Role of Surface Tension in Modelling Ship Waves

Xiao-Bo Chen

Research Department, Bureau Veritas, 17bis, Place des Reflets, 92400 Courbevoie (France)
Fax: +33-1-4291.3395 Email: xiao-bo.chen@bureauveritas.com

The surface tension is often ignored in describing water waves around large floating bodies, since its effect is considered to be significant only for rather short waves commonly called ripples whose wavelength is of order in centimeters. However, the theory of gravity waves may yield waves of very short length which cannot be ignored and cause substantial difficulties in modelling them. The work on the wave pattern due to a steady-moving concentrated pressure on the free surface by Ursell (1960) showed that the wave elevation near the track of pressure point from the linearized *pure-gravity* theory oscillates with indefinitely increasing amplitude and indefinitely decreasing wavelength. For the more general case of a point source both pulsating and advancing at a uniform speed, similar behavior is revealed in a recent work by Chen & Wu (2001). These singular and highly oscillatory properties being manifestly non-physical, it is expected that the surface tension plays an important role in modelling ship waves, at least part of them. The description of ship waves including the effect of surface tension is summarized here.

We consider the steady free-surface potential flow generated by a source advancing at constant horizontal speed. The dispersion function associated with the free-surface boundary condition including the surface tension is written as

$$D(\alpha,\beta) = \alpha^2 - k - \sigma^2 k^3 \tag{1}$$

in which (α,β) are the speed-scaled Fourier variables associated with the Froude number $F = U/\sqrt{gL}$, and $k = \sqrt{\alpha^2+\beta^2}$ the wavenumber. Here, U is the forward speed, g the acceleration due to gravity and L a reference length. Furthermore, the parameter σ is defined by

$$\sigma = \sqrt{T/(\rho g L^2)}/F^2 \tag{2}$$

where T is the surface tension[1]. The characteristic wavenumber of capillary waves according to Lighthill (1978) is $\sqrt{\rho g/T}$ while the fundamental wavenumber of *gravity* ship waves is g/U^2, so that the parameter σ is the ratio of both since

$$\sigma = k_0/k_m \qquad \text{with} \quad k_0 = 1/F^2 \quad \text{and} \quad k_m = L\sqrt{\rho g/T} \tag{3}$$

It's evident to consider σ as an important parameter in characterizing the capillary-gravity ship waves.

The dispersion curves defined by $D = 0$ are symmetrical with respect to both axes $\alpha = 0$ and $\beta = 0$. In the quadrant $\alpha \geq 0$ and $\beta \geq 0$, the dispersion curve is defined explicitly in polar coordinates (k,θ)

$$k(\theta) = \begin{cases} k_g(\theta) = 2/(\cos^2\theta + \sqrt{\cos^4\theta - 4\sigma^2}) & k \leq k_\sigma \\ k_T(\theta) = (\cos^2\theta + \sqrt{\cos^4\theta - 4\sigma^2})/(2\sigma^2) & k \geq k_\sigma \end{cases} \tag{4}$$

with $k_\sigma = 1/\sigma$. The curve described by (4) is a closed one limited in the region

$$0 \leq \theta \leq \theta_\sigma \qquad \text{with} \quad \theta_\sigma = \arctan[\sqrt{(1-2\sigma)/(2\sigma)}] \tag{5}$$

At $\theta = \theta_\sigma$ we have $k = k_\sigma$. At $\theta = 0$, we define

$$k_g^0 = 2/(1+\sqrt{1-4\sigma^2}) \quad \text{and} \quad k_T^0 = (1+\sqrt{1-4\sigma^2})/(2\sigma^2) \tag{6}$$

so that the dispersion curve intersects the α-axis at $\alpha = k_g^0$ and $\alpha = k_T^0$.

The dispersion curves given by (4) are depicted on the *left* part of Figure 1 at a Froude number $F = 0.1$ (using $L = 1$ m) for $\sigma = 0$ when the surface tension is ignored and $\sigma = 0.275$ when the surface tension is included. The dispersion curve without the surface tension ($\sigma = 0$) represented by the dashed line is given by $k = 1/\cos^2\theta$ and corresponds to the case usually called Neumann-Kelvin ship waves. It is an open curve as $k \to \infty$ when $\theta \to \pi/2$. The dispersion curve with the surface tension ($\sigma \neq 0$) is a *closed* one with a maximum wavenumber k_T^0 defined by (6). The point $(k_\sigma, \theta_\sigma)$ divides the dispersion curve into two portions :

[1] The surface tension T takes the value 0.074 N/m for the air-water interface at 20° C.

one ($k_g < k_\sigma$, thick solid line) along which the effect of gravity is dominant and another ($k_T > k_\sigma$, thin solid line) along which the capillarity is dominant.

The asymptotic analysis based on the stationary-phase argument realized in Chen & Noblesse (1997) gives a direct relationship between the dispersion curves in the Fourier plane and the corresponding wave systems on the free surface. Indeed, the constant-phase curve (crestlines) and related wavelength, directions of wave propagation, cusp angles, and phase and group velocities can be determined explicitely from the dispersion function. In particular, the crestlines are given by a simple formula

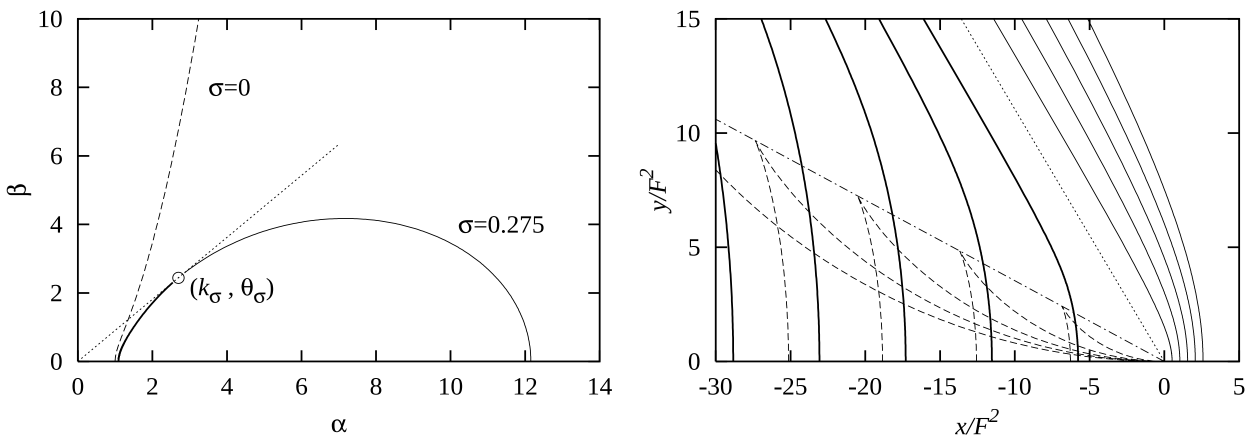
$$(x, y)_n = C_n(D_\alpha, D_\beta)/(\alpha D_\alpha + \beta D_\beta) \tag{7a}$$

with the phase constant C_n defined by

$$C_n = -\operatorname{sign}(\alpha^2 D_\alpha + \alpha\beta D_\beta) \cdot (2\pi)\, n \tag{7b}$$

in our case of steady ship waves. On the *right* part of Figure 1, we depict the crestlines for $n = (1, 2, \cdots, 5)$ defined by (7) associated with the dispersion curves plotted on the left part of the figure. The Neumann-

Figure 1: Dispersion curves (left) and crestlines (right) of capillary-gravity ship waves at $F = 0.1$



Kelvin ship waves represented by dashed lines composed of transverse and divergent waves are present only in the downstream and limited by a cusp line (dot-dashed line). The ship waves including the surface tension are present in both upstream and downstream. The upstream crestlines associated with the part of dispersion curve at $k_T > k_\sigma$ are capillary waves and plotted by thin solid lines. The wavelength of upstream capillary waves is of order $2\pi F^2/k_T^0$.

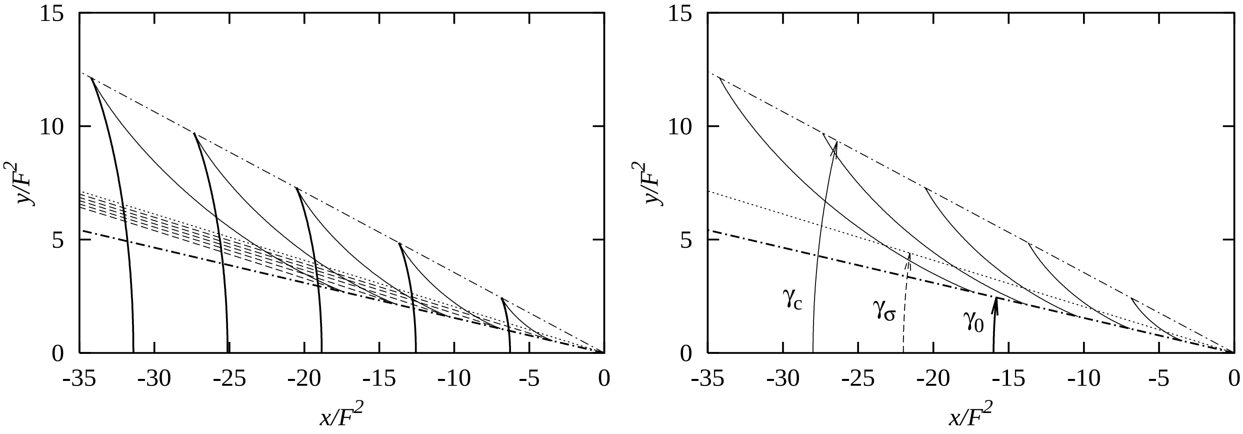
The downstream crestlines (thick solid lines) associated with the part of dispersion curve at $k_g < k_\sigma$ are gravity-dominant waves. Comparing to the pure-gravity waves (dashed lines), the transverse waves keep the same profile with a slight shorter wavelength $2\pi F^2/k_g^0$ instead of $2\pi/k_0$ (eqs. 3 and 6). The most striking feature concerns the divergent waves which disappear completely at this value of σ (in fact for $\sigma > \sigma_0$ given in the following) due to the effect of surface tension. In their place, the transverse waves are extended smoothly outward to a region limited by the ray (dotted line) forming an angle γ_σ with the negative-x axis defined in

$$\gamma = \arctan[y/(-x)] \leq \gamma_\sigma = \pi/2 - \theta_\sigma \tag{8}$$

The crestlines for $n = (1, 2, \cdots, 5)$ are depicted on Figure 2 for $\sigma = 0.02$ (left part). Only those of downstream waves are drawn for the sake of clarity. The transverse waves are represented by thick solid lines and the divergent waves by thin solid lines, while the rest of capillary-gravity waves by dashed lines limited by the dotted ray ($\gamma = \gamma_\sigma$). The variation of γ_σ given in (8) is plotted on the left part of Figure 3. $\gamma_\sigma = 0$ at $\sigma = 0$ means that no capillary waves exist since the effect of surface tension is ignored. At $\sigma = \sigma_m = 1/2$, the dispersion curve reduces to a point $(2, 0)$ and $\gamma_\sigma = \pi/2$ which means that all steady waves disappear (no wavy deformation of the free surface) since ship's speed is less than the minimum velocity of capillary-gravity waves so that waves propagating at ship's speed cannot be generated.

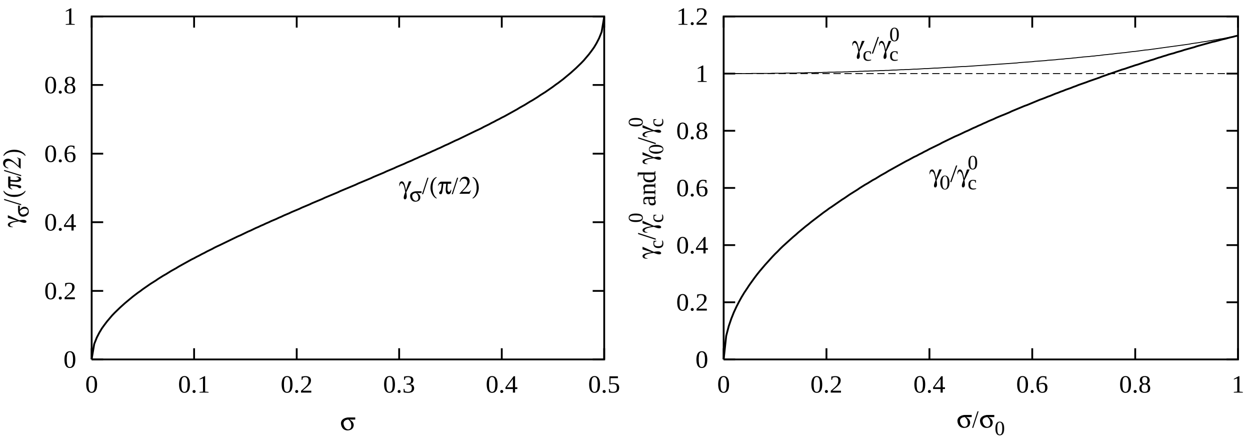
There are two other important rays, more evident on the right part of Figure 2 on which only crestlines of divergent waves are kept. One represented by the thin dot-dashed line is close to the cusp line of Kelvin ship waves, and another by thick dot-dashed line. We denote the two rays respectively by $\gamma = \gamma_c$ and $\gamma = \gamma_0$ the angles forming with the negative-x axis. Same as γ_σ, the ray-angles γ_c and γ_0 are function of the

Figure 2: Crestlines of capillary-gravity ship waves at $\sigma = 0.02$ (left) and definition of rays (right)



parameter σ. Following the work presented in [4], the value of γ_c is associated with the normal direction at the first point of inflection along the dispersion curve, which is quite close to that for the Neumann-Kelvin ship waves. There exists a second inflection point along the dispersion curve of capillary-gravity ship waves at low values of σ. The value of γ_0 is given by the normal direction at this second point of inflection. The ray-angle γ_c becomes the cusp angle $\gamma_c^0 = \gamma_c(\sigma = 0) \approx 19°28'$ of pure-gravity ship waves when $\sigma \to 0$ while γ_0 tends to zero. The variation of γ_c and γ_0 is shown on the right part of Figure 3. The value of $\sigma_0 \approx 0.133$ at which $\gamma_0 = \gamma_c$ is used to rescale the σ-axis on the figure. The cusp angle γ_c (thin solid line) is slightly larger than that of pure-gravity ship waves. The ray-angle γ_0 (thick solid line) increases rapidly for $\sigma > 0$ and touches the same value as γ_c for $\sigma = \sigma_0$.

Figure 3: Ray-angles γ_σ (left), γ_c and γ_0 (right) dependent on the parameter σ



It is shown that the divergent waves can be found only in the region ($\gamma_0 \leq \gamma \leq \gamma_c$) where transverse waves appear as well. In the region near the ship's track ($0 \leq \gamma \leq \gamma_0$), only transverse waves are present. Since γ_0 increases significantly with increasing σ (corresponding to the decrease of forward speed), the region ($\gamma_0 < \gamma < \gamma_c$) where divergent waves appear is more and more reduced. At $\sigma = \sigma_0 \approx 0.133$ (corresponding to $U = U_0 \approx 0.450$ m/s), there does not exist any divergent wave.

These results concerning divergent waves are welcome in the modelling of ship waves, since we know from [1] and [2] that, by excluding the surface tension, the part of divergent waves becomes extremely oscillatory and singular so that substantial difficulties arise in their numerical computations. Following the development presented in [5], we may write the wave component of capillary-gravity steady flow by the sum of two parts :

$$G^W = (G_g^W + G_T^W)/(\pi F^2) \tag{9a}$$

in which the gravity-dominant waves G_g^W and the capillarity-dominant waves G_T^W are expressed by a single integral along the part of dispersion curve $k_g < k_\sigma$

$$G_g^W = \int_0^{\theta_\sigma} (1 - S_g)\, \frac{k_g\, d\theta}{|D_k|_g}\, e^{zk_g}\, \sin(xk_g \cos\theta) \cos(yk_g \sin\theta) \tag{9b}$$

and along the part $k_T > k_\sigma$

$$G_T^W = \int_0^{\theta_\sigma} (1 - S_T) \frac{k_T \, d\theta}{|D_k|_T} e^{zk_T} \sin(xk_T \cos\theta) \cos(yk_T \sin\theta) \tag{9c}$$

respectively, and with

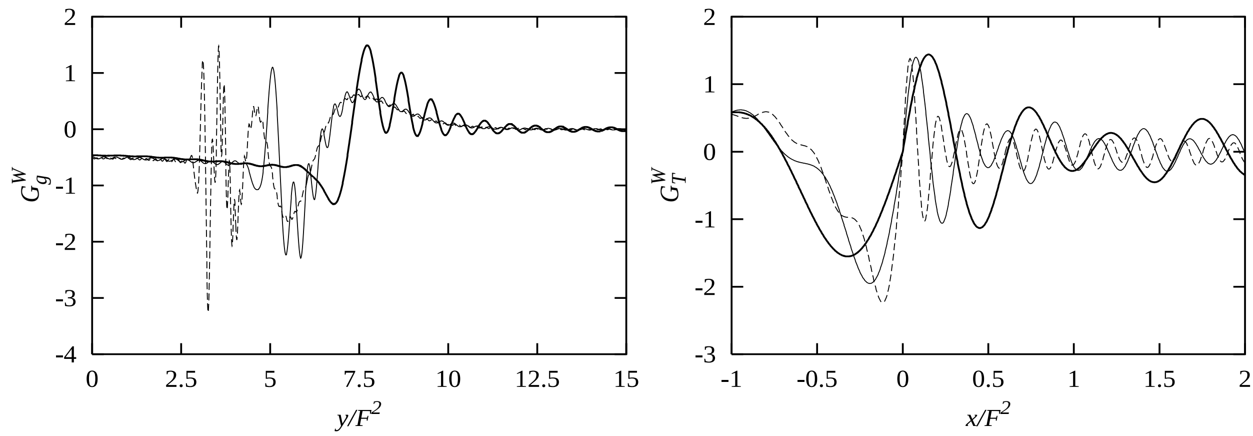
$$S_g = \mathrm{sign}(xD_\alpha + yD_\beta)|_{k=k_g} \quad ; \quad S_T = \mathrm{sign}(xD_\alpha + yD_\beta)|_{k=k_T} \tag{9d}$$

and

$$|D_k|_g = |2k_g \cos^2\theta - 1 - 3\sigma^2 k_g^2| \quad ; \quad |D_k|_T = |2k_T \cos^2\theta - 1 - 3\sigma^2 k_T^2| \tag{9e}$$

The single integrals (9b) and (9c) are *convergent* even for $z = 0$, i.e., when both the source and field points are located at the free surface, since the amplitude of oscillatory integrands is of order $O(1/k)$ instead of $O(k)$ if the surface tension is excluded. Furthermore, both k_g and k_T have finite value for $\sigma > 0$ so that G_g^W and G_T^W are not singular. The ship waves given by (9b) and (9c) are illustrated on Figure 4. The gravity-

Figure 4: Ship waves along a transversal cut (left) and a longitudinal cut (right)



dominant waves G_g^W are plotted on the left part of the figure along a transversal cut ($x = -20F^2$). The waves at $\sigma = 0.1, 0.05$ and 0.02 are plotted with thick solid lines, thin solid lines and dashed lines, respectively. The capillarity-dominant waves G_T^W are plotted on the right part of the figure along a longitudinal cut ($y = 0$). The waves at $\sigma = 0.275, 0.2$ and 0.15 are plotted with thick solid lines, thin solid lines and dashed lines, respectively. The source and field points are both located at the free surface ($z = 0$). These results confirm that waves generated by a point source steadily advancing on the free surface are not singular on the free surface including the region close to the track of the source.

In summary, the steady ship waves including the surface tension are analyzed using directly the relationship between the dispersion relation and far-field waves - results obtained in [4]. It is shown that the role of surface tension in modelling ship waves is twofold. Firstly, including the effect of surface tension yields more realistic description of ship waves. Especially for low forward speed, the divergent waves are largely compressed and appear only in a zone between two rays : the line ($\gamma = \gamma_0$) and the cusp ($\gamma = \gamma_c$). At lower speed of $U \leq U_0 \approx 0.450$ m/s (corresponding to $\sigma = \sigma_0 \approx 0.133$), no divergent waves exist. When $U < U_m \approx 0.232$ m/s (corresponding to $\sigma = \sigma_m = 0.5$), no wave can be generated. Secondly, introducing of surface tension in the formulation of ship waves eliminates the singularity of the Green function when both the source and field points are at the free surface. These benefits will be much more enjoyed in the numerical development of practical computation methods.

References

[1] Ursell F. (1960) On Kelvin's ship-wave pattern. *J. Fluid Mech.* **8**, 418-431.

[2] Chen X.B. & Wu G.X. (2001) On singular and highly oscillatory properties of the Green function for ship motions. *J. Fluid Mech.* **445**, 77-91.

[3] Lighthill J. (1978) *Waves in Fluids.* Cambridge University Press.

[4] Chen X.B. & Noblesse F. (1997) Dispersion relation and far-field waves. *Proc. 12th Intl Workshop on Water Waves and Floating Bodies*, Carry-Le-Rouet (France), 31-35.

[5] Chen X.B. (1999) An introductory treatise on ship-motion Green functions. *Proc. 7th Intl Num. Ship Hydrodynamics*, Nantes (France).