Smoothed Particle Hydrodynamics for Naval Hydrodynamics

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Introduction

The design of naval vehicles relies heavily on numerical techniques to model the flow patterns around the craft; for example Direct Numerical Simulation (DNS) of the Navier-Stokes equations, the boundary-element method, and Large-Eddy Simulations. However, these techniques are limited in the maximum Froude number the numerical schemes can model accurately (based on the length dimension of the ship). Recent advances in computational resources have facilitated the development of alternative Lagrangian techniques for modelling fluid motion. In particular, particle methods such as Smoothed Particle Hydrodynamics (SPH) have become increasingly popular to model free surface and rotational flows as they do not require special numerical treatment to capture large amplitude waves and nonlinearities. SPH does not suffer theoretically from any Froude number limitation and is presented herein as a promising technique to model naval hydrodynamics for the design of ship hulls.

Smoothed Particle Hydrodynamics is based on a representation of the fluid body by large parcels of water that are subject to Newton's Second Law. The technique was developed by Lucy (1977) and Gingold and Monaghan (1977) for the simulation of nonaxisymmetric phenomena in astrophysics. A major attraction of the SPH technique is that the need for fixed computational grids is removed when calculating spatial derivatives. Instead, estimates of derivatives are provided by analytical expressions. The technique has since been extended to complicated free surface flows including solitary wave propagation over a planar beach (Monaghan and Kos 1999), plunging breakers (Tulin and Landrini 2000), solid bodies impacting on the surface (Monaghan 2000) and dam break simulations (Monaghan 1994).

A drawback of SPH is its inherent difficulty when modelling boundaries. The ability to account correctly for the interaction between a boundary and the flow of nearby particles is of paramount importance for the prediction of the thin boundary layer around a naval craft and is considered in detail in the present work. Hence, we examine several techniques including large numbers of particles, variable particle resolution lengths and particle refinement whereby particles are split or merged depending on pre-specified criteria in order to provide sufficient resolution (Kitsionas and Whitworth 2002).

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Numerical Methodology

The basis of the SPH method is the approximation of particle properties by an integration of the properties of all particles in the domain. This can be expressed approximately by a weighted summation of the nearest neighbouring particles only:

$$\zeta(\mathbf{r}) = \sum_j \zeta_j \; h^n \; W_{ij} \tag{1}$$

where in this case ζ is the variable being approximated at a position vector $\mathbf{r}$ in n dimensions, h is a particle dimension and W_{ij} is a rapidly-decaying radial weighting or kernel function that approximates to a delta function in the limit as its radius of influence decreases to zero. In Lagrangian form, each particle i can described by a fixed mass m_i, density ρ_i, position $\mathbf{r}_i$ and velocity $\mathbf{v}_i$. Thus, the equations describing conservation of mass and momentum can be recast as ordinary differential equations (Monaghan 1992):

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i + e \sum_j m_j \left(\frac{\mathbf{v}_{ji}}{\rho_{ij}} \right) W_{ij} \tag{2}$$

$$\frac{d\rho_i}{dt} = \sum_j m_j \left(\mathbf{v}_i - \mathbf{v}_j \right) \nabla_i W_{ij} \tag{3}$$

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} + \mathbf{F}_i \tag{4}$$

where $\mathbf{F}_i$ is a body force, Π_{ij} is a viscosity term and ϵ $(0 \leq \epsilon \leq 1)$ is a constant that moves each particle with a velocity closer to the average velocity in its neighbourhood. For nearly incompressible flows, an equation of state is used to close the problem and provide an estimate for the fluid pressure that allows numerical time stepping determined by the local speed of sound.

To treat particles in close proximity to a boundary, the technique of Dalrymple and Knio (2001) is utilized where the boundary is represented by stationary particles that are not subjected to the equations of motion. This idea is further extended to model flow in a numerical wave tank with open boundaries so that flow around a moving vehicle can be computed in a frame of reference that coincides with the craft. This requires implementation of transmissive inlet and outlet boundaries such that particles can enter the computational domain upstream, undergo flow transformation past the vehicle, and finally exit at the downstream boundary. The open boundaries are described by two columns of fixed particles that attempt to mimic the effect of a continuation of the computational domain in both the upstream and downstream boundaries. At these particles, the appropriate boundary conditions are prescribed depending on the speed of the flow and assumed hydrostatic flow conditions.

Flow over a Submerged Hydrofoil

Duncan (1983) conducted a series of experiments to observe the steady wave profile generated behind a moving hydrofoil in 24m long wave tank of width 0.61m. His tests consisted of a submerged NACA

0012 hydrofoil of chord length 0.203m being pulled along metal runners using an external motor-driven belt and pulley system at a constant speeds of 80-100 cm/s with varying angles of attack $(5-10^{\circ})$, and different submergence depths.

Figure 1 shows the computed results for Duncan's hydrofoil moving at a speed of 80 cm/s and angle of attack of 5° at a submergence depth of 0.436m in water of total depth 0.261m. There are approximately only 1400 particles and it can be seen clearly that the flow over the hydrofoil has generated steady waveforms downstream with progressively decreasing amplitude in close agreement with the experimental results. The figure also shows a plot of one particle's path - a feature that is easily facilitated by the SPH technique. Importantly however, a very large boundary effect is clearly evident in the vicinity of the hydrofoil that effectively creates a much larger object. It is this effect and how to avoid it that will be the focus of much of the forthcoming paper.

Following model verification with comparison between the numerical model and the experiments of Duncan (1983), preliminary numerical results will be presented and comparison made with the extensive experimental data for the benchmark of flow past different types of Wigley hulls (Journe 1992).

Conclusions

The paper presents a numerical model utilising Smoothed Particle Hydrodynamics to study free surface flows past naval vehicles. A numerical investigation into flow over a submerged hydrofoil towed along a tank indicates the sensitivity and accuracy of the model to particle number and particle size. Particle refinement whereby the particles are split or merged according to some criterion will be shown to provide sufficient resolution to model waveforms induced in the wake produced by the hydrofoil.

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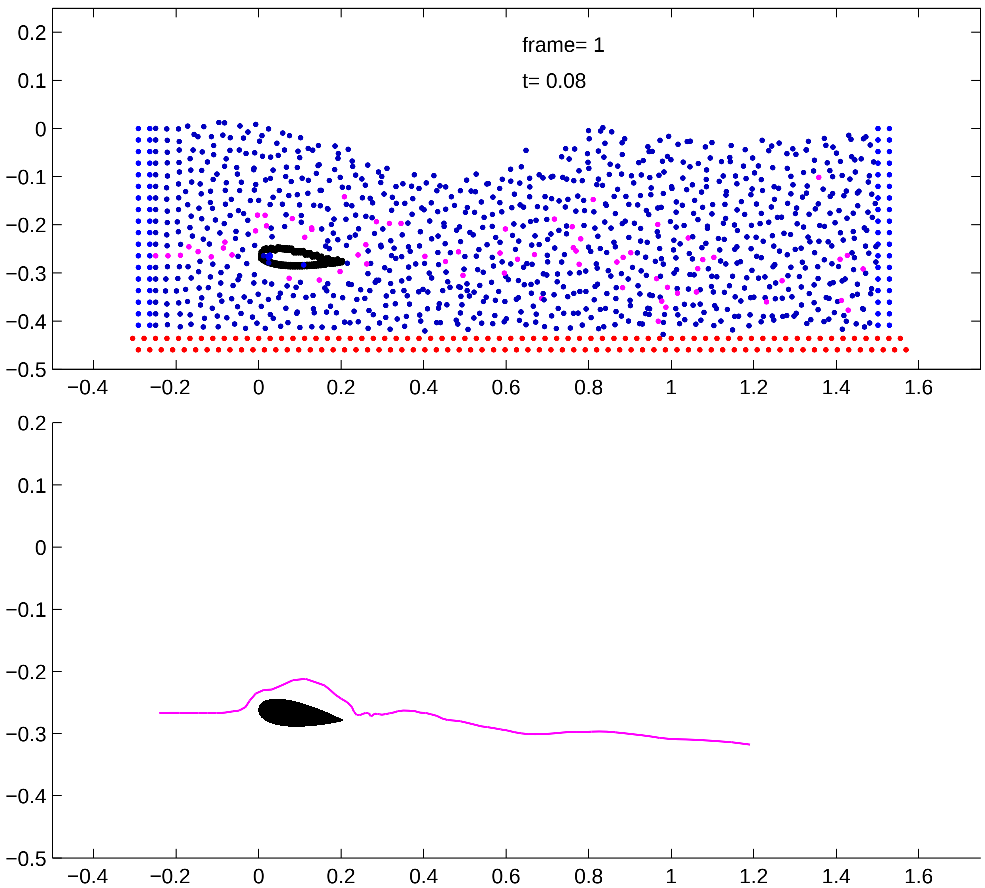

Figure 1: Flow over a submerged hydrofoil. Top: Snapshot of particles with foil, showing free surface deformation. Bottom: A streamline.