Approximations of the low-frequency second-order wave loads: Newman versus Rainey

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The numerical prediction of the slow-drift motion of moored structures requires the knowledge of the second-order wave loads taking place at the difference frequencies of the incoming wave-system. It is quite a numerical endeavor to compute these QTF's and, most often, use is made of the so-called Newman's approximation whereby the QTF's are based upon the drift forces in regular waves. This procedure applies well to massive structures such as barges or FPSO's, which have very low natural frequencies in surge, sway and yaw, and for which the drift forces in regular waves are large.

In this paper we are concerned with more slender structures such as semi-submersibles, TLP's or spar towers, which consist of cylinders with diameters in the 10-30 m range. In 12-18 s wave periods, this means that ka varies from near 0 up to 0.5, k being the wave number and a the radius. In this ka range the first-order wave loads and responses can be fairly well approximated through a Morison type approach, where only the inertia term is retained. To obtain the drift forces diffraction-radiation analysis becomes necessary. As these drift forces are small at low ka values, and we are concerned not only with surge, sway and yaw, but also roll, pitch and possibly heave, which have natural frequencies much closer to the wave frequencies, it turns out that Newman's approximation does not provide good estimates of the QTF's around the resonant frequencies.

A second-order extension of the inertia term in the Morison equation is provided by Rainey's equations (1989). They have been applied by Ma & Patel (2001) to study the slow surge and pitch motions of a spar platform (see also Kim & Chen, 1994). The questions that arise are, how accurate this procedure is, and whether it can be related to the exact QTF calculation or to Newman's approximation. Here we suggest that the two approaches can likely be used in combination and that they somehow supplement each other.

For the sake of simplicity, we deal with fixed bodies. The correct way to derive the QTF's is to solve the first-order diffraction problem, in a bichromatic wave system, and then derive the hydrodynamic loads to second-order. The main numerical difficulty stems from the second-order diffraction potential.

We assume uni-directional sea-states propagating along the Ox axis. The first-order velocity potential, including incident and diffracted contributions, can be written

$$\Phi^{(1)}(x,y,z,t) = \Re\left\{A_1\,\varphi_1^{(1)}(x,y,z)\,e^{-i\,\omega_1 t} + A_2\,\varphi_2^{(1)}(x,y,z)\,e^{-i\,\omega_2 t}\right\} \tag{1}$$

while the second-order potential, at the difference frequency $\omega_1-\omega_2$, consists of an incident and a diffracted component

$$\Phi^{(2)}(x,y,z,t) = A_1\,A_2\,\Re\left\{\left(\varphi_I^{(2)}(x,y,z)+\varphi_D^{(2)}(x,y,z)\right)\,e^{-i\,(\omega_1-\omega_2)\,t}\right\}. \tag{2}$$

A_1 and A_2 are the wave amplitudes.

In arbitrary waterdepth h the incident component $\varphi_I^{(2)}(x,y,z)$ has a rather complicated expression, so we do not reproduce it here. The diffracted component obeys a boundary value problem with the conditions $\partial\varphi_D^{(2)}/\partial n = -\partial\varphi_I^{(2)}/\partial n$ at the mean body surface and $g\,\partial\varphi_D^{(2)}/\partial z - (\omega_1-\omega_2)^2\,\varphi_D^{(2)} = \alpha^{(2)}$ at the mean free surface.

The second-order horizontal load, at the difference frequency $\omega_1-\omega_2$, consists of 5 terms:

$$F_-^{(2)} = 2A_1A_2\,\Re\left\{f_-^{(2)}\,e^{-i\,(\omega_1-\omega_2)\,t}\right\} = 2A_1A_2\,\Re\left\{\left(f_Q^{(2)}+f_{WL}^{(2)}+f_I^{(2)}+f_{D1}^{(2)}+f_{D2}^{(2)}\right)\,e^{-i\,(\omega_1-\omega_2)\,t}\right\} \tag{3}$$

where

$$f_Q^{(2)} = \frac{1}{2}\rho\iint_{S_{C_0}} \nabla\varphi_1^{(1)}\cdot\nabla\varphi_2^{(1)^*}\,n_{0x}\,dS \tag{4}$$

$$f_{WL}^{(2)} = -\frac{\rho}{2\,g}\,\omega_1\,\omega_2 \int_{\Gamma_0} \varphi_1^{(1)}\,\varphi_2^{(1)*}\,n_{0x}\,d\Gamma_0 \qquad (5)$$

$$f_I^{(2)} = -i\,\rho\,(\omega_1-\omega_2) \iint_{S_{C_0}} \varphi_I^{(2)}\,n_{0x}\,dS \qquad (6)$$

$$f_{D1}^{(2)} = i\,\rho\,(\omega_1-\omega_2) \iint_{S_{C_0}} \nabla\varphi_I^{(2)}\cdot\vec{n_0}\;\psi\,dS \qquad (7)$$

$$f_{D2}^{(2)} = \frac{i\,\rho\,(\omega_1-\omega_2)}{g} \iint_{z=0} \alpha^{(2)}\;\psi\,dS \qquad (8)$$

where S_{C_0} is the mean wetted hull, Γ_0 the mean water line, $\vec{n_0}$ the normal vector (into the fluid), ψ the radiation potential in surge at frequency $\omega_1-\omega_2$ and * designates the complex conjugate.

When $\omega_1-\omega_2 \to 0$ the QTF reduces to the drift force $f_d(\omega) = f_Q^{(2)}(\omega,\omega) + f_{WL}^{(2)}(\omega,\omega)$. This is the basis for Newman's approximations (after his 1974 paper) which consist in approximating $f_-^{(2)}$ from f_d. For instance the following approximation has been proposed (Molin & Bureau, 1980):

$$f_-^{(2)}(\omega_1,\omega_2) \simeq \sqrt{f_d(\omega_1)\,f_d(\omega_2)}\;\mathrm{sign}(f_d) \qquad (9)$$

For a fixed vertical cylinder, standing on the sea-floor, the components $f_Q^{(2)}$, $f_{WL}^{(2)}$, $f_I^{(2)}$ and $f_{D1}^{(2)}$ can be obtained analytically. The free surface integral (8) requires some effort. We checked that its contribution is negligible in the practical application considered here.

According to Rainey, the horizontal force acting on the cylinder is given by

$$F_{\text{Rainey}} = \rho\,\pi\,a^2 \int_{-h}^{\eta_I} \left(2\,\dot{U} + U\,U_x + 2\,W\,U_z\right)\,dz \qquad (10)$$

where we have discarded the "point load" at the free surface intersection, of third-order in the wave amplitude. There is no point load at the bottom since the cylinder is standing on the sea-floor.

In this equation U and W are the horizontal and vertical components of the flow velocity associated with the incoming waves. In bichromatic seas the second-order force at the difference frequency can easily be derived. When the waterdepth is large at first-order ($k_1 h$ and $k_2 h > 3$), one gets

$$\begin{aligned} F_{-\text{Rainey}}^{(2)} &= \rho\,\pi\,a^2\,A_1\,A_2\,(k_1-k_2)\left[\frac{3}{2}\,\frac{\omega_1\,\omega_2}{k_1+k_2} - g\right]\sin(\omega_1-\omega_2)t \\ &\quad -2\,\rho\,\pi\,a^2\,A_1\,A_2\,(\omega_1-\omega_2)\,(k_1-k_2)\left[\int_{-h}^{0}\varphi_I^{(2)}(0,0,z)\,dz\right]\sin(\omega_1-\omega_2)t \qquad (11) \end{aligned}$$

It is easy to check that, when $|k_1-k_2|\,a \ll 1$ (in practice $|k_1-k_2|\,a < 0.5$), the second term in the expression above provides a fairly good approximation of $f_I^{(2)} + f_{D1}^{(2)}$.

Now we compare the first term in Rainey's expression (11) to the exact calculation of $f_Q^{(2)} + f_{WL}^{(2)}$ and to Newman's approximation (9). We take a waterdepth h equal to 20 times the radius a, and we vary $(k_1-k_2)/k_1$ from 0 to 0.5 for $k_1 a = 0.1$, 0.2 and 0.4. The results are shown in figures 1 through 3. The QTF's are made non-dimensional by division by $\rho\,g\,a$. It can be observed that, all over the range of $k_1 a$ and $k_2 a$ values, Newman agrees well with the real part of $f_Q^{(2)} + f_{WL}^{(2)}$, while Rainey agrees with the imaginary part: on this particular case of a standing cylinder Newman and Rainey are complementary. The intuitive, if not physical, interpretation, is that Newman takes care of what happens to the wave field at infinity, while Rainey, which is waveless, takes care of the local nonlinearities.

In figures 1 and 2 it can be noticed that the imaginary part of the QTF quickly becomes dominant as the difference frequency increases from zero. $f_I^{(2)}$ and $f_{D1}^{(2)}$ also contribute only to the imaginary part. This means that Newman's approximation alone provides a poor estimate of the QTF when the difference frequency is not very small.

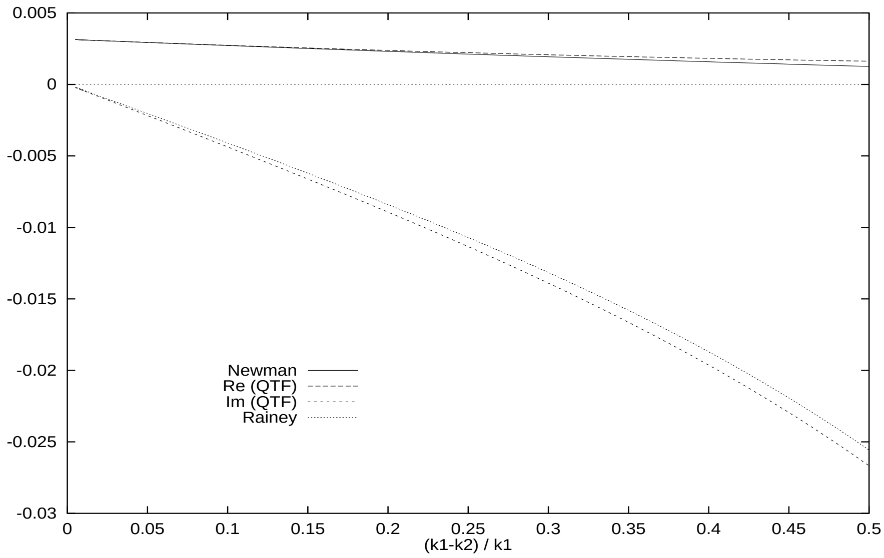

Figure 1: Exact and approximate values of $f_Q^{(2)} + f_{WL}^{(2)}$ for $k_1 a = 0.1$.

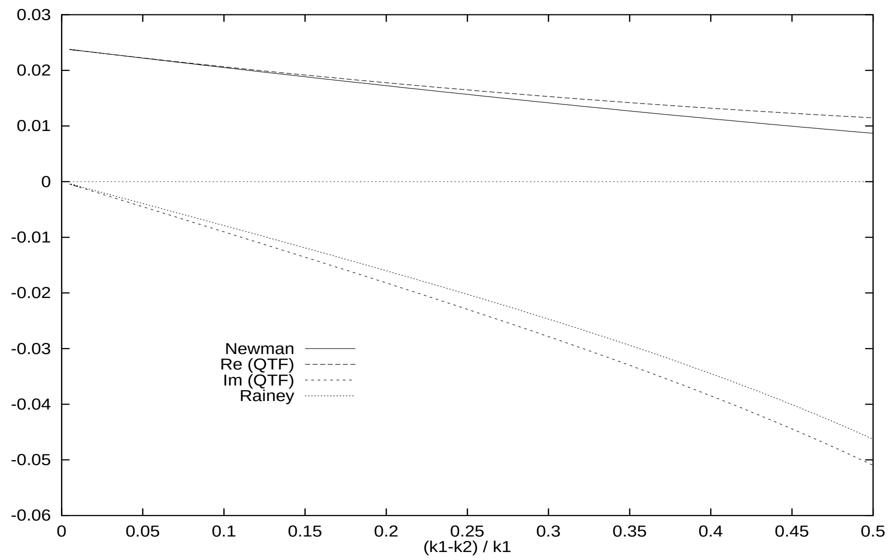

Figure 2: Exact and approximate values of $f_Q^{(2)} + f_{WL}^{(2)}$ for $k_1 a = 0.2$.

It may be wondered what is the degree of generality of the result that we have obtained. For instance what happens if the cylinder is free to move, or truncated, or completely submerged. And whether it also applies to the other components of the QTF's (in heave, pitch and roll).

The 2D case of a fixed horizontal cylinder, completely submerged, is easy to tackle. It was considered by Rainey (1992) in regular waves (see also Ogilvie, 1963). In regular waves, when $ka \ll 1$ and the cylinder is not too close to the free surface, one obtains that the vertical drift force is approximately given by

$$F_{dz} = 2\,\rho\,g\,\pi\,a^2\;k^2\;A^2\;e^{-2k\,d} \qquad (12)$$

(where d is the immersion) and that Rainey provides the same result. In this case Newman's approximation and Rainey's equations are redundant. The interpretation is that this vertical drift force has nothing to do with the wave-field at infinity. It is just related to a local mean vertical acceleration in the fluid kinematics (Rainey 1992).

This case suggests that, in combination with Rainey's equations, Newman's approximation should be applied only to that part of the drift force that one obtains through the far-field method.

Going back to the vertical cylinder, we can wonder whether this procedure would work when it is free to surge and pitch. A question that arises is whether Rainey's equations will yield any drift force in regular waves. The answer is no, provided the cylinder motion be in phase with the wave action: the radiation damping must be zero (which is fine under Rainey's low ka assumption) and the cylinder should extract no energy from the waves (meaning no other source of damping). Under this condition, like for the fixed cylinder, Rainey will contribute only to the imaginary part of the QTF.

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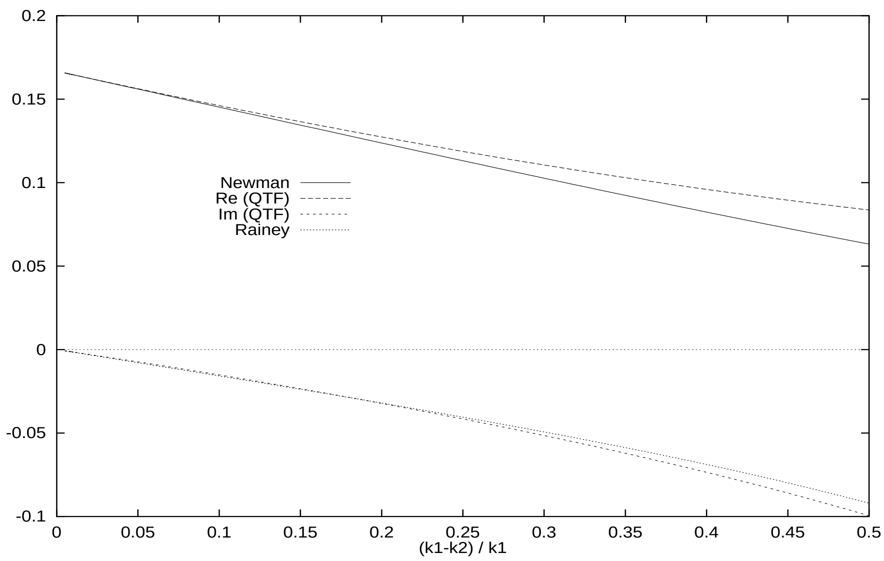

Figure 3: Exact and approximate values of $f_Q^{(2)} + f_{WL}^{(2)}$ for $k_1 a = 0.4$.

Discussion Sheet

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First Author :	Molin, B. and Chen, X-B.		
Discusser :	Rod C.T. Rainey		

Questions / Comments :

The 1989 JFM paper of mine that you referred to is correct (I believe), but difficult. I later realised how much simpler that result could be made - the simpler version is in Proc. Roy. Soc. A450 pp391-416.

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