. The agreement between them is excellent.

Figure 2 shows the comparison between the HOS simulation results and wave-basin measurements (Liagre 1999) of the three-dimensional steep bull's eye waves. Snapshots of the wave-field are shown in figures 2a and 2b from which it is seen that the reconstructed wave-field contains all key features of the bull's eye waves in experiments. The simulation results of the wave elevations at two positions near the convergence center of the bull's eye wave-field are compared to the experimental data in figures 2c and 2d. Excellent agreement between the HOS simulations and experimental measurements is again obtained.

5 Reference

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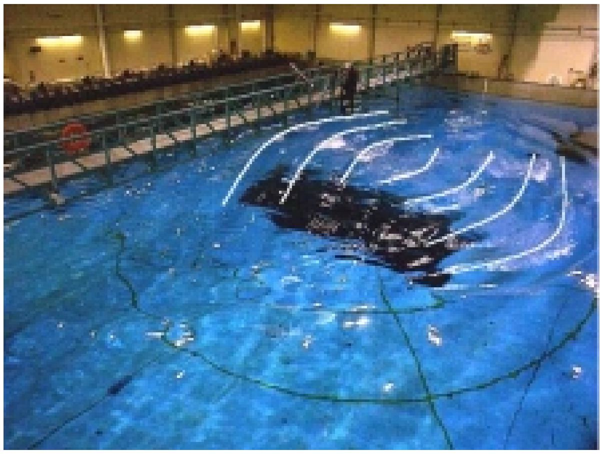
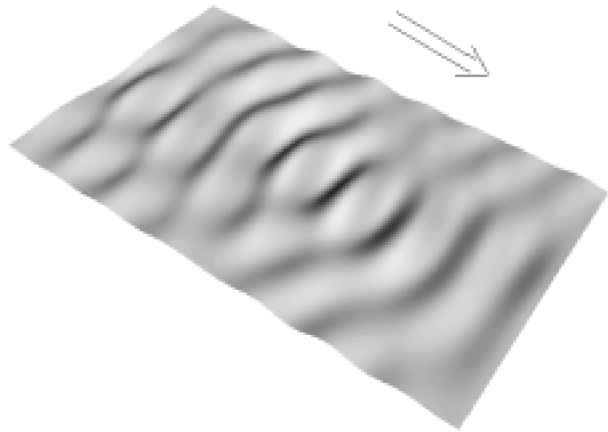


(a)



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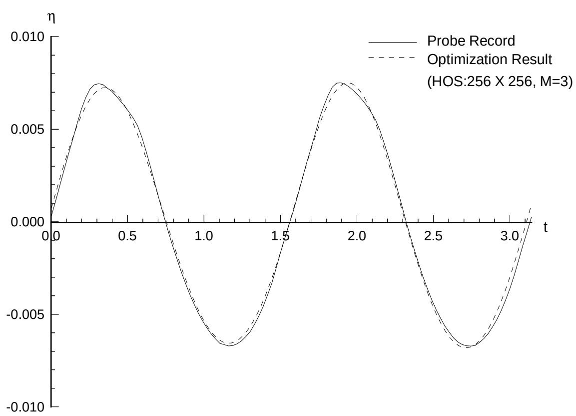
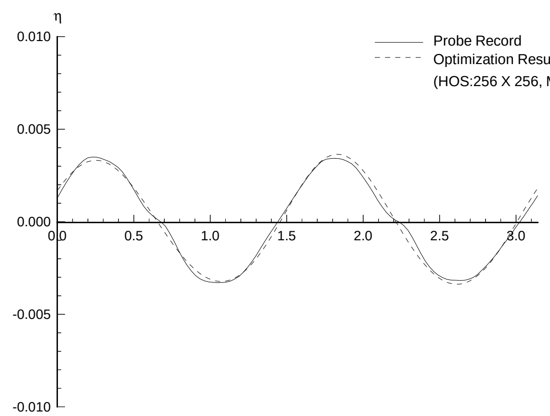


(c)



(d)

Figure 2: Comparisons of the reconstructed three-dimensional bull's eye wave-field to the wave-basin measurements (Liagre 1999): free-surface snapshots of the wave-field from the experiment (a) and the HOS simulation with N_H=256 $\times$ 256 modes and M=3 order (b); and time history of the wave elevations at specified two positions of the wave-field, (c) and (d), from the measurements (——) and the HOS simulation (- - -).