Water entry of a perforated wedge

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1. Introduction

Since the pioneering works of von Karman and Wagner the water entry of solid bodies has received a considerable attention.
In this paper we apply Wagner's approach to two-dimensional bodies which are perforated. This problem has some relevance in coastal engineering, to study wave impact upon perforated breakwaters. One may also conceive of using perforated shrouds as outer protections for other bodies subjected to wave impacts, like the under-sides of the decks of offshore platforms, or for bodies entering the free surface at large speeds.
Another reason to get interested into water entry of perforated bodies is that the jets that occur through the openings directly reflect the extension of the wetted area, and that their velocities are in proportion with the ambient pressures. With high speed cameras of appropriate resolution it should be possible to quantify, in time, the locations and velocities of the jets. Comparisons between theoretical and experimental results could then be easier than with solid bodies.

2. Formulation of the problem

The boundary condition on the wetted part of the entering body is inspired by previous work carried out by the first author on the hydrodynamics of perforated bodies (Molin, 1992). It is assumed that the flow through the openings results into pressure drops, which are proportional to the square of the local traversing velocities (relative to the body surface). The relationship between pressure drops and traversing velocities is taken in an averaged sense (over a large number of perforations), yielding:

$$p^- - p^+ = \rho \, \frac{1-\tau}{2\,\mu\,\tau^2} \, v \, |v|, \tag{1}$$

where τ is the porosity ratio (area of the perforations divided by total area) and μ a discharge coefficient, usually close to 0.5.

Here we consider the initial stage of the water entry of a porous blunt shape into still water. Some fluid leaks through the porous surface (as small jets through the openings), but the upper side remains at atmospheric pressure. Equation (1) then reduces to the simple form (after linearization à la Wagner)

$$-\Phi_t(x,0,t) = \frac{1-\tau}{2\,\mu\,\tau^2} \, (U + \Phi_y(x,0,t))^2, \tag{2}$$

where $y = 0$ is the initial level of the free surface, U is the vertical velocity of the body and $\Phi(x,y,t)$ is the velocity potential.

If the body is solid, then $\tau = 0$ and the boundary condition (2) resumes to the usual one: $\Phi_y = -U$. If the body is porous, then $\tau \neq 0$ and (2) can be interpreted as an evolution equation for Φ.

In the case of symmetric body shape, $y = f(x) = f(-x)$, the boundary-value problem with respect to the velocity potential has the form

$$\begin{array}{rcllll}
\Delta\Phi & = & 0 & y < 0 & & \\
\Phi_t & = & -\alpha\,(U + \Phi_y)^2 & |x| < c(t) & y = 0 & \\
\Phi & = & 0 & |x| > c(t) & y = 0 & \\
\nabla\Phi & \to & 0 & x^2 + y^2 \to \infty & &
\end{array} \tag{3}$$

where $\alpha = (1-\tau)\,/\,(2\,\mu\,\tau^2)$.
The wetted length $2\,c(t)$ of the body is obtained from the Wagner condition of continuous joining of the free surface and the surface of the entering body at $x = \pm c(t)$

$$f(c) = U\,t + \int_0^t \Phi_y(c, 0, T)\; dT, \tag{4}$$

where $t = 0$ is the impact instant and $\Phi_y(c(t), 0, T)$ is the vertical velocity of the free surface at the time instant T at the point $x = c(t)$.

3. Self-similar solution for a porous wedge

We consider the case of a wedge, with deadrise angle β, entering vertically the free surface with constant speed U. The self-similar problem is derived by introducing the non-dimensional variables:

$$x = \gamma\,U\,t\,X \qquad y = \gamma\,U\,t\,Y \qquad \Phi = \gamma\,U^2\,t\,\phi(X, Y) \qquad c = \gamma\,U\,t. \tag{5}$$

In the stretched variables the boundary condition on the wetted part of the wedge (2) and the Wagner condition (4) take, respectively, the forms

$$\phi - X\,\phi_X = -\frac{\alpha}{\gamma}\,(1+\phi_Y)^2 \qquad |X| \le 1 \qquad Y = 0, \tag{6}$$

$$\gamma\,\tan\beta = 1 + \int_1^\infty \frac{\phi_Y(u, 0)}{u^2}\, du. \tag{7}$$

The numerical difficulties mostly reside with the nonlinearity of equation (6), and with the fact that γ is not known a priori. To overcome them, an iterative resolution method is used with equation (6) being presented as

$$\phi^{(n)} - X\,\phi_X^{(n)} + \frac{\alpha}{\gamma^{(n-1)}}\,(2+\phi_Y^{(n-1)})\,\phi_Y^{(n)} = -\frac{\alpha}{\gamma^{(n-1)}} \tag{8}$$

This means that an equation of the type

$$\phi - X\,\phi_X + f(X)\,\phi_Y = k \qquad |X| \le 1 \qquad Y = 0 \tag{9}$$

has to be solved at each iteration.

This scheme turned out to be efficient for small values of the parameter $\alpha\,\tan\beta$. At large values of the parameter it is more expedient to reverse equation (6), writing it under the form

$$\phi_Y = -1 + \sqrt{\frac{\gamma}{\alpha}\,(X\,\phi_X - \phi)} \tag{10}$$

and to solve it iteratively through the scheme

$$\phi_Y^{(n)} = -1 + \sqrt{\frac{\gamma^{(n-1)}}{\alpha}\,\left(X\,\phi_X^{(n-1)} - \phi^{(n-1)}\right)}. \tag{11}$$

The choice between either iterative scheme is decided upon the value of the product $\alpha\,\tan\beta$. When $\alpha\,\tan\beta$ is smaller than 1, the first scheme is used. When it is larger than 1, the second one is followed.

To present preliminary results, we use a very crude numerical method. That is, we bound the domain at some distance $X = \pm L$ and use eigenfunction expansion of the potential

$$\phi(X, Y) = \sum_{n=1}^{N} A_n\,\cos\lambda_n X\; e^{\lambda_n Y} \tag{12}$$

in the region $-L \le X \le L$ and $-\infty < Y \le 0$, where $\lambda_n = (2\,n-1)\,\pi/(2\,L)$ so that $\Phi(\pm L, Y) \equiv 0$.

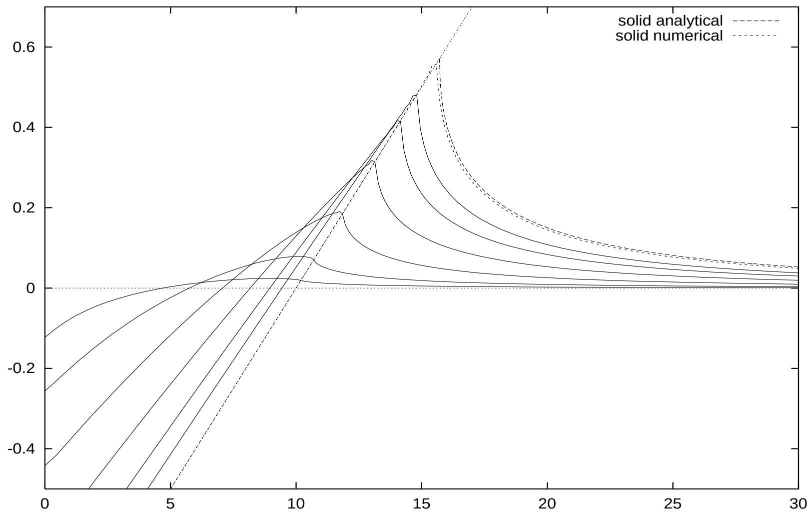

Figure 1: Porous wedge. $\cot\beta = 10$. Free surface elevation obtained from self-similar solution for $\alpha = 1, 4, 16, 64, 256, 1024$ and for a solid wedge.

A Galerkin procedure is then followed to build up a linear system which is solved by a standard Gauss routine to yield the A_n coefficients. This procedure is repeated until convergence, which is always attained within a few iterations.

When convergence has been reached the free surface position in the self-similar variables is given by

$$E(X) = \frac{1}{\gamma}\, X \int_X^\infty \frac{\phi_Y(u,0)}{u^2}\, du = \frac{1}{\gamma} \sum_{n=1}^{N} \lambda_n^2\, A_n\, X \int_{\lambda_n X}^\infty \frac{\cos v}{v^2}\, dv \qquad (X > 0). \tag{13}$$

The free surface elevation $\eta(x,t)$ is related to $E(X)$ through

$$\eta(x,t) = \gamma\, U\, t\, E\left(\frac{x}{\gamma\, U\, t}\right).$$

Finally the vertical force on the porous wedge is obtained as

$$f_y = -\rho\, \frac{d}{dt} \left\{ \int_{-\infty}^{\infty} \Phi(x,0,t)\, dx \right\} = -2\,\rho\,\gamma^2\, U^3\, t \int_{-L}^{L} \phi(X,0)\, dX = 4\,\rho\,\gamma^2\, U^3\, t \sum_{n=1}^{N} \frac{(-1)^n\, A_n}{\lambda_n}. \tag{14}$$

In the case of small porosity, $\tau \ll 1$, the iterative scheme (11) can be followed to derive the asymptotic behaviour of the solution. Then it is found that the ratio $f_y(\tau)/f_y(0)$ (that is the force on the porous wedge divided by its value for the solid one) behaves as $1 - b\sqrt{\gamma/\alpha}$, where $b = 1.113$, and the quantity γ is $\pi^2\alpha[\sqrt{a^2 + 2\pi\alpha\tan\beta} + a]^{-2}$, where $a = 0.712$, for large values of the product $\alpha\tan\beta$.

We first present results for a wedge with a deadrise angle β such that $\cot\beta = 10$. The half-length L of the numerical domain is taken equal to 8 and the truncation order N of the series (12) is $N = 800$. Figure 1 shows the free surface elevation $\gamma\, E$ plotted versus $\gamma\, X$ (that is $\eta(x,t)/(U\,t)$ plotted against $x/(U\,t)$), for different α values: $\alpha = 1, 4, 16, 64, 256$ and 1024 (or, roughly, porosity ratios of 62, 39, 22, 12, 6 and 3 %). As the porosity increases, the profiles of the liquid surface (inside and outside of the wedge) are getting smoother and the points of maximum elevation move inside the wedge.

It should be noted that the profiles obtained inside the porous wedge reflect the (locally) averaged amount of water that has leaked through it. The actual elevations of the tips of the jets that have flowed through the openings are given by

$$\eta_j(x,t) = (\gamma\ \tan\beta - 1)\, \frac{x}{\gamma} + \frac{U\,t}{\tau} \left(1 - \frac{x}{\gamma\, U\, t}\right) \left[1 - \tau + E\left(\frac{x}{\gamma\, U\, t}\right)\right]$$

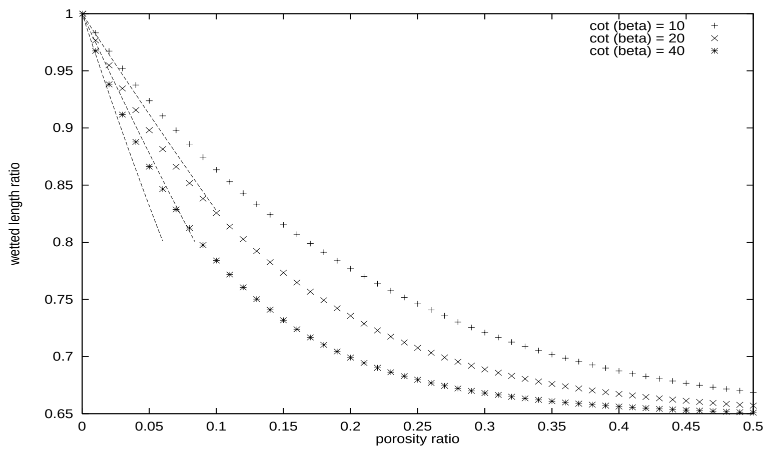

Figure 2: Porous wedge. Wetted length ratio $\gamma(\tau)/\gamma(0)$ as a function of the porosity ratio τ, for different deadrise angles.

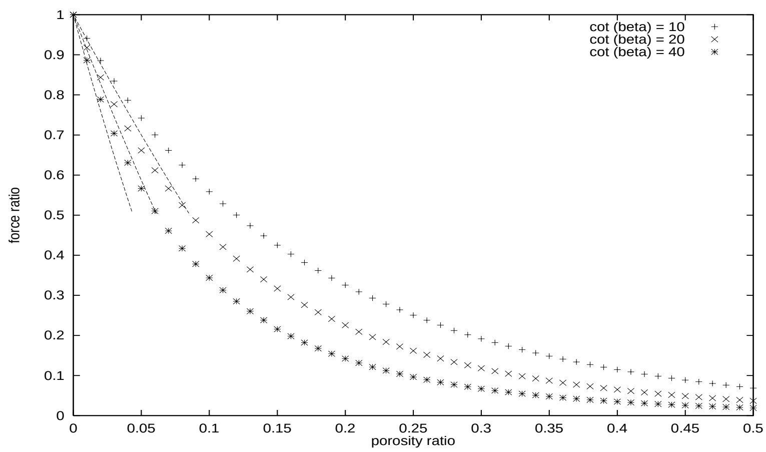

Figure 3: Porous wedge. Vertical force ratio $f_y(\tau)/f_y(0)$ as a function of the porosity ratio τ, for different deadrise angles.

The profiles related to a solid wedge ($\alpha = \infty$) are also shown, as obtained analytically and with the proposed numerical method; it can be seen that they are in fair agreement.

Then we give results relative to three deadrise angles β such that $\cot\beta$ takes the values 10, 20 and 40. The discharge coefficient μ is taken equal to 0.5. Figure 2 gives the wetted length ratio $\gamma(\tau)/\gamma(0)$ for porosity factors τ ranging from 0 (solid wedge) to 0.5. Figure 3 gives the vertical force ratio $f_y(\tau)/f_y(0)$. Also shown on these figures are the values provided by the asymptotic expressions given above, with a fair agreement at low porosity ratios. It can be observed that a porosity of 10 % reduces the force by a factor of 3 for the flattest wedge, as compared to the solid case. With 20 % porosity the reduction factor comes up to 7.

Reference

Molin B. 1992 Motion damping by slotted structures. In *Hydrodynamics: Computations, Model Tests and Reality, Developments in Marine Technology, 10.* Elsevier.