Parabolic Approximation of the Hydro-elastic Behavior of a Very Large Floating Structure in Oblique Waves

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1.INTRODUCTION

Recently the estimation of elastic motion of a very large floating structure (VLFS) has been carried out for the Mega-Float project in Japan. The latest version of the computer code is based on the Finite Element Method. This code can deal with arbitrary shape of the structure and the topology of the sea bottom, however the code is time consuming and is not useful for the conceptual design. Actuary, the committee of the project regards this code as a program for the detail design.

It is obvious that we need a method to estimate the effects from all environments, for example the bottom topology, break water, geometry of the structure and so on, in the conceptual design. The method should be easy to handle and should not be time consuming. In this aspect, Takagi and Kohara[2000] proposed an application of the ray-theory to hydro-elastic behavior of VLFS. The theory itself is based on the classical ray theory. The hydro-elastic behavior of VLFS is treated as wave propagation in the platform. The wave field around the platform and in the platform is represented as a summation of wave rays.

The shortcoming of the ray theory is that corners of the platform are singular point. Takagi [1999] solved the corner problem that is the wave propagation from the water region to the semi-infinite quarter plane covered with the elastic platform and it is found that the corner effects is inversely proportional to the square root of the distance from the corner. Therefore, the corner effect is limited around the corners. Another shortcoming is that the wave amplitude is suddenly changed along a ray that passes through a corner. This shortcoming is overcome by applying the parabolic approximation in this paper.

2.FORMULATION OF THE RAY THEORY

It is well known that the water wave problem is greatly simplified by the shallow water approximation, since all evanescent terms are vanished. Although extension of the ray theory to the finite depth problem or the varying depth problem has no essential problem, the shallow water approximation is employed in this paper for the simplification of the problem. The velocity potential satisfies the modified continuity equation

$$\frac{K}{h}\phi + [1 + M\nabla^4]\nabla^2\phi = 0, \tag{1}$$

where $K = \frac{\omega^2}{g}$, $M = D/\rho g$ and h is the water depth. D is the flexural rigidity of the platform, g is the gravitational acceleration and ρ is the density of the water. (1) gives the dispersion relation in the platform.

$$K = (1 + M\alpha^4)\alpha^2 h \tag{2}$$

It is well known that (2) has six roots, and three of them are suitable for present problem. We call these roots as α_n $(n = 0, 1, 2)$ herein after.

The ray tracing is performed, according to the characteristic form of the conservation of the wave number

$$\frac{d\boldsymbol{\alpha_0}}{ds} = -\frac{\partial\Omega}{\partial\boldsymbol{r}} \qquad \text{on} \qquad \frac{d\boldsymbol{r}}{ds} = \frac{\partial\Omega}{\partial\boldsymbol{\alpha_0}}. \tag{3}$$

where $\Omega(\alpha_0) = \sqrt{g\alpha_0(1 + M\alpha_0^4)\alpha_0^2 h}$.

If the water depth and the flexural rigidity were constant, the problem would be very simple. We just take the refraction of the ray into account at the edge of the platform. The difficulty is that the displacement of the platform is suddenly changed along a ray that passes through a corner as stated previously. In order to overcome this difficulty, we discuss the asymptotic form of the exact representation of the wave propagation in the platform.

3.ASYMPTOTIC FORM IN THE PLATFORM

According to the boundary integral formulation derived by Takagi [1999], the hydro-elastic behavior of the platform is represented by the sinusoidal distribution of the Green function.

$$\begin{aligned} I &= \lim_{\epsilon\to 0}\int_0^\infty e^{-i(k_y - i\epsilon)\eta} G(x, 0, y, \eta)d\eta \\ &= \lim_{\epsilon,\iota\to 0}\frac{1}{2\pi}\int_0^\infty e^{-i(k_y - i\epsilon)\eta}\int_0^\infty \frac{1}{k}\left(\frac{K}{(K - i\iota) - (1 + Mk^4)k\ 2h} - 1\right) J_0(kR')dkd\eta, \end{aligned} \tag{4}$$

where ϵ and ι ensure the radiation condition.

Applying a contour integral to (4), a component of plane waves can be derived. The detail of the derivation is found in Takagi[1999].

$$I = -\frac{i}{2\pi}\frac{K}{\alpha_0\Lambda'(\alpha_0)}\int_{-\infty}^{\infty}\frac{e^{-i\alpha_0 R\cosh\theta}}{k_y-\alpha_0\sin(\beta+i\theta)}d\theta-\frac{i}{2\pi}\frac{K}{\alpha_1\Lambda'(\alpha_1)}\int_{-\infty}^{\infty}\frac{e^{i\alpha_1 R\cosh\theta}}{k_y+\alpha_1\sin(\beta+i\theta)}d\theta$$
$$-\frac{i}{2\pi}\frac{K}{\alpha_4\Lambda'(\alpha_4)}\int_{-\infty}^{\infty}\frac{e^{-i\alpha_4 R\cosh\theta}}{k_y-\alpha_4\sin(\beta+i\theta)}d\theta+C_{I1}+C_{I2}+C_{I3}, \qquad (5)$$

where,

$$C_{I1} = \begin{cases} 0 & \text{when } 0>\beta>\mu_0 \\ -i\dfrac{Ke^{-iR\alpha_0\cos(\mu_0-\beta)}}{\alpha_0^2\Lambda'(\alpha_0)\cos\mu_0} & \text{when } \frac{\pi}{2}<\beta<\mu_0 \end{cases}, \qquad (6)$$

$$C_{I2} = -i\frac{Ke^{-iR\alpha_1\cos(\mu_1-\beta)}}{\alpha_1^2\Lambda'(\alpha_1)\cos\mu_1} \qquad (7)$$

$$C_{I3} = \begin{cases} 0 & \text{when } \beta<\Re[\mu_4] \\ -i\dfrac{Ke^{-iR\alpha_4\cos(\mu_4-\beta)}}{\alpha_4^2\Lambda'(\alpha_4)\cos\mu_4} & \text{when } \beta>\Re[\mu_4] \end{cases}, \qquad (8)$$

$$\frac{k_y}{\sqrt{\alpha_n^2-k_y^2}} = \tan\mu_n, \qquad \left.\begin{matrix} x \\ y \end{matrix}\right\} = R\left\{\begin{matrix}\cos\beta \\ \sin\beta\end{matrix}\right. \qquad (9)$$

and $\Lambda(\alpha)=(1+M\alpha^4)\alpha^2 h$. It is noted that C_{I1} denotes plane progressive waves and it does not affect the other edge i.e. on the line $y=0$. C_{I2} and C_{I3} are also plane progressive waves, however their wave number is a complex number and these waves decay quickly as the coordinate x becomes large. The first three terms in (5) represent the end effect and the first term is asymptotically proportional to $1/\sqrt{\alpha_0 R}$. When the observation point is far from the corner, the end effect vanishes and the solution coincides with the solution of the semi-infinite half platform problem.

A difficulty is expected when θ equals to β, because the stationary phase method can not be applied to the first term of (5). Therefore, the asymptotic form along this line is discussed. The following coordinate is defined

$$\left.\begin{matrix} x' \\ y' \end{matrix}\right\} = R\left\{\begin{matrix}\cos(\beta-\mu_0) \\ \sin(\beta-\mu_0)\end{matrix}\right., \qquad (10)$$

and the order of y' is assumed to be $O(\alpha_0^{-1/2})$. It is also noted that the we are considering the region $R=O(1)$.

After some manipulations we obtain

$$\frac{\alpha_0\cos\mu_0}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\alpha_0 R\cosh\theta}}{k_y-\alpha_0\sin(\beta+i\theta)}d\theta+i\frac{\alpha_0^2\Lambda'(\alpha_0)}{K}C_{I1}$$
$$=e^{-i\alpha_0 x'}\frac{1+i}{2}\left[\frac{1}{2}+C(\sigma)-i\left(\frac{1}{2}+S(\sigma)\right)\right]+O(\alpha_0^{-1}), \qquad (11)$$

where $\sigma=\alpha_0 y'/(\pi\alpha_0 x')^{1/2}$. It is obvious that the right hand side of (11) is the anti-symmetric solution of the parabolic approximation that is found in May[1989].

4.PARABOLIC APPROXIMATION

According to the previous analysis, it is reasonable to assume the velocity potential of the form

$$\phi(x,y)=\psi(x,y)e^{-i\alpha_0(x\cos\mu_0+y\sin\mu_0)}. \qquad (12)$$

If the angle of wave propagation μ_0 is large enough, ψ would be the anti-symmetric type solution of the parabolic approximation as stated previously. On the contrary, when $\mu_0=0$ (head sea case), Ohkus[1999] showed that ψ is the symmetric solution of the parabolic approximation. A difficulty may be expected, when $\mu_0=O(\alpha_0^{-1/2})$. In this case, the parabolic approximation leads

$$-2i\alpha_0\left(\frac{\partial\psi}{\partial x}-\mu_0\frac{\partial\psi}{\partial y}\right)+\frac{\partial^2\psi}{\partial y^2}=0, \qquad (13)$$

in which the all terms are equal in order of magnitude when $x = O(1)$ and $y = O(\alpha_0^{-1/2})$. In such a order-assumption, the relative error of (13) is $O(\alpha_0^{-1})$.

To solve this equation, we require the conditions,

$$\psi(x,y) \to 1, \qquad \text{when} \quad y \to +\infty, \tag{14}$$

$$\psi(x,y) = 0, \qquad x < 0. \tag{15}$$

It is noted that these boundary conditions are not enough to solve (13). Another boundary condition is required along the line $y = 0$. This boundary condition is given from the matching condition that will be discussed later. Only the following condition is necessary here that guarantees the radiation condition of the solution in the water region.

$$\psi(1 + iT\alpha_0\mu_0) = T\frac{\partial \psi}{\partial y}, \qquad \text{on} \quad y = 0, \tag{16}$$

where T is a constant value.

The Fourier transformation technique is convenient to solve (13). The solution is represented in the Fourier transformation space,

$$\mathcal{F}[\psi(x,y) - H(x)] = A(l)e^{(i\alpha_0\mu_0 - \sqrt{2\alpha_0}\sqrt{l - \alpha_0\mu_0^2/2})y}. \tag{17}$$

To satisfy the boundary conditions, $A(l)$ shoul have the form

$$A(l) = -i\frac{1 + iT\alpha_0\mu_0}{l\left(1 + T\sqrt{2\alpha_0}\sqrt{l - \alpha_0\mu_0^2/2}\right)}. \tag{18}$$

The inverse Fourier transformation gives the solution

$$\begin{aligned}\psi(x,y) = &\frac{1+i}{2}\left[\frac{1}{2} + C(\sigma) - i\left(\frac{1}{2} + S(\sigma)\right)\right] \\ &-\frac{1-i}{4\sqrt{\pi}}\int_0^x \left(\frac{\sqrt{x-\xi}}{T\sqrt{\alpha_0}} - i\frac{\sqrt{\alpha_0}y}{\sqrt{x-\xi}}\right)\frac{[V_1(\xi) + V_2(\xi)]}{x-\xi}e^{-i\alpha_0\{y-(x-\xi)\mu_0\}^2/2(x-\xi)}d\xi,\end{aligned} \tag{19}$$

where

$$V_1(x) = \frac{T\alpha_0\mu_0}{i + T\alpha_0\mu_0}erfc\left[\frac{(1+i)\sqrt{\alpha_0\mu_0^2}}{2\sqrt{2}}\sqrt{x}\right],$$

$$V_2(x) = \frac{i}{i + T\alpha_0\mu_0}e^{-i(1+T^2\alpha_0^2\mu_0^2)x/2T^2\alpha_0}erfc\left[\frac{1-i}{2\sqrt{\alpha_0}T}\sqrt{x}\right], \tag{20}$$

and $erfc(x)$ is the complementary error function.

It is obvious that the first term of (19) remains when θ approaches to infinity. This is the anti-symmetry solution of the parabolic approximation that obtained in the previous section.

We also have the alternative form of (19),

$$\psi(x,y) = 1 - \frac{1-i}{2\sqrt{\pi}}\int_0^x \left(\frac{1}{T\sqrt{\alpha_0}}[V_1(\xi) + V_2(\xi)] + i\sqrt{\alpha_0}\mu_0\right)\frac{e^{-i\alpha_0\{y-(x-\xi)\mu_0\}^2/2(x-\xi)}}{x-\xi}d\xi. \tag{21}$$

This form shows that the solution approaches to the symmetric solution of the parabolic approximation when θ is very small.

In order to achieve the matching with the inner solution that will be discussed in the next section, a function $U(x)$ is defined as

$$U(x) = 1 - \frac{1-i}{2\sqrt{\pi}}\int_0^x \left(\frac{1}{T\sqrt{\alpha_0}}[V_1(\xi) + V_2(\xi)] + i\sqrt{\alpha_0}\mu_0\right)\frac{e^{-i\alpha_0(x-\xi)\mu_0/2}}{x-\xi}d\xi. \tag{22}$$

It is apparent that ψ approach to $U(x)$ when y approaches to zero.

5.INNER SOLUTION

It is noted that (19) does not satisfies the edge conditions, and the parabolic approximation ignores the exponentially-decay term that is one of fundamental solutions in the platform region. This fact implies the necessity of the inner solution, and the inner solution is valid in the region of $y = O(\alpha_0^{-1})$.

It is easily obtained that *psi* should satisfy the following equation in this region.

$$\frac{K}{h}\psi + \left[1 + M\left(-\alpha_0^2 - 2i\alpha_0\mu_0\frac{\partial}{\partial y} + \frac{\partial^2}{\partial y^2}\right)^2\right]\left(-\alpha_0^2 - 2i\alpha_0\mu_0\frac{\partial}{\partial y} + \frac{\partial^2}{\partial y^2}\right)\psi = 0. \tag{23}$$

The solution of (23) in the platform region is supposed to have the form

$$\psi_I(x,y) = U(x)\sum_{n=1}^{2} B_n e^{\left(\alpha_0 \sin\mu_0 - \sqrt{\alpha_n^2 - \alpha_0^2\cos{\mu_0}^2}\right)y} + B_0\psi(x,y). \tag{24}$$

It is obvious that the matching condition requests

$$B_0 = 1 \tag{25}$$

On the other hand the inner solution in the water region should satisfy (23) of the case $M = 0$. Therefore, the inner solution in the water region has the form

$$\phi_I = U(x)\sum_{n=1}^{2} B_n e^{-ik(x\cos\chi_R - y\sin\chi_R)}, \tag{26}$$

where k is the solution of dispersion relation in the water region $K/h = k^2$ and $\chi_R = Cos^{-1}\alpha_0\cos\mu_0/k$.

The continuity of the flux along the line $y = 0$ requires

$$\frac{1}{T} - i\alpha_0\mu_0 + i\sum_{n=1}^{2} B_n\sqrt{\alpha_n^2 - \alpha_0^2\cos^2\mu_0} = ik\sin\chi_R\sum_{n=0}^{2} B_n. \tag{27}$$

The so-called free-free boundary condition at the edge of platform requires two conditions

$$\sum_{n=1}^{2}\left[\alpha_n^4 + \alpha_0^2\cos^2\mu_0(1-\nu)\alpha_n^2\right]\sqrt{\alpha_n^2 - \alpha_0^2\cos^2\mu_0}B_n + \left(\frac{1}{T} - i\alpha_0\mu_0\right)\alpha_0^4[1+\cos^2\mu_0] = 0 \tag{28}$$

$$\sum_{n=0}^{2}\left[\alpha_n^4 - \alpha_0^2\cos^2\mu_0(1-\nu)\alpha_n^2\right]B_n = 0. \tag{29}$$

Now we have three equations and three unknowns, thus the inner solution is completely determined with a simple linear algebra.

6.CONCLUDING REMARKS

It was found that the hydro-elastic behavior of a very large floating structure is represented with a simple equation by applying the parabolic approximation. It seems that this result is also extended to the case $\mu_0 < 0$ and to the improvement of the sudden change of wave elevation that appears in the water region. These will be the future works. Finally, it should be noted that the corner effect is still the problem, and further study is necessary since the magnitude of motion at corners of the platform is usually big.

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