Third-order interactions and wave run-up

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In 2001, within the scope of a French research project on the roll motion of barges, some model tests were conducted in the facilities of Cehipar (in El Pardo, Spain). A large rectangular barge model (length: 5 m, width: 1.2 m, draft: 0.24 m) was submitted to regular and irregular beam waves. A quite noticeable, and somewhat unexpected, phenomenon was observed: a large run-up occurring right in the middle of the hull, on the weather side. As a matter of fact, the open-deck model nearly sank, because of water over-flowing, and the test program had to be revised to milder wave conditions. In these tests the wave periods covered a narrow range around the roll natural period, at 1.6 s.

Run-up calculations were subsequently performed with a 3D linear diffraction-radiation code. They showed nothing like had been observed during the experiments.

More recently, another series of tests was performed at the wave-tank BGO-First in la Seyne sur mer, first on a model homothetic to the Cehipar model (at a scale 1:2), then on a shortened version. Similar effects were observed.

Subsequently the barge model was replaced with a rigid vertical plate, attached and perpendicular to one of the side-walls. The plate extended all throughout the water-depth (3 m) and had a width of 1.2 m. By geometric symmetry this is equivalent to a 2.4 m wide plate in the middle of a 32 m wide tank. The distance from the plate to the wavemaker was about 20 m. The instrumentation consisted in capacitive wave gauges along the plate, and at different locations in the basin. Tests were done in regular waves with wavelengths ranging from 1.2 up to 3 m, and steepnesses H/L ranging from 2 to 6 %.

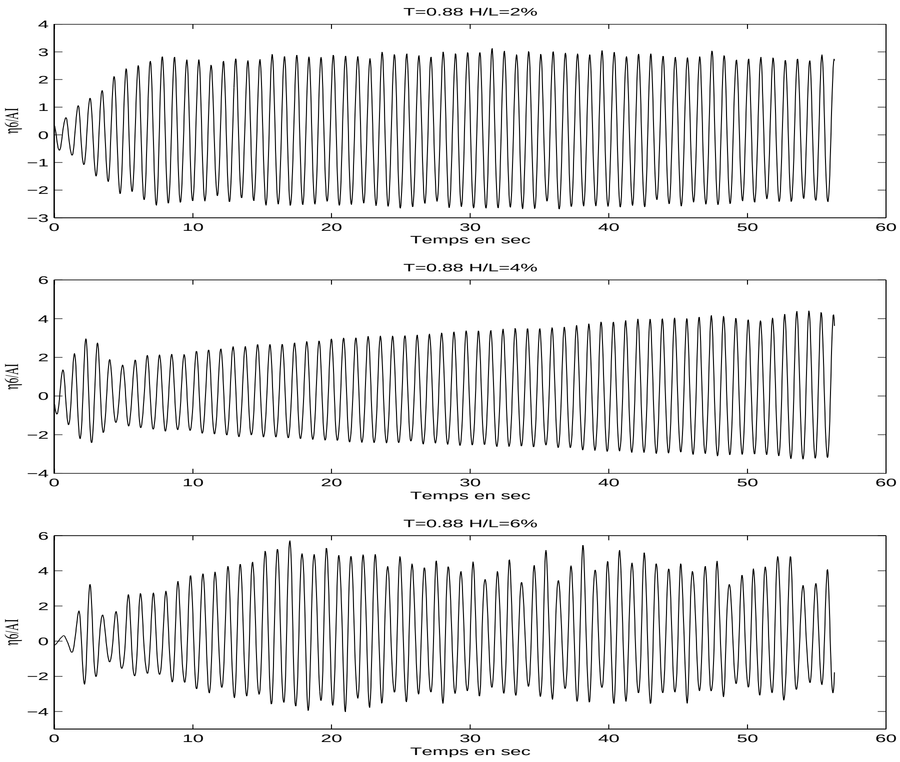

Figure 1: Time series of the free surface elevation at the plate.

Figure 1 shows typical time series of the free surface elevation measured at the corner in-between the plate and the wall. They refer to a wave period of 0.88 s (a wavelength of 1.2 m) and steepnesses of 2, 4 and 6 %. The free surface elevations have been normalized by the amplitude of the incoming waves. They are shown from the time the wave front meets the plate, until the time waves reflected by the plate and re-reflected by the wavemaker arrive at the plate again. According to linear theory, at this wave period, the RAO of the free surface motion at this location is about 1.7, this means the time series should oscillate, roughly, in-between -2

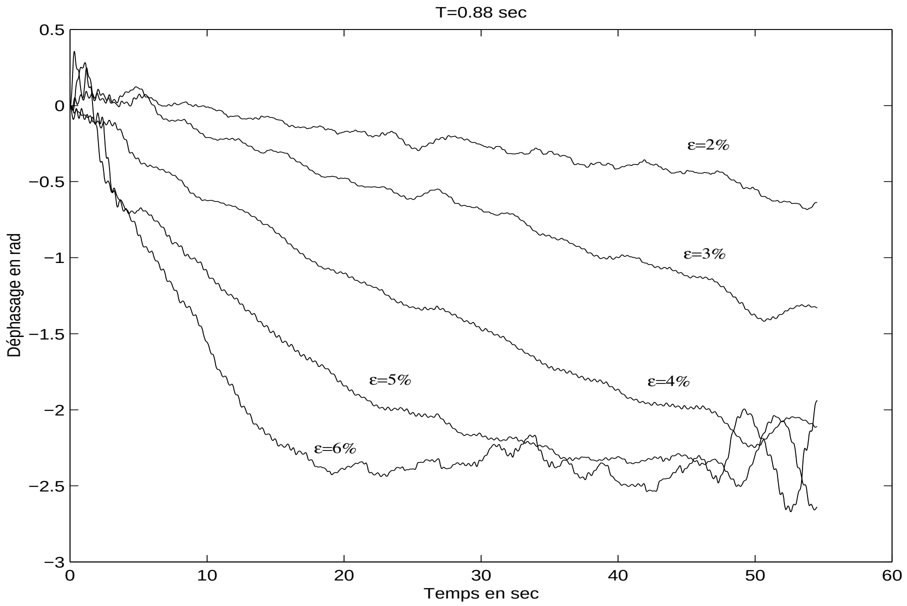

Figure 2: Phase lags vs time.

and +2 (accounting for some harmonic contributions). It can be observed that much larger values are attained as the wave steepness increases, exceeding 5 at $H/L = 6$ %. A striking feature of the records at $H/L = 4$ % and $H/L = 6$ % is the long duration of the transient part of the time series. Actually it looks like no steady state is being reached.

Figure 2 complements figure 1 by showing the phase difference between the free surface elevation at the plate and the incident wave elevation, as measured by the opposite wall of the tank. It is given for different wave steepnesses ranging from 2 to 6 %. The phase lag was obtained through Fourier analysis over a sliding window, one period long. It can be observed that the phase differences steadily decrease from zero toward negative values, at a rate that depends on the wave steepness. Negative phase difference means that the wave elevation at the plate is lagging behind the incident wave elevation away from the plate. At the larger steepnesses the phase lag seems to stabilize at a value of about −2.5 rd.

An intuitive interpretation of these results is that the incoming wave system is being slowed down as it progresses toward the plate, through interaction with the reflected wave system. As the reflected wave system gradually spreads over the basin, the interaction area increases in time. This would provide an explanation for the transient character of the experimental records.

To render such interaction effects, one needs to pursue the hydrodynamic analysis to third-order in the wave steepness. In the following we attempt at proving that these tertiary wave effects are responsible for the localized run-up phenomenon.

A pioneering paper on changes of phase velocity through third-order effects is due to Longuet-Higgins & Phillips (1962). Many others have followed. In Longuet-Higgins & Phillips (1962) the wave system consists of two plane waves with frequencies ω_1, ω_2, and arbitrary directions of propagation. Here we are concerned with waves of the same frequency ω. From Longuet-Higgins & Phillips (1962) we easily obtain that, given two plane waves of identical frequency ω, with amplitudes A_1, A_2 and directions 0, β, third-order interactions result in a modification of the wave number of the first component given by

$$\Delta k_1 = k^3\, A_2^2\, f(\beta) + \frac{1}{2}\, k^3\, A_1^2\, f(0) = k^3\, A_2^2\, f(\beta) - k^3\, A_1^2 \tag{1}$$

where $f(\beta)$ is given by

$$f(\beta) = (\cos\beta - 1)\,\sqrt{2 + 2\,\cos\beta} - 2\,\cos\beta - \frac{1}{2}\sin^2\beta - \frac{2\,(1-\cos\beta)}{\sqrt{2+2\,\cos\beta} - 4}\,\left(1 + \cos\beta + \sqrt{2 + 2\,\cos\beta}\right) \tag{2}$$

and shown in figure 3.

This result applies to two plane wave systems which have interacted for a sufficiently long time that a steady state has been reached. It is not clear how far it can be applied to our transient problem where the reflected wave field varies in amplitude and direction, both over time and space.

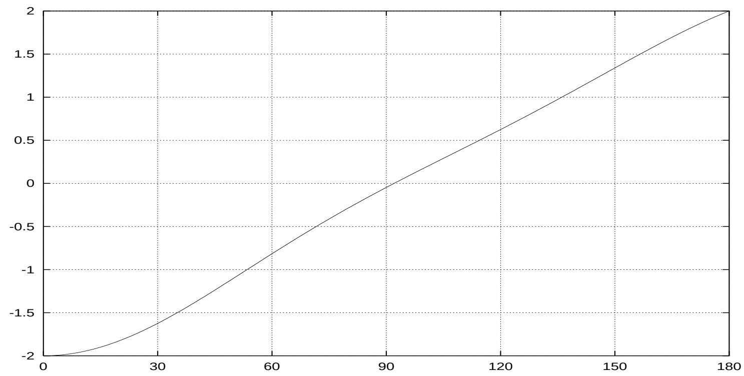

Figure 3: Interaction function $f(\beta)$.

If we assume that we can make use of it, then we can simply estimate the phase delay of the incoming wave system. We write its velocity potential under the form

$$\Phi_I = \frac{A_I\, g}{\omega}\, \mathrm{e}^{k\,z}\, \Re\left\{\mathrm{e}^{\mathrm{i}\,k\,x - \mathrm{i}\,\omega\,t - \mathrm{i}\,\psi(x,y,t)}\right\} \tag{3}$$

where the phase ψ is supposed to vary slowly in space and time. The coordinate system is such that $y = 0$ is the tank wall (the vertical plane of symmetry) and $x = 0$, $|y| \leq 1.2$ m is the plate.

We consider that the reflected wave-field is locally equivalent to a plane wave with amplitude $A_R(x, y, t)$ and direction $\beta(x, y, t)$. Then, neglecting the interaction of the incoming wave with itself (it results in identical phase change all across the tank), we obtain from (1) that ψ satisfies

$$\frac{\partial \psi}{\partial x} = -k^3\, A_R^2\, f(\beta) \tag{4}$$

If we focus on the first instants after the incident waves reach the plate, we can (over)simplify the situation by stating that the reflected wave-field progresses at the group velocity back toward the wave-maker. Hence the phase lag at $y = 0$ is given by

$$\psi(0, 0, t) = -k^3\, f(\pi) \int_{-C_G t}^{0} A_R^2(x, 0, t)\, \mathrm{d}x \tag{5}$$

Taking $A_R \sim A_I$ as a first approximation we get

$$\psi(0, 0, t) = -k^2 A_I^2\, \omega\, t \tag{6}$$

This agrees only qualitatively with the phase variations given in figure 2.

In a second step we have solved exactly the linear, frequency domain, diffraction problem, through eigenfunction expansions. Then we have mimicked the time development of the reflected wave-field by considering that it extends to a distance $d(t) = C_G\, t$ (in the x direction) from the plate. We have computed the phase lag $\psi(0, y)$ for increasing values of d, by identifying, locally, the reflected wave system with a plane wave and integrating (4) in x, from $-d$ to 0. The results are shown in figure 4 for a steepness of 4 %. We can notice that the phase difference, between the end and the center of the plate, stabilizes to about 1.2 rd after 8 m.

What we have done here is incorrect in many ways. First we have assumed that the incident and reflected wave systems progress as Heaviside functions in the (positive/negative) x direction. Secondly we have not updated the reflected wave system as the incident one undergoes modifications.

The latter problem can be grossly remedied through the following procedure: an interaction length d is chosen, for instance 20 m (the distance from the plate to the wave-maker). This gives the phase function $\psi(0, y)$ at the plate. The diffraction problem is then solved again with a modified incident potential expressed as (3). This gives a new reflected wave system, hence new phase lags, etc. The process is repeated until convergence is reached. The choice of the distance d appears to matter little as long as it is much larger than the plate length.

We give in figure 5 the obtained RAO's of the free surface elevation along the plate, now for a wavelength of 1.8 m ($T = 1.07$ s), obtained through this iterative procedure, for different steepnesses.

It can be observed that the run-up gets strongly enhanced at the wall, around $y = 0$, as the steepness increases. The curvature of the crests of the incoming waves results into focusing toward the center of the plate. In the

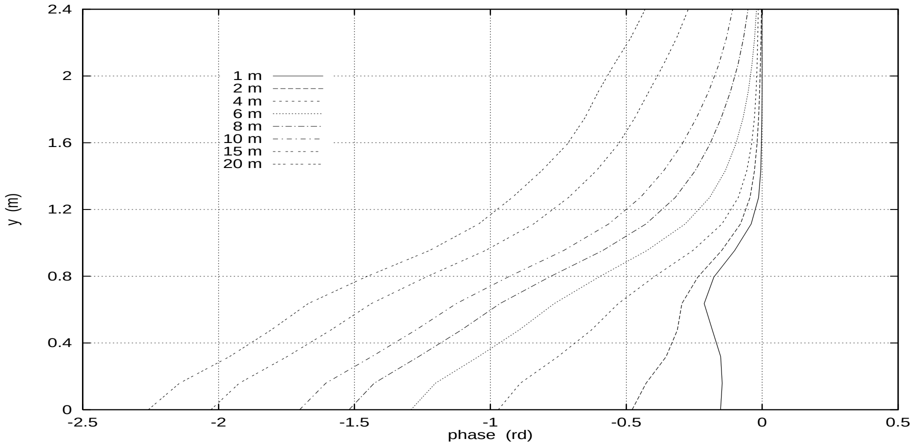

Figure 4: Phase angles for $L = 1.2$ m ($T = 0.88$ s) and $H/L = 4$ % and for interaction lengths from 1 to 20 m.

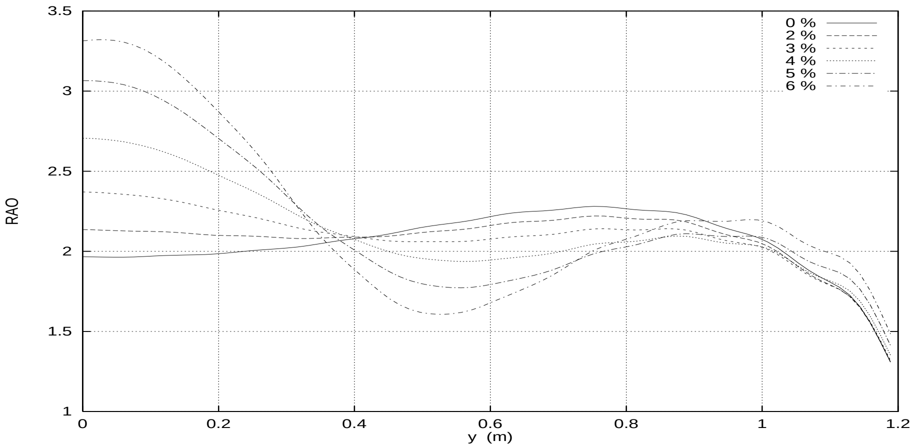

Figure 5: RAO of the free surface elevation along the plate for a wavelength of 1.8 m and for different steepnesses.

tests with the barge models, this could be clearly seen as two waves travelling from bow and stern toward midship and colliding there.

A RAO above 3 at a steepness of 6 % means very steep waves. Local non-linear effects are likely to come into play and to enhance even further the run-up. It seems to us, however, that the major role is being played, not by local non-linear effects, but by disseminated ones, that is the tertiary wave interactions between incident and reflected wave-fields on the weather side of the structure. This would be a key mechanism in scenarios of water-shipping in beam waves.

To confirm these findings, numerical simulations with the non-linear code *XWAVE* of the last author are under way. Results will be given at the workshop.

Acknowledgments

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Reference

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