# The Influence of Viscosity on the Wavemaking of a Model Catamaran

**Lawrence J. Doctors**
The University of New South Wales, Sydney, NSW 2052, Australia
Email: `L.Doctors@UNSW.edu.au`

**Summary**

Turbulent (or eddy) viscosity is included here in the computation of the linearized wave pattern generated by a high-speed catamaran when travelling in water of finite depth and restricted width. The predictions are applied to the case of a 1.5 m long model towed in a model test basin. Four different depths, three different spacings between the demihulls and 13 different speeds are considered.

There is excellent correlation between the traditional inviscid predictions for the wave elevation and the experimental measurements, but only for Froude numbers typically above 0.35 in value and also not in the vicinity of the critical depth Froude number. Inclusion of the viscous effects improves the predictions at high Froude numbers and also provides much closer correlation between theory and experiment down to a Froude number of 0.2 in value.

## 1 Introduction

In the present investigation, we consider the matter of the influence of the viscosity of the water on the wave generation of a model vessel towed in a model test basin. It is, of course, well known that viscosity has been ignored in classical ship hydrodynamics.

Regarding the purely inviscid calculation of the wave elevation, one may consult the work of Tuck, Scullen, and Lazauskas (2000) and Doctors and Day (2000). In the current effort, we shall build upon the research published by Tuck, Scullen, and Lazauskas (2002), where viscosity was included for the case of deep water.

## 2 Mathematical Formulation

The coordinate system and principal parameters defining the problem are shown in Figure 1(a). The vessel has a waterline length $L$, a waterline demihull beam $B$, and a draft $T$. The spacing between the demihulls is $s$. The width of the channel is $w$ and the depth of the water is $d$. The acceleration due to gravity is $g$, and $U$ is the speed of the vessel.

Tuck, Scullen, and Lazauskas (2002) proposed the following free-surface condition for the potential $\phi$:

$$g\phi_z + U^2\phi_{xx} - 4\nu_t U\phi_{xzz} = 0 \,. \qquad (1)$$

Here, $\nu_t$ is the turbulent viscosity. The last term introduces an imaginary component to the wave number of the free-surface waves and results in a spatial damping factor which corresponds to the temporal damping factor derived by Bassett (1888), for waves travelling in water of finite depth, and by Lamb (1961), for waves travelling in water of infinite depth.

We now consider the potential for a finite-depth wave of the form:

$$\phi = \phi_0 \frac{\cosh[(k(z+d)]}{\cosh(kd)} \exp[\mathrm{i}k(x\cos\theta + y\sin\theta)] \,, \qquad (2)$$

where $k$ is the circular wave number and $\theta$ is the wave angle. This equation satisfies the Laplace equation as well as the kinematic condition on the bottom of the channel.

Substitution of Equation (2) into Equation (1) yields the viscous dispersion relationship:

$$k - k_0\sec^2\theta\tanh(kd) - \frac{4\nu_t\mathrm{i}}{U}k^2\sec\theta = 0 \,. \qquad (3)$$

We may now define the complex wave number

$$k^* = k + \mathrm{i}\delta \,, \qquad (4)$$

where $k$ is the standard (inviscid) solution of Equation (3). Substituting this definition into Equation (3) and keeping the leading-order terms in $\delta$ yields the imaginary component of the wave number:

$$\delta = \frac{4\nu_t}{U}k^2\sec\theta/[1 - k_0 d\sec^2\theta\,\mathrm{sech}^2(kd)] \,. \qquad (5)$$

In the case of deep water, $d \to \infty$, the result of Tuck, Scullen, and Lazauskas (2002) is recovered.

The viscous damping factor that is to be included for each component of the wave spectrum is then

$$V = \exp[-\delta(|x^*|\cos\theta + |y^*|\sin\theta)] \,. \qquad (6)$$

The offsets $x^*$ and $y^*$ are the distances from the source point (approximated by the center of buoyancy) to the field point.

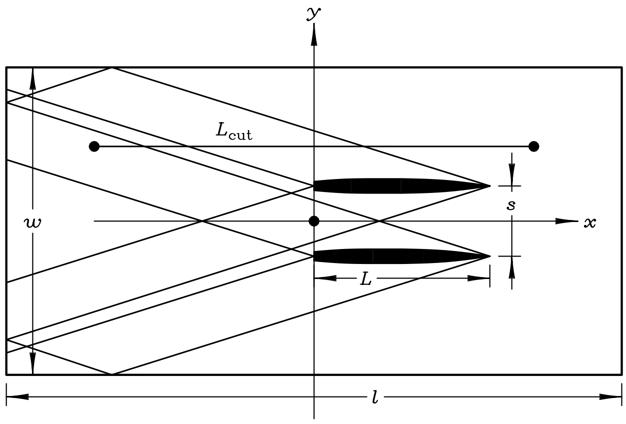


**Figure 1: Definition of the Problem**
**(a) Principal Features**

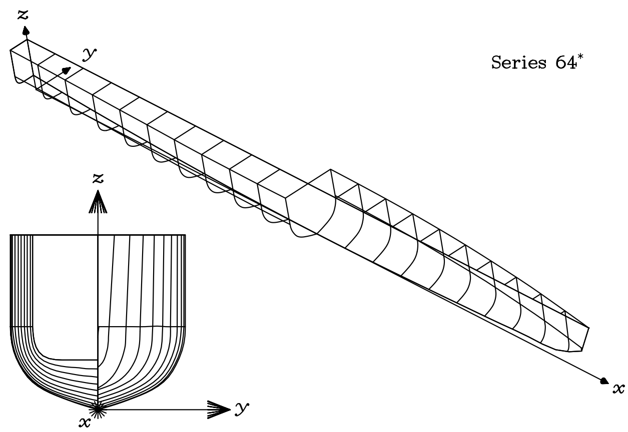


**Figure 1: Definition of the Problem**
**(b) Modified Series 64 Demihull**

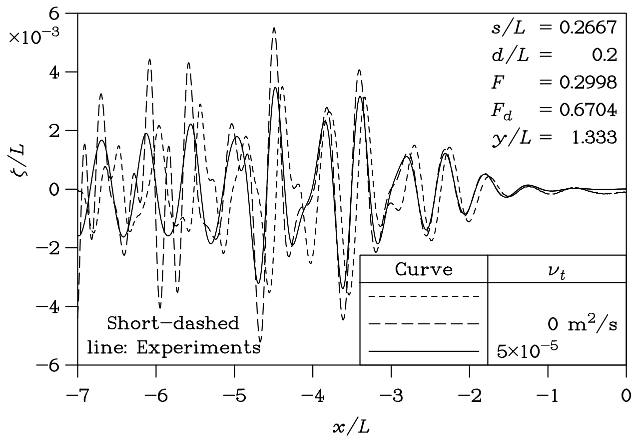


**Figure 2: Effect of Viscosity on Wave Profiles**
**(a)** $d/L = 0.2$ **and** $F = 0.2998$

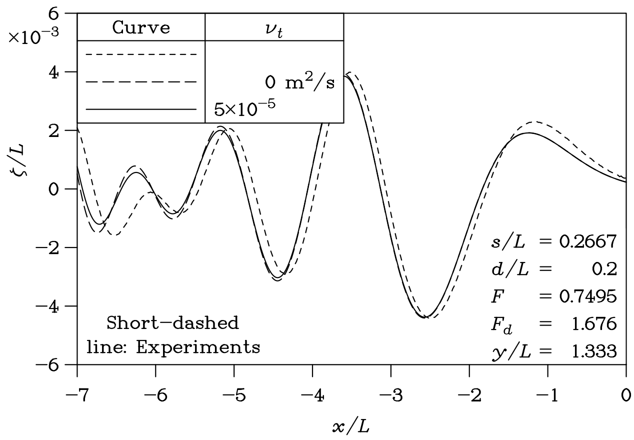


**Figure 2: Effect of Viscosity on Wave Profiles**
**(b)** $d/L = 0.2$ **and** $F = 0.7495$

## 3 Computer Program

Advantage was taken of the obvious spatial recursion relationship between two corresponding terms in the summation for the wave elevation, for two points in the wave field. In this way, considerable computational effort was saved.

Calculations were extended to a distance of 7.0 model lengths downstream of the transom stern, corresponding to the useful length of the longitudinal wave cuts, which could be obtained after the model had reasonably achieved a steady-state condition.

## 4 Model Vessel

A modified version of the Series 64 hull defined by Yeh (1965) was tested. Two such demihulls were constructed and the spacing between them could be set as required. The hull is characterized by its high-speed form with a transom stern. It is shown in Figure 1(b).

The waterline length of the model was 1.500 m and the three demihull spacings that were considered were 0.300 m, 0.400 m, and 0.500 m.

The tank width was 12.0 m and the four water depths employed were 0.300 m, 0.450 m, 0.600 m, and 0.900 m.

## 5 Influence of Viscosity on the Wave Elevation

Figure 2 presents a comparison of the experimental and theoretical wave elevation $\zeta$ for two values of the Froude number $F$. It can be seen that the traditional inviscid result $\nu_t = 0$ is quite acceptable for the higher Froude number and also for the first few waves at the lower Froude number. It can also be seen that choosing a turbulent viscosity $\nu_t = 5 \times 10^{-5}$ m$^2$/s considerably improves the theoretical prediction for the lower Froude number; this value of $\nu_t$ is less than the value suggested by Tuck, Scullen, and Lazauskas (2002) — for a larger model — by a factor of 4.0.

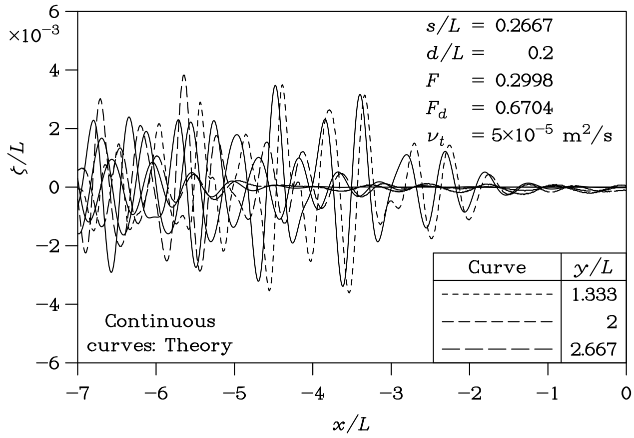


**Figure 3: Comparison of Wave Profiles (a) $d/L = 0.2$ and $F = 0.2998$**

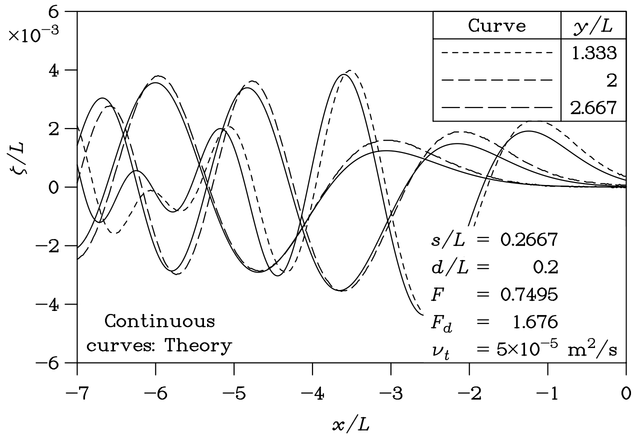


**Figure 3: Comparison of Wave Profiles (b) $d/L = 0.2$ and $F = 0.7495$**

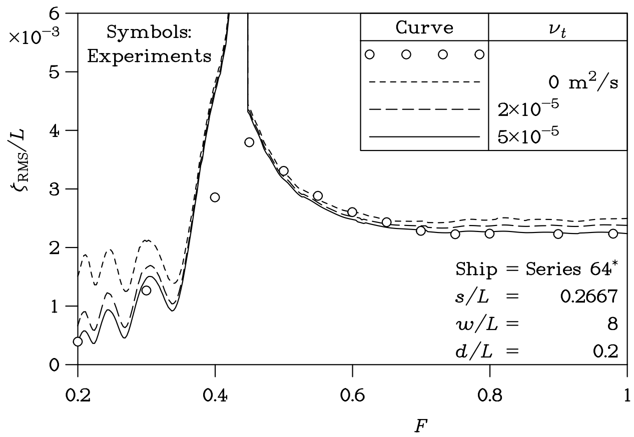


**Figure 4: Effect of Viscosity on RMS Wave Elevation (a) $d/L = 0.2$**

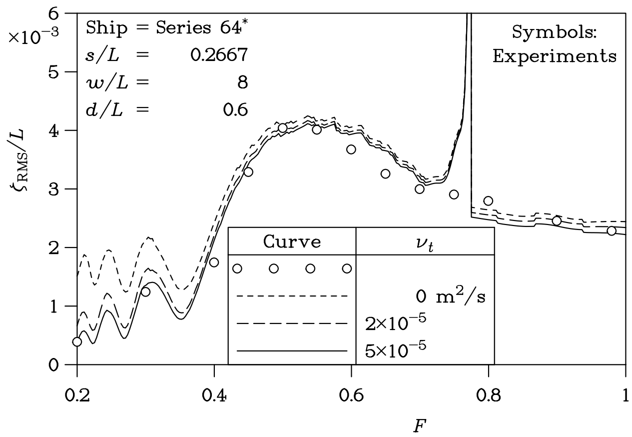


**Figure 4: Effect of Viscosity on RMS Wave Elevation (b) $d/L = 0.6$**

## 6 Comparison of Wave Profiles

The two parts of Figure 3 show a comparison of the experimental and viscosity-corrected theoretical curves, for three different longitudinal profiles and two Froude numbers. One can confirm again that there is acceptable correlation between theory and experiment.

## 7 Root-Mean-Square Wave Elevation

The root-mean-square wave elevation $\zeta_{\text{RMS}}$ over all eight longitudinal wave cuts is plotted in Figure 4 for the shallowest $d/L = 0.2$ and for the deepest $d/L = 0.6$ conditions. Use of viscous damping in the theory greatly improves the prediction at all Froude numbers. The well-known problematic situation at the critical speed can also be observed.

Next, Figure 5 illustrates the effect of experimenting with the transverse-wave term in the theory. The purpose of this exercise was to study the problem of performing such tests in which the tank length is severely limited. In such cases, one should expect to encounter persistent unsteady effects at the critical speed. It can be observed that ignoring this term in shallow water, on the one hand, or assuming an increased tank width in deep water, on the other hand, greatly improves the correlation between the experiments and the theory.

Finally, Figure 6 presents the effect of demihull spacing on the RMS wave elevation. It is seen that increasing the spacing results in a lower wave generation.

## 8 Conclusions

This work has demonstrated that the inclusion of turbulent viscosity in the theory greatly enhances the accuracy of the predictions. Further work should involve a study of the appropriate turbulent viscosity at different vessel sizes. Such research could be done using larger models and full-scale vessels.

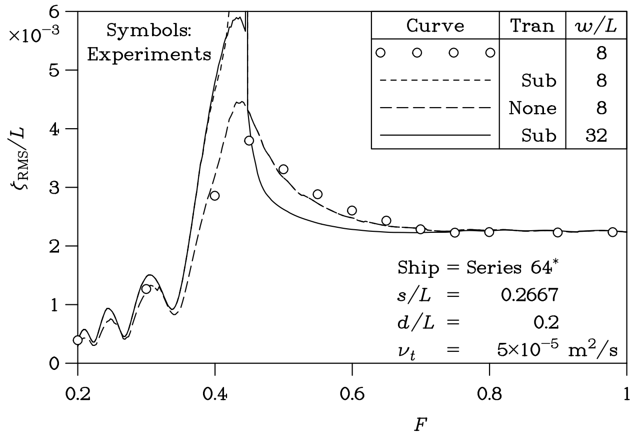


**Figure 5: Effect of Transverse Wave on RMS Wave Elevation (a) $d/L = 0.2$**

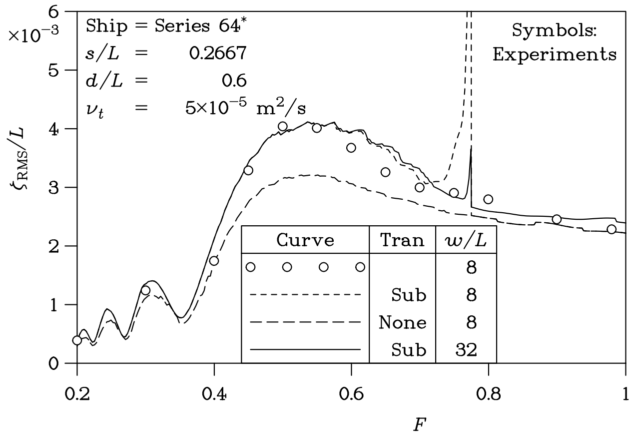


**Figure 5: Effect of Transverse Wave on RMS Wave Elevation (b) $d/L = 0.6$**

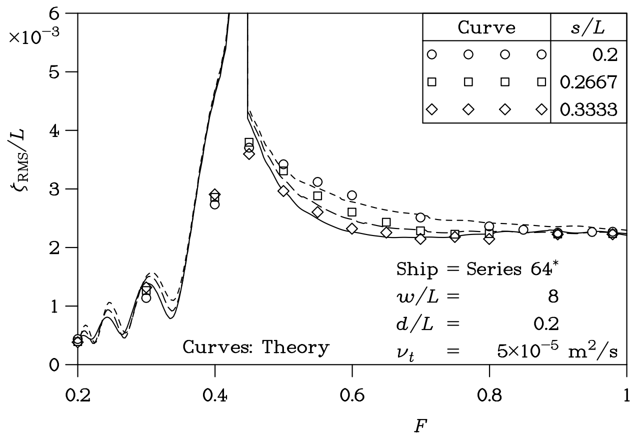


**Figure 6: Effect of Demihull Separation on RMS Wave Elevation (a) $d/L = 0.2$**

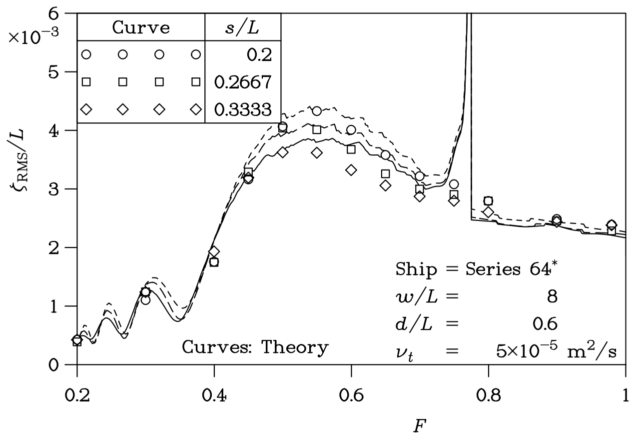


**Figure 6: Effect of Demihull Separation on RMS Wave Elevation (b) $d/L = 0.6$**

## 9 Acknowledgments

The tests were performed in the Model Test Basin at the Australian Maritime College (AMC) by Mr Nigel Lynch, student at UNSW, under the able supervision of Mr Gregor Macfarlane of the AMC. Professor Ernie Tuck of The University of Adelaide provided much useful theoretical advice.

The author gratefully acknowledge the assistance of the Australian Research Council (ARC) Discovery-Projects Grant Scheme (via Grant Number DP0209656). He is also appreciative to The University of New South Wales for infrastructure support.

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**Question by** : X.B. Chen
I'd like to understand the additional term in (1) representing the viscous effect. In the classical studies, that is written like $-2\upsilon U w_{xz}$ (with $\upsilon$ kinematic viscous coefficient and w the vertical fluid velocity). Your $-4\upsilon_t U\phi_{xzz}$ is supposed to include much more effect from other dissipating effects, is not it?

**Author's reply**:
The presence of this extra( viscous) term in the otherwise inviscid linearized free-surface condition has been derived by a number previous researchers, including those references in the paper. For example it can be shown that if the viscous effect is weak, then the entire influence is felt only as a boundary layer at the free-surface, rather than throughout the fluid domain. Hence, curiously, the (inviscid) Laplace equation is still applicable.

The discussion by Tuck, Scullen and Lazauskas (2002) suggested that the additional factor of 2 can be explained by the difference between the phase velocity and the group velocity.

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**Question by** : T. Miloh
The linearized free-surface boundary condition including a correction for the dynamic viscosity has been known long before Tuck et al. (2002). As to Dr. Chen's question why the $\phi_{xzz}$ term is proportional to $4\upsilon$ instead of $2\upsilon$, I would like to draw your attention to two papers one by Ruvinsky & Friedman, and the other by Longuet-Higgins (both JFM late 70's or early 80's) in which this viscous term is specifically derived by considering a shear layer on the free-surface and applying the boundary condition over the displaced surface.

**Author's reply**:
I would like to thank Prof. Miloh for the additional references in the J. Fluid Mechanics which should shed further light on the question by Dr. Chen.

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**Question by** : M. Tulin
A very interesting paper. In comparing measurements with linearized predictions how can you distinguish between the effect of finite beam (i.e non-linear wave resistance effects) at low Froude number with the viscous propagation effect?

**Comment by** : M. Tulin
In this connection, it would be interesting to have more closely spaced measurements at the lower Froude numbers in order to see if the predicted crests and troughs in the resistance curve actually exists.

**Author's reply**:
Prof. Tulin's question is a very probing one. In general, of course, one can never be absolutely certain when adding a particular refinement to a theory that the refinement is indeed the explanation of any improved correlation between theory and experiment. In the present case, the viscous effect eliminates the high frequency waves at both low and high Froude numbers

resulting in better agreement in these two extreme limits; this is a strong indication that the physics of the problem are better represented in this manner.

It is true that the nonlinear finite-beam effects are ignored in the theory and that they might also possibly explain the difference between the classic inviscid linear theory and the experiments. Interestingly, it seems possible that the current viscous effects in the theory might eliminate the unrealistic linear high-frequency waves and thus might provide an engineering model for improving the predictions.

I agree with the suggestion of obtaining more data points at low Froude numbers and these tests are currently in the planning stage.