Wave-Drift Added Mass of Bodies with Slow Drift Motions

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1.INTRODUCTION

Ocean structures are usually constrained by mooring or tether systems, which supply relatively weak restoring forces in the horizontal plane. Under the slowly varying drift forces exerted by ocean waves, these structures may undergo low-frequency resonant oscillations in the horizontal motion modes, i.e. surge, sway and yaw. The nonlinear wave loads are proportional to the square of the wave amplitude in magnitude and occur at a frequency σ that is the difference between each pair of frequencies, say ω_i and ω_j, in the components of ocean wave spectrum, i.e. $\sigma =| \omega_i - \omega_j |$. Conventional added mass and damping can be obtained by solving a linear radiation problem in which the body of the structures is forced to oscillate in the calm water. In the case when the frequency σ of the oscillation is very small, the wave-radiating damping vanishes with an order of $O(\sigma^7)$ in the horizontal motion modes while the added mass tends to the same order of the displaced water mass. However, with the presence of the incident waves, there exists another kind of added mass and damping that is caused by the nonlinear interaction between waves and slow oscillations. As part of the nonlinear wave loads, their magnitude is proportional to the square of the wave amplitude, which is different from the conventional added mass and damping, and they are called wave drift added mass and wave drift damping respectively. Recently, many studies have been made to evaluate and measure the wave drift damping which is more significant compared with the conventional wave-radiation damping and plays a key role in the simulation of slow drift motions, especially in estimating the resonant response. On the other hand, wave drift added mass is considered less important and relatively less attention has been paid to it. Nevertheless, it has been reported that the added mass increases significantly when measured in waves [1]. Therefore, to simulate slow drift motions accurately and to determine the resonant frequency, it is worth investigating the magnitude of the wave drift added mass and how much it would affect the slow drift motions.

In the present work, the problem of interaction between slow horizontal oscillations of a body and ambient waves is considered. The approach used by Newman [2] to investigate wave drift damping is adopted. The feature of this method is that perturbation expansion based on two time scales is used to simplify nonlinear boundary conditions. The wave forces acting on the body are evaluated by integration of hydrodynamic pressure along the instantaneous wetted body surface. From the quadratic nonlinear forces in terms of the wave amplitude, the wave drift added mass is picked out by the component in contract phase to the acceleration of the slow oscillations. These results will be compared with the conventional linear added mass to examine their significance.

2.PERTURBATION EXPANSION OF THE POTENTIAL

We are going to consider the problem of a body slowly oscillating in a train of regular waves with a wave number k_0 and incident angle β. The nonlinear interaction among slow oscillation modes is not considered so that each mode can be studied separately. The body is restrained from the linear responses to the incident waves. The frequency of the slow oscillations is assumed to be σ, which is much smaller than the incident wave frequency ω. The oscillatory displacement and velocity are assumed to be $\mathrm{Re}\{i\xi_j e^{-i\sigma t}\}$ and $\mathrm{Re}\{\sigma\xi_j e^{-i\sigma t}\}$ respectively, where the subscript j= 1, 2 and 6 denoting surge, sway and yaw respectively. Following the approach of Newman's [2], the total velocity potential can be expressed by the following perturbation expansion up to the quadratic order in wave amplitude A:

$$\Phi(\boldsymbol{x},t)=\mathrm{Re}\left\{\phi_1 e^{-i\omega t}+\phi_2^{(0)}+\ldots+\xi_j\left[\phi_{0j}e^{-i\sigma t}+\phi_{1j}^{(+)}e^{-i(\omega+\sigma)t}+\phi_{1j}^{(-)}e^{-i(\omega-\sigma)t}+\phi_{2j}^{(0)}e^{-i\sigma t}+\ldots\right]\right\} \tag{1}$$

The potentials on the right-hand side of eqn. (1) depends only on the space position $\boldsymbol{x}$. The number in the subscript indicates the order in wave amplitude while the letter j is related to the corresponding slow motion modes. Superscripts are used if necessary to denote harmonic time dependence in the respective frequencies. Here, potentials associated with double wave frequency are omitted since they will not contribute to the wave-drift added mass and damping. Substituting the above expansion into the boundary conditions satisfied by the total velocity potential and resorting terms with the same order, the boundary value problem governing each order of potentials can be obtained. Detail deduction is referred to Newman's work [2].

3.CALCULATION OF THE WAVE-DRIFT ADDED MASS

The wave forces are evaluated by the integration of the hydrodynamic pressure along the instantaneous wetted body surface and expanded in the same way as the velocity potential, i.e.

$$F_i(t)=\mathrm{Re}\left\{F_{1i}e^{-i\omega t}+F_{2i}^{(0)}+\ldots+\xi_j\left[F_{0ij}e^{-i\sigma t}+F_{1ij}^{(+)}e^{-i(\omega+\sigma)t}+F_{1ij}^{(-)}e^{-i(\omega-\sigma)t}+F_{2ij}^{(0)}e^{-i\sigma t}+\ldots\right]\right\} \tag{2}$$

Here, the subscript i=1, 2, and 6 denoting the force component in surge, sway and yaw direction respectively. In eqn. (2) F_{0ij} is the linear force in i-th direction per unit motion of ξ_j and is related to the linear added mass A_{0ij} and wave-radiating damping B_{0ij} as follows:

$$F_{0ij}=-\left(-i\sigma^2 A_{0ij}+\sigma B_{0ij}\right)=i\sigma\rho\int_{S_0}\phi_{0j}n_i ds \tag{3}$$

where S_0 is the mean wetted body surface. In the limiting case that σ tends to zero, the radiation potential ϕ_{0j} of the slow oscillation tends to satisfy a rigid wall condition on the free surface. It is further normalized as $\phi_{0j}=\sigma\varphi_j$ where φ_j is a real function. Hence, as σ tends to zero, the linear wave- radiating damping B_{0ij} vanishes while the added mass tends to

$$A_{0ij} = \rho \int_{S_0} \varphi_j n_i ds \tag{4}$$

On the other hand, $F_{2ij}^{(0)}$ is a force component in quadratic order of wave amplitude, which can also be separated into two parts that is in phase with the acceleration and the velocity of the slow oscillation respectively, i.e.

$$F_{2ij}^{(0)} = -\left(-i\sigma^2 A_{2ij} + \sigma B_{2ij}\right) \tag{5}$$

The formula to evaluate the wave-drift added mass is given by

$$\begin{aligned} A_{2ij} = \mathrm{Im}\Big\{ &-\rho \int_{C_0} \left[-i\sigma\phi_{2j}^{(0)} + \tfrac{1}{2}\nabla\left(\phi_{1j}^{(+)} - \phi_{1j}^{(-)}\right)\cdot\nabla\phi_1^* + \nabla\phi_{0j}\cdot\nabla\phi_2^{(0)} + \tfrac{1}{2}i\nabla\phi_1\cdot D_j\left(\nabla\phi_1^*\right)\right] n_i ds \\ &+ \frac{\rho}{2g}\int_{C_0} \left[\omega^2\left(\phi_{1j}^{(+)} - \phi_{1j}^{(-)}\right)\phi_1^* + \sigma\omega\left(\phi_{1j}^{(+)} + \phi_{1j}^{(-)}\right)\phi_1^* - \tfrac{1}{2}i\sigma\phi_{0j}\nabla\phi_1\cdot\nabla\phi_1^* + i\omega^2\phi_1 D_j\left(\phi_1^*\right)\right. \\ &\left. - \tfrac{1}{2}i\omega\phi_1\nabla\left(\phi_{0j} - \phi_{0j}^*\right)\cdot\nabla\phi_1^* + i\sigma\omega^2\left(\tfrac{1}{2}\phi_{0jz} + v\phi_{0j}\right)\phi_1\phi_1^*\right] n_i dl \Big\} \end{aligned} \tag{6}$$

where C_0 is the mean water line of the body and D_j is a derivative operator which is defined as $D_1 = \partial/\partial x$, $D_2 = \partial/\partial y$ and $D_6 = x\partial/\partial y - y\partial/\partial x = \partial/\partial\theta$. The asterisk * in the superscript denotes the complex conjugate. The letter *z* in the subscript means the derivative with respect to it.

When σ tends to zero, we define $\phi_{1j}^{(+)} - \phi_{1j}^{(-)} = P_j$ and $\phi_{1j}^{(+)} - \phi_{1j}^{(-)} = \sigma Q_j$ where P_j is related to the linear wave potential ϕ_1 as $P_j = -iD_j(\phi_1) - \kappa_j$ with $\kappa_1 = k_0\cos\beta$, $\kappa_1 = k_0\sin\beta$ and $\kappa_6 = i\partial/\partial\beta$.

Then, in this limiting case the wave drift added mass A_{2ij} can be evaluated simply by an integral along the mean water line of the body:

$$A_{2ij} = \mathrm{Im}\left\{\frac{\rho}{2g}\int_{C_0}\left[\omega Q_j\phi_1^* + iv^2\varphi_j\phi_1\phi_1^* - \tfrac{i}{2}\varphi_j\nabla\phi_1\cdot\nabla\phi_1^*\right] n_i dl\right\} \qquad \text{with } v = \omega^2/g \tag{7}$$

4.EXAMPLES AND DISCUSSION

A circular cylinder with radius *a* is taken as an example to calculate the wave drift added mass when the incident wave angle is 0°. Shown in Fig. 1 are results of the wave-drift added mass A_{211} normalized by $\rho\pi a\zeta^2$ where ζ is the wave amplitude. It can be observed that the wave-drift added mass generally is the same order as the wave drift damping in magnitude. In order to compare with the linear added mass, the possible maximum wave amplitude before breaking, i.e. ζ=0.14π/k_0 , is used to renormalize the wave-drift added mass and the ratio of the wave-drift added mass A_{211} to the linear added mass A_{011} is plotted in Fig. 2. It can be seen that the contribution from the wave-drift added mass is not negligible if the wave amplitude is comparable to the dimension of the body.

REFERENCE

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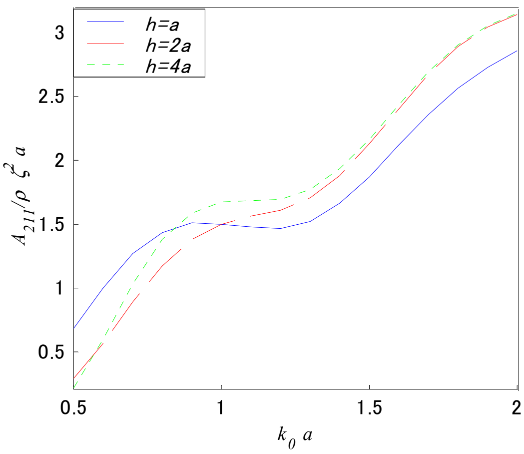

Fig.1a Wave-drift added mass A_{211} for a uniform cylinder in different water depth h.

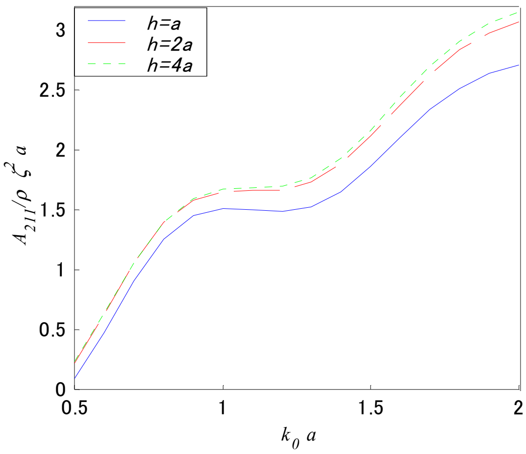

Fig.1b Wave-drift added mass A_{211} for a truncated cylinder with different draught d in a water depth h=4a.

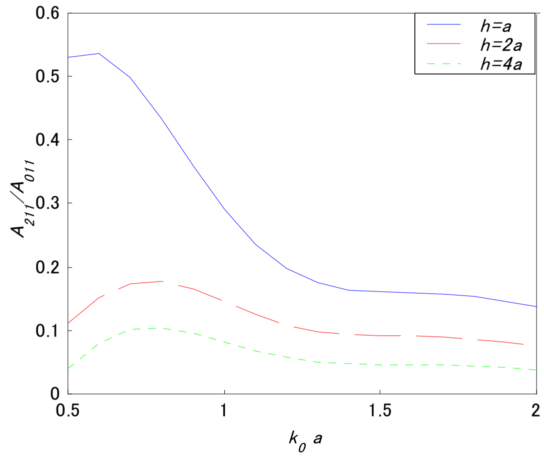

Fig.2a Wave-drift added mass A_{211} compared with linear added mass A_{011} for a uniform cylinder in different water depth h.

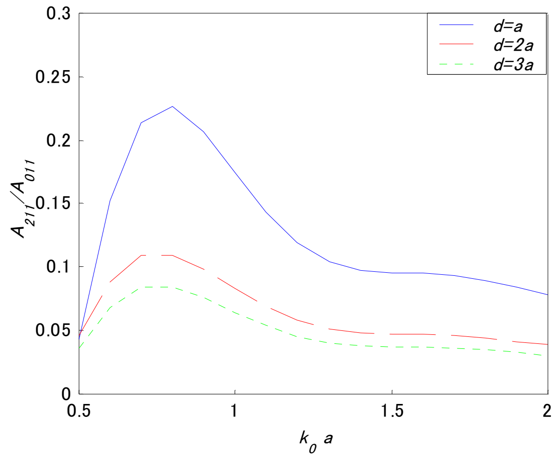

Fig.2b Wave-drift added mass A_{211} compared with linear added mass A_{011} for a truncated cylinder with different draught d in a water depth h=4a.