APPROXIMATE MODELS OF ELASTIC WEDGE IMPACT

T.I. Khabakhpasheva, A.A. Korobkin
Lavrentyev Institute of Hydrodynamics, Novosibirsk, Russia
E-mail: Tana@hydro.nsc.ru, KAA@hydro.nsc.ru

Summary

Analysis is presented for the impact of a wedge with elastic side walls dropped against liquid free surface. The deadrise angle of the initially undeformed wedge is small. The problem is considered within the Wagner theory [1]. The liquid flow is two-dimensional, symmetric and potential. The side walls of the wedge are modelled by simply supported Euler beams. Hydroelastic behavior of the beams is of main interest. In order to evaluate the maximum bending stresses, not only the impact stage but also the penetration stage, during which the wedge surface is totally wetted and the wedge platings continue to vibrate, are considered. Different approximate approaches are tested against the numerical solution obtained by normal mode method. It is shown that for thin platings the maximum bending stresses occur at the penetration stage but for the thick plating they can be evaluated from the solution of the problem at the impact stage. Approximate models are concern with the case of thick platings, when the wedge can be treated as almost rigid. It is demonstrated that the simplified decoupled model, which does not account for the beam inertia and assumes the hydrodynamic loads being calculated within the rigid-wedge impact problem, provides reasonable estimates of the maximum bending stresses for thick wedge platings.

Introduction

The plane unsteady problem of an elastic wedge penetrating an ideal and incompressible liquid is considered. Initially ($t' = 0$) the wedge touches the horizontal free surface of the liquid at a single point and starts to move down thereafter with a constant velocity V. The initial contact point is taken as the origin of the Cartesian coordinate system $x'Oy'$ (dimensional variables are denoted by a prime). The line $y' = 0$ corresponds to the liquid free surface at $t' = 0$. The flow caused by the wedge impact is symmetric with respect to the line $x' = 0$. Initial position of the wedge side walls is described by the equation $y' = |x'| \tan\gamma$, $|x'| < L\cos\gamma$, where γ is the deadrise angle of the equivalent rigid wedge and L is the length of the side walls. The side walls of the wedge are modelled by simply supported Euler beams. Due to the symmetry, the right side wall can only be considered. Normal deflection of the beam is denoted by $w'(s', t')$, where s' is the coordinate along the initially undeformed side walls, $s' = 0$ corresponds to the wedge tip and $s' = L$ to the beam end point. The beams are deforming owing to their interaction with the liquid.

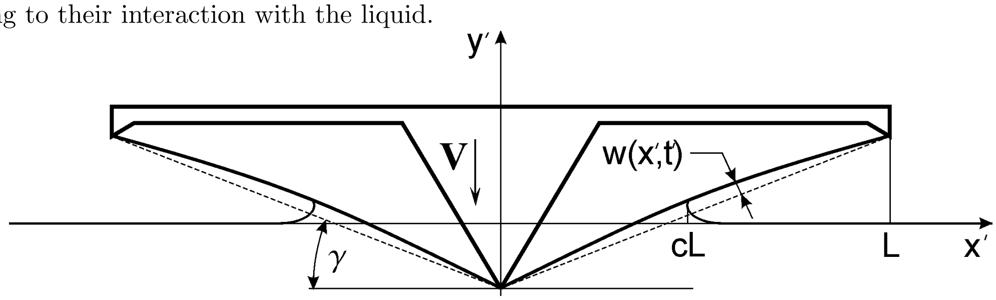

Figure 1. Scheme of the impact.

We shall determine the liquid flow, the pressure distribution in the liquid region, deflection of the wedge, the stress distribution in the wedge platings and the dimension of the wetted part of the entering wedge.

Non-dimensional variables are used below. The beam length L is taken as the length scale and the impact velocity V as the velocity scale of liquid particles. Assuming that the wedge is rigid and the free surface is undeformed during the penetration, we obtain that the wedge is totally wetted at instant $T = (L/V)\sin\gamma$ and the vertical displacement of the wedge is equal to $L\sin\gamma$ at this time instant. The quantity T is taken as the time scale and the product $L\sin\gamma$ as the displacement scale. We denote $\epsilon = \sin\gamma$ and consider the coupled problem of elastic wedge interaction with the liquid in the case $\epsilon \ll 1$. The pressure scale is $\rho V^2 \epsilon^{-1}$, where ρ is the liquid density.

Formulation of the Wagner problem

The plane and potential flow generated by the wedge penetration is described by the velocity potential $\varphi(x, y, t)$ which satisfies the following equations within the Wagner theory

$$\Delta\varphi = 0 \qquad (y < 0), \tag{1}$$

$$\varphi = 0 \qquad (y = 0, |x| > c(t)), \tag{2}$$

$$\varphi_y = -1 + w_t(|x|, t) \qquad (y = 0,\ |x| < c(t)), \tag{3}$$

$$p(x, y, t) = -\varphi_t(x, y, t) \qquad (y \le 0), \tag{4}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty). \tag{5}$$

Equations (1)-(5) represent the hydrodynamic part of the problem, which can be solved once the deflection $w(x, t)$ and the function $c(t)$, which describes the dimension of the contact region, are known. The beam deflection is governed in the non-dimensional variables by equations

$$\alpha \frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial^4 w}{\partial x^4} = p(x, 0, t) \qquad (0 < x < 1,\ t > 0), \tag{6}$$

$$w = w_{xx} = 0 \ \ (x = 0,\ x = 1,\ t \ge 0), \tag{7}$$

$$w = w_t = 0 \ \ (0 < x < 1,\ t = 0). \tag{8}$$

These equations represent the structural part of the problem, which can be readily solved once the hydrodynamic pressure $p(x, 0, t)$ is given along the wetted part of the wedge surface. Here $\alpha = (\rho_b h)/(\rho L)$, $\beta = EJ\epsilon^2/(\rho L^3 V^2)$, ρ_b is the density of the beam, h is the beam thickness, E is the elasticity modulus, J is the inertia momentum of the beam cross section, $J = h^3/12$ for the beam of constant thickness.

The boundary-value problem (1) - (5) is considered under the additional condition that the elevation of the free surface is equal to the vertical position of the deformed wedge at the intersection points. The Wagner condition can be reduced to the equation [2]

$$\int_0^{\frac{\pi}{2}} y_b[c(t) \sin\theta, t] d\theta = 0. \tag{9}$$

The bending stress distribution on the upper side of the beam $\sigma(s, t)$ is given in the dimensionless variables as $\sigma(x, t) = w_{xx}(x, t)/2$ with $2\epsilon Eh/L$ being the stress scale.

The hydrodynamic part (1) - (5), the structural part (6) - (8) and the geometrical part (9) of the Wagner problem are closely connected to each other and have to be treated simultaneously in general case. For almost rigid wedge, when the parameter β in (6) is large in some sense, which has to be clarified, the structural part of the problem can be approximately separated and treated after the solution of the hydrodynamic problem (1) - (5) and (9) has been obtained. In order to study applicability of this approximate solution and others available, one need to solve accurately the Wagner problem (1) - (9). This can be done with the help of the normal mode method.

Normal mode method

Within this method the beam deflection $w(x, t)$ is sought in the form

$$w(x, t) = \sum_{n=1}^{\infty} a_n(t) \psi_n(x), \tag{10}$$

where eigenfunctions $\psi_n(x)$ are given as

$$\psi_n(x) = \sin(\lambda_n x), \quad \lambda_n = \pi n \ \ (n = 1, 2, ..) \tag{11}$$

and represent the eigen modes of simply supported beam - so called 'dry' modes. It is convenient to take the principal coordinates $a_n(t)$ of the beam deflection as the new unknown functions and to express other quantities with their help [3,4].

As a result, we arrive at the infinite system of ordinary differential equations with respect to the principal coordinates $\vec{a} = (a_1, a_2, a_3, ...)^T$ and auxiliary vector-function $\vec{v} = (v_1, v_2, v_3, ...)^T$:

$$\frac{d\vec{a}}{dt} = (\alpha I + S)^{-1}(\beta D \vec{v} + \vec{f}), \tag{12}$$

$$\frac{d\vec{v}}{dt} = -\vec{a}, \tag{13}$$

where I is the unit matrix and D is the diagonal matrix, $D = \mathrm{diag}\{\lambda_1^4, \lambda_2^4, \lambda_3^4, ...\}$,

$$f_m(c) = \int_{-c}^{c} \sqrt{c^2 - x^2} \psi_m(|x|)\, dx, \qquad S_{nm}(c) = \int_{-c}^{c} \varphi_n(x, 0, c) \psi_m(|x|)\, dx.$$

By differentiating equation (9) with respect to time, we obtain

$$\frac{dc}{dt} = Q(\vec{a}, \vec{v}, c). \tag{14}$$

The initial conditions for the system of ordinary differential equations (12)-(14) are

$$\vec{a} = 0, \quad \vec{v} = 0, \quad c = 0 \quad (t = 0). \tag{15}$$

The initial-value problem t(12)-(15) is suitable for numerical simulations of hydroelastic behaviour of elastic wedge entering liquid. An efficient algorithm to compute the functions $S_{nm}(c)$ and $f_m(c)$, where $0 \le c \le 1$, has been developed in [5].

Approximate models

Equations (12)-(15) are referred to as the "full model" of the elastic wedge impact problem. Below the results obtained within the full model are used for testing several approximate models. Numerical results obtained within the full model are referred as exact ones. Three approximate models are considered:

A1. Within this model the added-mass matrix $S(c)$, which appears in equation (12), is evaluated with the help of approximate formula $S(c) = c^2 S(1)$. This approximation highly reduces the CPU time but is still in a good agreement with exact solution for all values of the parameters α and β.

A2. In the case of almost rigid wedge the corresponding decoupled problem, within which the hydrodynamic pressure and the dimension of the contact region are determined without account for elastic deflections of the wedge platings, can also be reduced to system (12),(13) with $S \equiv 0$ and $c = \frac{\pi}{2}t$. This model can be used only at the impact stage. At the penetration stage $S = S(1)$.

A3. Quasi-static model is derived from A2 in the limit $\alpha \to 0$. At the impact stage equation (6) gives

$$\beta w^{\mathrm{IV}} = p(x, 0, t) = \begin{cases} c\dot{c}(c^2 - x^2)^{-1/2} & (0 < x < c) \\ 0 & (c < x < 1) \end{cases} \tag{16}$$

and (7) provides

$$w = w'' = 0 \qquad (x = 0, 1). \tag{17}$$

Note that the hydrodynamic pressure distribution is taken from the solution of rigid-wedge entry problem, where $c = \frac{\pi}{2}t$.

Solution of the boundary-value problem (16), (17) was obtained in analytical form. At each time instant during the impact stage the maximum of non-dimensional bending stresses is

$$w''_{max}(c) = \max_{0<x<c} w''(x) = \frac{\pi c^2}{\beta} \sin^2\left(\frac{c}{2} - \frac{\pi}{4}\right).$$

The maximum occur at the point $x_{max} = c \cos c$, which is well behind the jet root region.

The absolute maximum of bending stresses occur at $c_* \approx 0.8$ and is given in microstrains as

$$\sigma_{max} = K 10^6 \frac{\varepsilon h}{2L\beta}, \qquad \text{where} \qquad K = \frac{\pi c_*^4}{(c_*^4 + 4)} = 0.2842. \tag{18}$$

Numerical results

The initial-value problems for the full model and the approximate models A1 and A2 are solved numerically by the Runge-Kutta method. In all cases $S = S(1)$ at the penetration stage.

Calculations were performed for impact of a wedge with the deadrise angle of 10^o. The wedge plating length is 0.5m and the beam thickness changes from 0.6cm to 3cm. The platings are made of mild steel with ρ_b=7850kgm^{-3} and E=21$\cdot$10^{10}Nm^{-2}, the impact velocity is 4ms^{-1}. Scaling is identical for any model in use. In transforming the measured strains to stresses, 1000 microstrains correspond to a stress level of $210N/mm^2$.

Maximum bending stresses (in microstrains) calculated with the help of the full model are presented in Fig.2 as a function of non-dimensional time. Upper line corresponds to $h = 0.6cm$, middle line is for $h = 1.2cm$ and the lower line is for $h = 2.5cm$. The vertical lines separate impact stage from the penetration stage.

Absolute maximum of the bending stresses as function of the beam thickness is shown in Fig.3 against the maximum stresses at the impact stage. It is seen that only the impact stage can be considered for estimating the stress maximum when $h > 1.5cm$ and a reasonable estimation can be obtained for $h > 1.2cm$. Dotted line presents maximum stresses obtained with the help of the model A3.

Fig.4 shows the maximum bending stresses at the impact stage as function of the beam thickness h (in meters). It is seen that the model A1 predicts the maximum bending stresses quite well. The decoupled model A2 can be used for $h > 1.2cm$. Surprisingly, the quasi-static model perform better then A2 and can be recommended for $h > 8mm$.

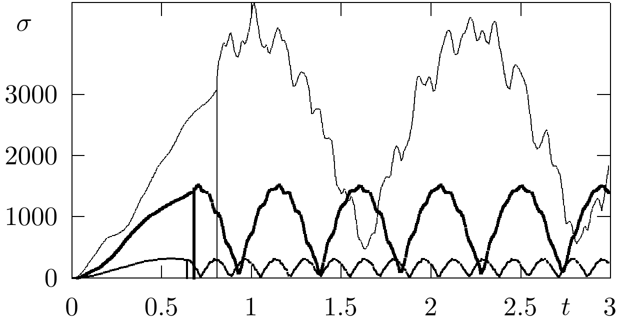

Figure 2. Maximum of stresses ($h = 0.6cm$, $h = 1.2cm$, $h = 2.5cm$).

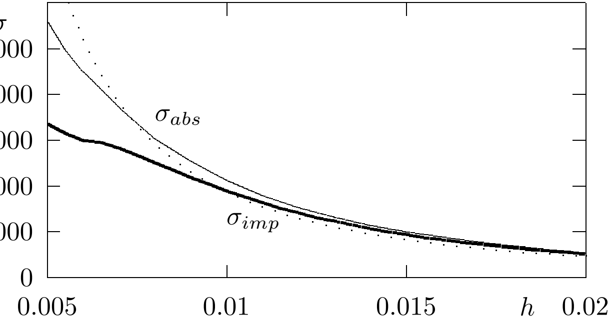

Figure 3. Maximum stresses at the impact stage (σ_{imp}) and absolute maximum of stresses (σ_{abs}).

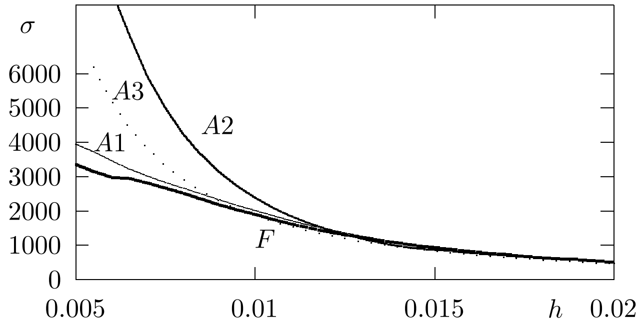

Figure 4. Maximum of stresses at the impact stage obtained by different approximate models.

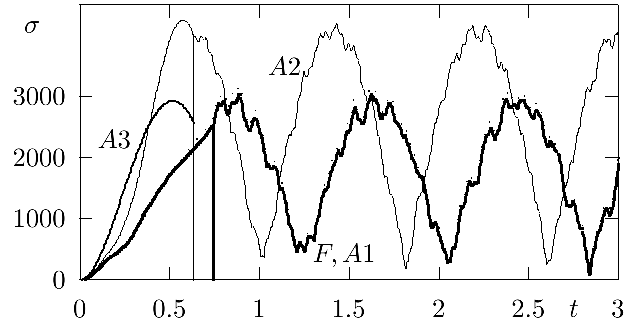

Figure 5. Maximum of stresses for $h = 0.8cm$.

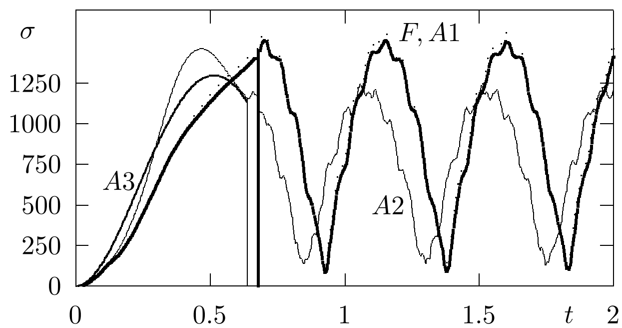

Figure 6. Maximum of stresses for $h = 1.2cm$.

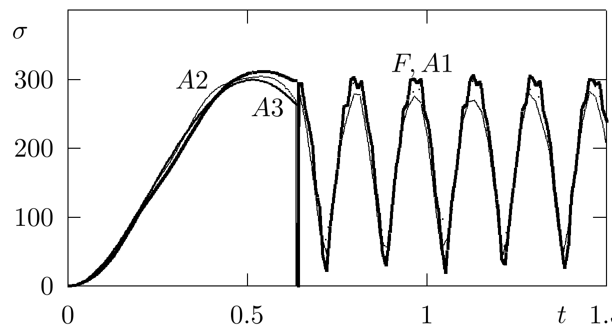

Figure 7. Maximum of stresses for $h = 2.5cm$.

Evolutions of maximum bending stresses calculated within different models are shown in Figures 5-7 for different beam thickness. We may conclude that the model A1 can be used instead of more time-consuming full model (F) almost for any impact conditions. The analytical model A3 correctly describes the stress evolution for thick platings.

Conclusion

Impact of elastic plates onto liquid free surface has been intensively studying within the hydroelasticity theory during last decade. The present analysis has the goal to clarify limits, beyond which simplified models provide reasonable estimates of maximum bending stresses in the impacting body. The analytical model A3, which is of common use at preliminary design stage, gives surprisingly acceptable estimation of absolute maximum of bending stresses. New simplified model A1 was derived, which may substitute the full model for any impact conditions.

References

1. Wagner, H. Über Stoss- und Gleitvergänge an der Oberfläche von Flüssigkeiten. *Z. Angew. Mach. Mech.* Vol.12. H.4. 1932. S.193-235.

2. Korobkin, A.A. Water impact problems in ship hydrodynamics. *Advances in Marine (Hydrodynamics), ed. M. Ohkusu.* Southampton. Computational Mech. Publ. 1996. pp.323-371.

3. Korobkin A. A. Wave impact on the centre of an Euler beam. *J.Appl.Mech.Tech.Phys.* 1998. V.39. N 5. P. 134-147.

4. Korobkin A.A., Khabakhpasheva T.I. Periodic wave impact onto an elastic pate. *Proceedings Seventh International Conf. on Numerical Ship Hydrodynamics.* Nantes, France 19-22 July 1999. P.(7.3-1)-(7.3-19).

5. Korobkin A.A. Elastic wedge impact. Lecture Notes. INSEAN. 2000.

Question by : T. Miloh

Is it (easily) possible to extend this 2nd solution to a 3rd axisymmetric cone, and if not, where do you expect to encounter the main difficulties?

Basically the parameter b is much larger than a. I believe that it is interesting to also investigate the limit of $b = O(a)$ where both $(a, b) \neq 0$ or ∞. Is there a boundary layer type behaviour at the edge?

Author's reply:

The axisymmetric cone problem will be considered in the next presentation by I.M. Scolan.

We did not considered the limit $(a, b) \to \infty$. On the other hand, you can see in my slide that solutions for $a \neq 0$ and $a = 0$ are close to the rigid plates walls. $\left(a = O(h), b = O(h^3)\right)$. We considered applicability domains for models without inertial terms $(a = 0)$ for the case of the floating plate and we found that a can be equal to 0, if $a << 1$, but it is not the case $a << b$. I believe that in this case we shall obtain the same results. In the future work, we will present plane (a, b) with applicability domains for various approximate approaches.
