INFLUENCE OF TRAPPED AIR ON THE WATER IMPACT ACTING ON A MASS AND SPRING SYSTEM

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SUMMARY

We describe a theoretical approach to a distorted plate penetrating calm water surface as a flow model of the water impact in rough seas. Further simplifications are employed that the structure of ship is modeled by a tandem mass and spring system and a sequence of circular hollows is used as a bottom shape of the body instead of the surface shape of short crested waves. The theory shows good agreement with a free fall experiment. Another result shows that the influence of the trapped air is big at the small-scale water impact.

1.INTRODUCTION

It is well known that the impact pressure due to the water impact of the flat plate is infinite in the Wagner's theory[1]. However, because of several cushioning effects, the impact pressure acting on a ship bottom in the rough sea is finite. The cushioning effects considered so far are the compressibility of the water, the air trapping, and the three-dimensional effect.

Another important issue is the elasticity of the hull and/or the surface plate. When the hull hits the surface of the water the energy of motion of the hull changes to the energy of the water and the strain energy of the structure. The strain energy by the water impact varies with the duration of impact, because the hull structure responds as a mass and spring system. When the water impact acts upon the hull, the hull structure responds as a mass and spring system with two degrees of freedom. There is no influence of the elasticity on the impact phenomena when the duration of impact approaches to zero. This is the case that the flat bottom hits the flat water-surface without air trapping and the response is regarded as an impulsive response. When the duration of impact is the order of the natural period of local structure, the strain energy decreases as the duration increases. This case can be seen in reality, since the water surface of the realistic sea is not flat and the air trapping is expected. However, the trapped air makes problem complicated, since these effects act as other mass and spring systems.

The estimation of the trapped air has another difficulty. When a body with very small dead-rise angle approaches to the flat water-surface, the air, which is pushed ahead of the body can't escape completely from the gap between the bottom and the water surface and, as a result, a cavity filled with the trapped air may be formed. However, it is often the case that the process of the cavity formation is unstable and depends very much on small variation of parameters such as attack angle, fine geometry of the bottom near the edge and so on. This fact makes theoretical and also experimental approach of the cushioning effect difficult. Thus, the total system is complicated and this complication makes understanding of the problem difficult.

However, it would be possible to employ some assumptions to make the problem simpler, if we observe the realistic sea. When a large ship experiences water impact, the condition of sea must be rough. High swells induce large ship motions and, as a result, slamming occurs at the bow of the ship. In this case, the surface of the sea is not smooth, since short crested wind-waves are added on the swells. It is apparent that this rough surface guarantees the cavity formation under the ship bottom in advance. In addition, the linearized formulation claims that the deformation of water surface is identical with the distortion of the bottom plate of a ship. This is a great advantage to solve the water impact problem in realistic sea, because we can use the combination of distorted body and flat water-surface instead of flat body and distorted water-surface.

In this paper, we describe a theoretical approach to a distorted plate penetrating calm water surface as a flow model of the above mentioned water impact in rough seas. In addition, we employ further simplifications that the structure of ship is modeled by a tandem mass and spring system and a sequence of circular hollows is used as a bottom shape of the body instead of the surface shape of short crested waves.

2.FORMULATION

The linear theory is based on the matched asymptotic expansion method and the perturbation parameter is preferred to be the ratio between the vertical scale of the distortion of the plate and the horizontal scale of it. Although the overall problem is composed of the outer problem and the inner problem, only the outer problem is considered here, because the impact force is not affected by the inner solution. We use the following non-dimensional space and time co-ordinate.

$$(x, y, z) = \left(\frac{X}{R}, \frac{Y}{R}, \frac{Z}{H}\right), \quad t = \frac{V_0 T}{H}, \tag{1}$$

where R is radios of the circular hollow, H is depth of the hollow and V_0 is initial drop-speed of the disk. The linearization is performed by assuming $H/R = \varepsilon << 1$.

The equation of motion of a tandem mass and spring system is represented as a system of linear differential equations.

$$m_1 \frac{d^2 z_1}{dt^2} = -k_1 z_1 + k_2(z_2 - z_1), \tag{2}$$

$$m_2 \frac{d^2 z_2}{dt^2} = f - k_2(z_2 - z_1). \tag{3}$$

This system is falling onto the water surface with a constant speed V_0. z_n denotes the vertical displacement of the mass m_n relative to the co-ordinate fixed on this system, and the positive sign means vertically upward. Where, all variables are non-dimensionalized as follows:

$$k_n = \frac{H^2 K_n}{\rho_w V_0^2 R^3}, \quad m_n = \frac{M_n}{\rho_w R^3}, \quad f = \frac{FH}{\rho_w V_0^2 R^3}. \tag{4}$$

K_n is the spring constant, M_n is the mass, ρ_w is the density of water and F is the impact force due to water pressure and trapped-air pressure.

We define the non-dimensional drop speed $v(t) = V(t)/V_0$ of the mass m_2 onto the water surface, which is initially calm. $v(t)$ has the following relation with z_2:

$$\frac{dz_2}{dt} = -v(t) + 1.0. \tag{5}$$

The mass m_2 is assumed to be a circular disk whose bottom surface $z = b(x, y)$ is distorted. In addition, it is assumed that the disk bottom has a circular hollow whose shape is represented by an elementary function for making the problem simpler.

$$b(x, y) = (x^2 + y^2 - 1)^2. \tag{6}$$

The impact force acting on the disk is obtained by solving an initial-boundary value problem, which is known as Wagner's theory. It is well known that this problem has a singularity at the inter section between the bottom surface and the water surface. Therefore, we employ the displacement potential, which has no singularity at the inter section. The displacement potential φ is defined as follows:

$$\varphi(x, y, z, t) = \int_0^t \phi(x, y, z, \tau) d\tau, \tag{7}$$

where ϕ is the velocity potential. The boundary conditions for the displacement potential are obtained by integrating Wagner's boundary conditions respect to time.

$$\nabla^2 \varphi = 0 \ \ for \ z < 0, \tag{8}$$

$$(\varphi + q(t)) \left(\frac{\partial \varphi}{\partial z} - \Xi(t) - b \right) = 0 \ \ on \ z = 0, \tag{9}$$

$$\varphi \leq -q(t), \ \ \frac{\partial \varphi}{\partial z} - \Xi(t) - b \leq 0 \ \ on \ z = 0. \tag{10}$$

The functions $q(t)$ and $\Xi(t)$ are defined as follows:

$$\begin{aligned} \frac{d^2 q}{dt^2} &= p(t) \quad for \ \ (x, y) \in \Omega_c, & (11) \\ q(t) &= 0 \qquad for \ \ (x, y) \in \Omega_f. & (12) \\ \frac{d\Xi}{dt} &= -v(t), \quad \Xi(0) = 0, & (13) \end{aligned}$$

where Ω_c denotes the projection of the cavity on the plane $z = 0$ and Ω_f denotes the projection of the outside free surface on the plane $z = 0$. Further assumption is that the air pressure in the cavity is taken in the Tate form.

$$\begin{aligned} p(t) &= \beta \left(\frac{1}{\nu(t)^n} - 1 \right), & (14) \\ \beta &= \frac{1}{n} \frac{\rho_a}{\rho_w} \left(\frac{c_0}{V_0} \right)^2 \frac{H}{R}, & (15) \end{aligned}$$

where n is a constant which is depend on the gas properties and in case of the air $n = 1.4$. The function $\nu(t)$ is the non-dimensional cavity volume, ρ_a is the density of air and c_0 is sound velocity of the air.

The initial-boundary value problem for the displacement potential is solved by means of the boundary element method in space and the Runge-Kutta method in time. It is apparent from the definition that the fluid pressure is obtained by differentiating the displacement potential twice respect to time. However, differential operation makes the numerical precision worse. Thus, a new scheme for estimation of the impact force is considered in which no differential operation is appeared, although one differential operation is required when the pressure distribution is needed.

Once we solve the initial-boundary problem for the displacement potential, the intersection between the body surface and the free surface is known. Thus, we define a boundary value problem for the velocity potential.

$$\begin{aligned} \nabla^2 \phi &= 0 & for \ \ z < 0, \quad & (16) \\ \phi &= -\frac{dq}{dt} & for \ \ (x, y) \in \Omega_c, \quad & (17) \\ \frac{\partial \phi}{\partial z} &= -v(t) & for \ \ (x, y) \in \Omega_b, \quad & (18) \\ \phi &= 0 & for \ \ (x, y) \in \Omega_f, \quad & (19) \end{aligned}$$

where Ω_b denotes the projection of the wetted part of the body on the plane $z = 0$. The impact force f is obtained by integrating the linear pressure distribution on the wetted body surface and the air pressure in the cavity.

$$f = -\int_{\Omega_b} \frac{\partial \phi}{\partial t} dS + \int_{\Omega_c} \beta \left(\frac{1}{\nu(t)^n} - 1 \right) dS \tag{20}$$

After some manipulation, the right-hand side of (20) is transformed.

$$f = -\frac{d}{dt} \left(\int_{\Omega_B} \phi dS \right) + \pi \frac{d}{dt} \left(a(t)^2 \frac{dq}{dt} \right), \tag{21}$$

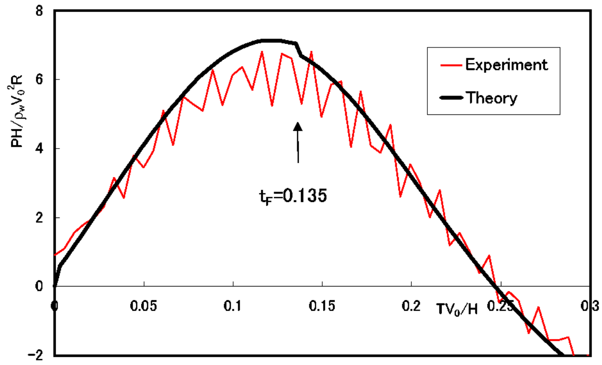

Figure 1: Comparison between theoretical and experimental result of the air pressure in the cavity:$\beta = 17.0$,$V_0 = 0.922m/s$, $R = 0.07m$ and $M = 3.38kg$

where $\sqrt{x^2+y^2} = a(t)$ is the intersection between the body surface and the free surface in the cavity. If we define the normalized velocity potential $\phi_0 = \phi/v(t)$ and substituting (21) into (3), $v(t)$ is obtained.

$$v(t) = \frac{m_2 + k_2 X(t) - \pi a(t)^2 \frac{dq(t)}{dt}}{m_2 - \int_{\Omega_b} \phi_0 dS}, \qquad (22)$$

where $X(t)$ has the following relation with z_1 and z_2.

$$\frac{dX}{dt} = z_2 - z_1, \quad X(0) = 0. \qquad (23)$$

The boundary value problem for the normalized velocity potential ϕ_0 is easily solved by means of the boundary element method.

3. PROBLEM OF THE WAGNER SCHEME

If you take $k_1 = k_2 = 0$ and add the gravity as an external force, the present formulation represents the free fall motion of the mass m_2. Fig.1 shows the comparison between theoretical and experimental result of the air pressure in the cavity. There is a small fluctuation on the time history of experimental data. This fluctuation is supposed to be due to vibration of the pressure gauge, since the natural period of pressure gage is almost as same as period of the fluctuation. Except for this point, the agreement between theory and experiment is good up to the time $t = t_F$. However, a problem happens at the time $t = t_F$ that the free surface in the cavity can't be attached at the intersection, then the Wagner scheme fails. The same situation has already been reported by Korobkin [2] in the case of two-dimensional water impact problem with attached cavity. Following his theory, we investigate the detail of this phenomenon.

If the drop speed of the body is a constant V_0, the non-dimensional elevation of the outer free surface is

$$Y(x,0,t) = \left[\frac{2}{\pi}\int_a^c y_b(\xi,t)\frac{\xi\sqrt{(c^2-\xi^2)(\xi^2-a^2)}}{\xi^2-x^2}d\xi - D\right]\left[(x^2-c^2)(x^2-a^2)\right]^{-1/2}, \qquad (24)$$

and of the inner free surface is

$$Y(x,0,t) = \left[-\frac{2}{\pi}\int_a^c y_b(\xi,t)\frac{\xi\sqrt{(c^2-\xi^2)(\xi^2-a^2)}}{\xi^2-x^2}d\xi + D\right]\left[(x^2-c^2)(x^2-a^2)\right]^{-1/2}, \qquad (25)$$

where $c(t)$ denotes the intersection point between the outer free surface and the body surface, and function $y_b(x,t) = (x^2-1)^2 - t$ denotes the instantaneous body shape. $D(t)$ is evaluated from the asymptotic form of vertical velocity far from the body. The vertical displacements of both the outer free surface and the inner one are bounded if

$$D + A^2(B-1) + A(B-1)^2 + \frac{A^3}{2} - At = 0, \qquad (26)$$

$$D + A^2(B-1) - A(B-1)^2 - \frac{A^3}{2} + At = 0, \qquad (27)$$

where $A = (c^2-a^2)/2$ and $B = (c^2+a^2)/2$. These two equation leads cubic equation of A^2:

$$A^6 - 2tA^4 + 2D^2 = 0 \qquad (28)$$

It is found that if $D > 4(t/3)^{3/2}$ A^2 has no positive root. This corresponds to the time $t = t_F$. After this instance, (26) and (27) are not able to satisfied simultaneously. Thus we must weaken the unbounded condition at the inter section point in the cavity $x = a$. It is noted that even if the elevation of the free surface is not bounded at $x = a$ estimation of the cavity volume is still possible, because the singularity at $x = a$ is proportional to $1/\sqrt{a-x}$. Provably, in this case, we need to consider new inner solution, which may satisfy the full nonlinear free surface condition. However, we don't discuss about this further.

Two artificial conditions are tested here, instead of the bounded condition at the intersection. The first one is rather simple that the intersection point $x = a$ is fixed after $t = t_F$. The second one is that the intersection point $x = a$ is determined so that the singularity at the intersection is minimized. It seems that the second condition is more reasonable than the first one. However, results show that the difference is not big. Thus, we decide that the first condition is used in the three-dimensional analysis.

4. SCALE EFFECT DUE TO TRAPPED AIR

It is very important to know the scale-effect between the tank test model and the full-scale ship. Thus, the investigation about the scale effect due to elasticity and the trapped air is focused on. We take

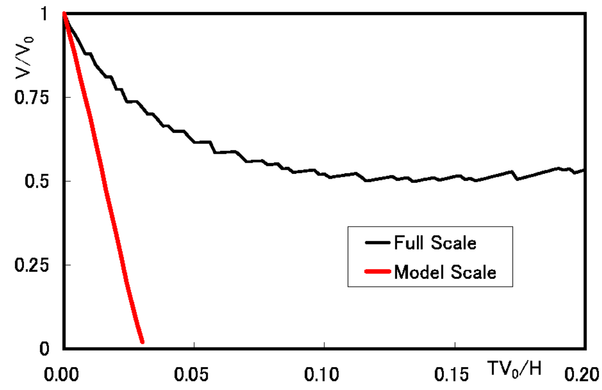

Figure 2: Drop speed of the mass m_2

	Full scale	Model scale
V_0	5.0 m/s	0.71 m/s
M_1	4.0×10^7 kg	320.0 kg
M_2	4.0×10^4 kg	0.32 kg
K_1	3.0×10^8 N/m	1.2×10^5 N/m
K_2	1.2×10^8 N/m	7.8×10^4 N/m.
T_1	2.30 sec.	0.325 sec.
T_2	0.115 sec.	0.0162 sec.
β	1.13	56.6

This corresponds to about 200m-long bulk-carrier in full scale and the model is 1/50 scale. The model values are according to Froude's law, and the spring constants are determined so that the natural period of the system in the air keeps Froude's law. T_1 and T_2 denote the natural period of the system. The non-dimensional natural period t_1 and t_2 are 8.20 and 0.409 respectively.

The drop speed of the mass m_2 is shown in Fig.2. The drop speed in the model scale is rapidly reduced after the impact compared with that in the full scale. The drop speed becomes zero at $t = 0.034$. Therefore, the curve of model scale mass-spring system is vanished at this time in all other figures. The impulse acting on the mass m_2 is shown in Fig.2. Where, the impulse I is

$$I = \int_0^t f dt = -\int_{\Omega_B} \phi dS + \pi a(t)^2 \frac{dq}{dt}. \qquad (29)$$

If you make $k_n = \infty$, the speed of mass would not be changed. This case is shown as 'Const. Speed' in the figure. It is apparent that the impulse in model scale is much bigger than that in the full scale. This makes speed reduction quicker. It is noted that this difference is due to trapped air. Interesting point is that the trapped air makes impulse larger. This means that the cushioning effect is obtained from the distortion of the water surface but the trapped air does not work as a cushion. The elasticity gives almost the same cushioning effect both in the full scale and the model scale. The same tendency is found in the time histories of the air pressure in the cavity, which are

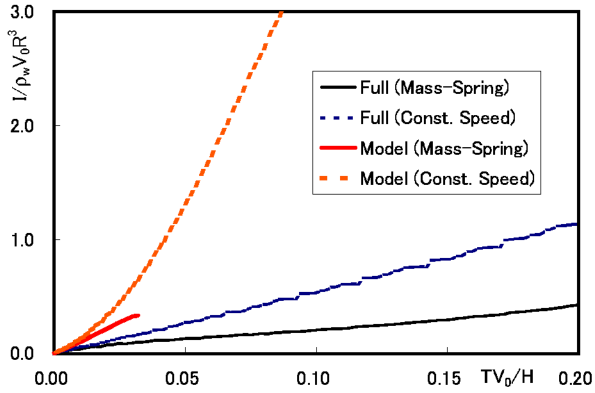

Figure 3: Impulse acting on the mass m_2.

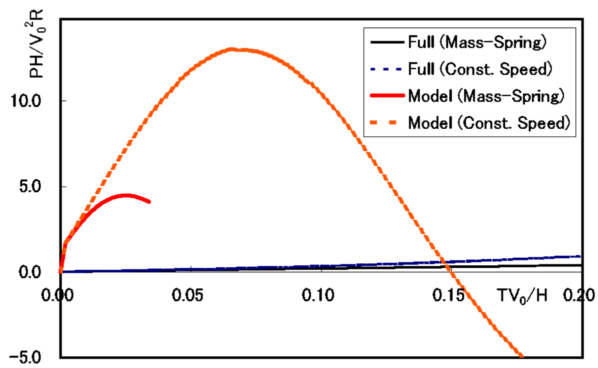

Figure 4: Air pressure in the cavity.

shown in Fig.4.

5. CONCLUSION

We try to describe the complicated ship-slamming problem by a simple mathematical model. In the case of free-fall motion, this model is in good agreement with an experimental result. Other results by this model show that the influence of the trapped air is big at the small-scale water impact.

6. REFERENCES

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