Geometrical-Optics for the Deflection of a Very Large Floating Flexible Platform

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Abstract

In an earlier paper, see HERMANS (2000), a derivation is given of an integro-differential equation for the determination of the deflection of a large floating flexible platform, excited by waves. In this paper we investigate the possibility to derive a short wave formulation for this problem. It will be shown that for a flexible half-plane and a strip in deep water the integral equation can be used to find such a formulation. The asymptotic results shall be compared with the numerical results obtained before.

1. Introduction

For some time there is great interest in the design of very large floating airports. One of the concepts is to build a very large mat-like structure, that is kept at station bye means of anchor lines or by means of a dynamic positioning system. One of the advantage of this kind of structures is that it can be towed toward its temporary destination. OHKUSU *et al* (1996) presented an asymptotic theory to descibe the deflection of the platform due to relatively short incident waves while it is positioned at shallow water. Due to the fact that the vertical dimension is averaged out it is relatively simple to derive a valid formulation. HERMANS (1997) derived a formulation for deep water. Unfortunately this formulation uses some non-physical boundary conditions and its applicability is questionable for this reason. Later HERMANS (2000) derived an exact differential-integral formulation for the deflection. In the latter paper the problem is solved numerically. Here we use that formulation so obtain asymptotic short-wave results.

2. Mathematical formulation

In this section we derive the general formulation for the diffraction of waves by a flexible platform of general geometric form. We restrict ourself to platforms with constant elastic properties. This restriction can be weakened later on.

The fluid is incompressible, so we introduce the velocity potential $\Phi(\boldsymbol{x},t) = \nabla \boldsymbol{V}(\boldsymbol{x},t)$, where $\boldsymbol{V}(\boldsymbol{x},t)$ is the fluid velocity vector. We assume waves in still water. Hence $\Phi(\boldsymbol{x},t)$ is a solution of the Laplace equation

$$\Delta\Phi = 0 \quad \text{in the fluid,} \tag{1}$$

together with the linearised kinematic condition, $\Phi_z = w_t$, and dynamic condition, $p/\rho = -\Phi_t - gw$, at the linearized free water surface $z = 0$, where $w(x,y,t)$ denotes the free surface elevation, and ρ is the density of the water. The linearised free surface condition outside the platform becomes:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0 \tag{2}$$

at $z = 0$ and $(x,y) \in \mathcal{F}$.

The platform is assumed to be a thin layer at the free-surface $z = 0$, which seems to be a good model for a shallow draft platform. The platform is modelled as an elastic plate with zero thickness. To describe the deflection w we apply the thin plate theory, which leads to an equation for w of the form

$$m\frac{\partial^2 w}{\partial t^2} = -D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w + p|_{z=0} \tag{3}$$

where m is the mass of unit area of the platform while D is its equivalent flexural rigidity.

We apply the operator $\frac{\partial}{\partial t}$ to (3) and use the kinematic and dynamic condition to arrive at the following equation for Φ at $z=0$ and in the platform area $(x,y)\in\mathcal{P}$:

$$\left\{\frac{D}{\rho g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2}+1\right\}\frac{\partial\Phi}{\partial z} + \frac{1}{g}\left\{\frac{\partial^2}{\partial t^2}\right\}\Phi = 0\ . \tag{4}$$

The free edges of the platform are free of shear forces and moment. We assume that the radius of curvature, in the horizontal plane, of the edge is large. Hence, the edge may be considered to be straight locally. We then approximate the boundary conditions at the edge by:

$$\frac{\partial^2 w}{\partial n^2}+\nu\frac{\partial^2 w}{\partial s^2}=0 \ \text{ and } \ \frac{\partial^3 w}{\partial n^3}+(2-\nu)\frac{\partial^3 w}{\partial n\partial s^2}=0 \tag{5}$$

where ν is Poisson's ratio, n is in the normal direction, in the horizontal plane, along the edge and s denotes the arc-length along the edge.

3. Semi-infinite plate

We restrict ourselves to the one-dimensional situation with waves perpendicular to the edge of a semi-infinite plate. In Hermans (2000) we derived the following integro-differential equation for the deflection $w(x)$:

$$2\pi\left(\mathcal{D}\frac{\partial^4 w(x)}{\partial x^4}-(\mu-1)w(x)\right) = k_0\int_{\mathcal{P}}\left(\mathcal{D}\frac{\partial^4 w(\xi)}{\partial \xi^4}-\mu w(\xi)\right)\mathcal{G}(x,0;\xi,0)\ \mathrm{d}\xi + 2\pi\zeta_\infty\,\mathrm{e}^{ik_0x} \tag{6}$$

where ζ_∞ is the wave height of the incident wave and k_0 its wave-number. We also introduced the parameters

$$\mu=\frac{m\omega^2}{\rho g},\quad \mathcal{D}=\frac{D}{\rho g}\ ,$$

together with the boundary conditions

$$\frac{\partial^2 w}{\partial x^2}=\frac{\partial^3 w}{\partial x^3}=0 \quad \text{at } x=0.$$

For large values of x the solution is finite and consists of outgoing waves. The Green's function, obeying the radiation condition, has the form

$$\begin{aligned}\mathcal{G}(\boldsymbol{x},\boldsymbol{\xi})-\ln r &= -\ln r_1 - 2\int_{\mathcal{L}'}\frac{1}{k-k_0}\,\mathrm{e}^{k(z+\zeta)}\cos k(x-\xi)\ \mathrm{d}k \\ &= -\int_{\mathcal{L}'}\left(\frac{k+k_0}{(k-k_0)k}\,\mathrm{e}^{k(z+\zeta)}\cos k(x-\xi)+\frac{1}{k}\,\mathrm{e}^{-k}\right)\mathrm{d}k\end{aligned} \tag{7}$$

where the contour $\mathcal{L}'$ is in the complex k-plane from $k=0$ to $k=\infty$ that passes, due to the radiation condition, underneath the pole of the integrand $k=k_0=\omega^2/g$, see figure 1.

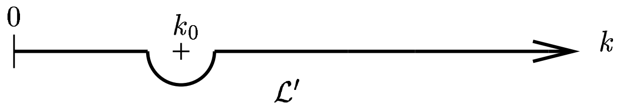


Figure 1: Contour of integration

For the half-plane problem the solution will be sought as a superposition of exponential functions of the form:

$$w(x) = \frac{i}{\omega}\phi_z^- = w_1(x) + w_2(x) = \sum_n a_n \exp\{i\kappa_n x\} + w_2(x) \tag{8}$$

where for the platform with homogeneous physical properties it is expected that the constant '**amplitudes**' a_n and '**wave numbers**' κ_n can be determined. It turns out that, for large values of $\mathcal{D}$, $w_1(x)$ is a good numerical approximation of the geometrical-optics solution. If the physical parameters are not constant the method has to be extended according to the more general '**ray**' approach. Due to the fact that we consider a half-plane the real part of κ_n has to be negative or if the real part equals zero it must obey the outgoing condition. The inhomogeneous term in the equation behaves like $\exp(ik_0x)$ this does not indicate that the solution behaves accordingly. The physics of the problem shows that, for the semi-infinite plate, the wavy part of the solution in the far field has a different real wave-number κ_1, as has been made visible in the numerical simulations as well.

We introduce (8) in (6) and carry out the integration with respect to ξ. This leads to the relation:

$$\sum_n a_n \left(\mathcal{D}\kappa_n^4 - (\mu - 1)\right)\ \mathrm{e}^{i\kappa_n x} =$$

$$i\sum_n a_n \frac{k_0}{2\pi}(\mathcal{D}\kappa_n^4 - \mu)\int_{\mathcal{L}'} \frac{1}{k-k_0}\left(\frac{\mathrm{e}^{ikx}}{k-\kappa_n} - \frac{\mathrm{e}^{-ikx}}{k+\kappa_n}\right)\mathrm{d}k + \zeta_\infty\ \mathrm{e}^{ik_0x} \tag{9}$$

The integral has to be evaluated vor positive values of x. We transform the integral to integrals along the vertical axis in the complex k-plane, we then obtain the contribution to the modes in $w_1(x)$ explicitly. It turns out that the well known dispersion relation see HERMANS (1997) is recovered. There are three physically realistic solutions for κ:

$$\left(\mathcal{D}\kappa^4 - \mu + 1\right)\kappa = \pm k_0 \tag{10}$$

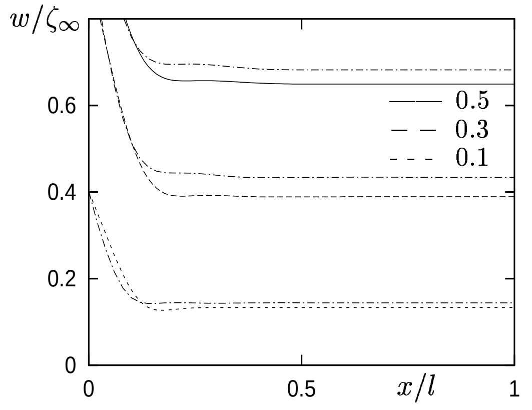
The two boundary conditions give two relations for the unknown values of the 'amplitudes' a_n and $w_2(0)$. In the first integral we also obtain a contribution of the pole, $k = k_0$, of the integrand. The contribution of this pole has to cancel the inhomogeous term $\zeta_\infty\ \mathrm{e}^{ik_0x}$. This leads to a third relation for a_n ,we obtain:

$$\sum_{n=1}^{3} k_0(\mathcal{D}\kappa_n^4 - \mu)\frac{1}{\kappa_n - k_0}a_n + \mathcal{I}\{w_2\} + \zeta_\infty = 0 \tag{11}$$

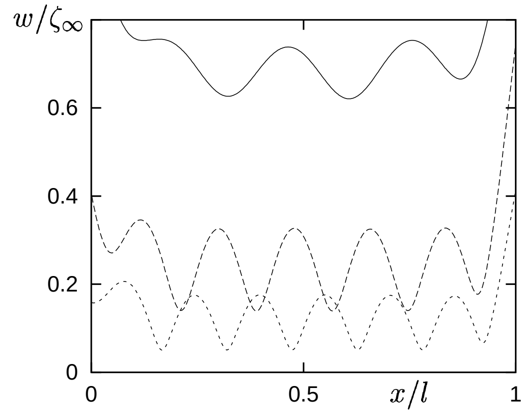
The influence of $w_2(x)$ can be taken into account iteratively. Extension to a finite width problem is straight forward. It will be explained how three-dimensional cases and the case of obligue incident waves can be treated.

4. Results

In the figures below a comparison is shown of the results obtained, for the semi-infinite plate, by this method where the first iterate of $w_2(x)$ is taken into account with results obtained by TKACHEVA by means of an asymptotic evaluation of Wiener-Hopf results. The numerical results obtained by means of a boundary element method for a finite platform of width ($l = 300$ m. and $D/\rho g = 10^5$ m^4.) are shown. Next the asymptotic values for the reflection

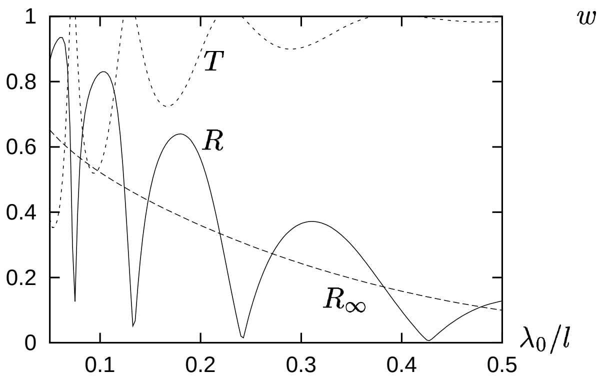


(a) asymptotic results for a semi-infinite platform (TKACHEVA $- \cdot -$)

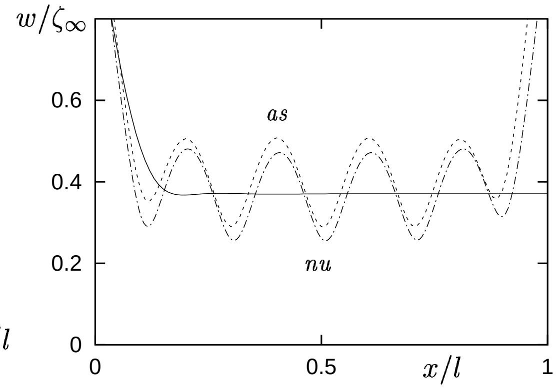


(b) numerical results for a platform with length $l = 300$ m.

and transmission coefficients for the finite plate, where we neglected the influence of $w_2(x)$ are shown. The final result is a comparison of the values of the deflection near the resonance frequency are shown. In figure (c) one also gets an impression about the accuracy of the zeroth order method. The coefficients may deviate about 5% from those obtained by the direct computations.



(c) Reflection and transmission coefficients for finite ($l = 300$ m.) and semi-infinite plate



(d) Comparison of the numerical and the asymptotic results for a finite ($l = 300$ m.) and a semi-infinite platform with $\lambda_0/l = 0.2425$.

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