A new type of trapped mode and its relevance to the forces on parallel arrays of breakwaters

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The classical linearised equations and boundary conditions governing water waves and their interaction with obstacles permit few explicit analytical solutions. The exceptions include Havelock's 1929[5] solution for a vertical wavemaker and the solution derived from this by Ursell in 1947[17] for the scattering of two-dimensional water waves by a thin partly-immersed barrier in deep water. Susequently a number of authors, including Mei[12] and Porter D.[15] showed that an explicit analytical solution was possible for the scattering by or radiation from, any number of thin vertical barriers positioned on the same vertical line.

Later Levine & Rodemich[8] showed that the scattering by a pair of identical parallel partly-immersed vertical barriers could also be solved explicitly but the usefulness of their solution was restricted by its complexity. In contrast the simpler case of a pair of identical parallel vertical barriers extending indefinitely into the fluid from a point beneath the free surface was solved by Jarvis[6] and useful results on the transmission and reflection coefficients obtained.

In 1972 Evans & Morris[3] revisited the case of the surface-piercing pair of barriers and utilising a powerful variational method were able to get accurate results for the scattering coefficients. In particular they were able to prove that for certain spacings, depths of immersion and incident wave frequency, a wave could be completely reflected by the pair of barriers, the first time this had been observed in classical water wave theory. The same phenomenon did not hold true for the submerged pair of barriers although in both cases the more common phenomenon of complete transmission did occur.

The result of zero transmission was subsequently confirmed by Newman[13] using matched asymptotics for closely-spaced barriers and by McIver[11] for finite water depth using matched eigenfunction expansions. Finally Porter R. & Evans[16] confirmed McIver's results using an accurate complementary variational approach.

The importance of the zero transmission result to Evans & Morris lay in the possibility of designing efficient breakwaters although the closeness in frequency of complete transmission for the same barrier configuration ruled this out as a practical breakwater in mixed seas. Further examples showed that zero transmission does not require a pair of separated surface-piercing bodies for it to occur. Thus a careful scrutiny of the curves of transmission published by Haren & Mei in 1979[4] in their paper on the scattering by a Salter duck wave-energy device shows that it occurs here also whilst more recently Parsons & Martin[14] have shown numerically that it occurs in the scattering of waves by an inclined partly-immersed thin plate provided the plate is not vertical.

Speculation over the question of the uniqueness or otherwise of the two-dimensional water wave problem led to a revival in interest in zero transmission configurations. Thus Evans argued that at that frequency and configuration an identical pair of barriers positioned at an appropriate and sufficiently large distance from the first pair would totally reflect a wave incident upon them from the direction of the other pair and that the reflected wave would itself be totally reflected on reaching that pair and so on so that the net effect would be a standing wave between two identical widely-spaced pairs of barriers and only a local evanescent field outside each pair. Such a motion would provide an example of a non-uniqueness as any multiple of this solution could be added to the solution to the scattering of a wave incident on the two pairs of barriers from the region exterior to both pairs.

The uniqueness question was put to rest in 1996 on the publication by McIver M.[10] of an explicit example of non-uniqueness obtained by superposing two identical line sources in the free

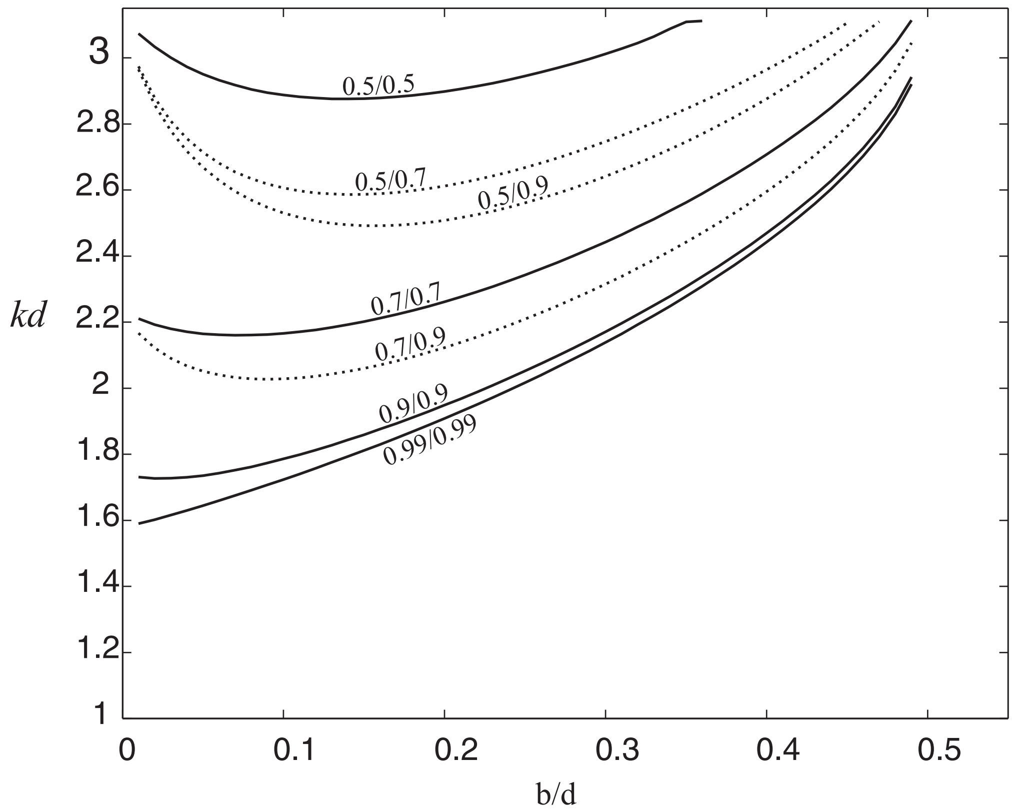

Figure 1: Curves of zero transmission past 2 barriers. Pairs of numbers next to curves indicate ratios of barrier widths to channel width d. b is the barrier spacing and k is the wave number related to the radian frequency through the relation $\omega^2 = gk \tanh kH$ where H is the depth of water in the channel.

surface at a spacing which cancelled their net far field. She showed that certain streamlines of the resulting field entered the sources from above the free surface and could be replaced by pairs of rigid bodies entirely enclosing the singularities with an open free surface between them. The rigid bodies all have the property that they intersect the free surface non-vertically a property shared also by the Salter duck thus adding weight to the Evans argument but the authors are not aware as to whether a single McIver body is capable of demonstrating zero transmission.

Recently Kuznetsov et al[7] have solved the four-barrier problem numerically using the variational approximations described by Porter & Evans and have confirmed that there are solutions describing standing waves or trapped modes between the pairs of barriers with just a local decaying field outside the pairs and that these solutions do indeed occur close to the spacings and frequencies predicted by the wide-spacing arguments of Evans.

The existence of trapped modes in an infinite wave channel containing a vertical bottom-mounted surface-piercing circular cylinder was first proved by Callan et al[1] and their relevance to the large forces on large finite arrays of identical cylinders was pointed out by Maniar & Newman[9] at a previous Workshop. The existence of trapped modes about an arbitrary cross-sectional cylinder in a channel was proved by Evans et al[2]. In all cases the condition satisfied on the centre-line was the Dirichlet condition of the vanishing of the potential, corresponding to a fundamental sloshing motion.

In the present work we obtain for the first time solutions describing trapped modes for bodies on the centre-line of an infinite channel which satisfy a Neuman condition on the centre-line and which can therefore be accessed by an incident plane wave from infinity. The method is a direct extension of the ideas on zero transmission described above.

Thus we first show that two identical vertical thin plates with axes perpendicular to the walls of the channel can if suitably spaced, when partly spanning the channel, exhibit zeros of transmission at certain frequencies. Figure 1 gives an example of how these frequencies depend upon spacing and span.
Armed with this information it will be shown that trapped modes exist in the interior fluid region between this pair of plates and its mirror image at suitable spacings and frequencies which approach the values corresponding to zero transmission as the spacing between the two pairs increases. The relevance of the results to large double arrays of breakwaters and the forces they might experience will be discussed. Curves showing the variation of the trapped mode frequencies with the geometry of the configuration will be presented. An extension to the case of Rayleigh-Bloch waves between a double row of identical parallel plates with gaps will be discussed showing for the first time the existence of such waves at wave numbers above the fundamental wavenumber associated with Rayleigh-Bloch waves.

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