The Influence of Surface Tension and Viscosity on the Wavemaking of a Model Catamaran

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Summary

Both surface tension and molecular viscosity are included here in the computation of the linearized wave pattern generated by a high-speed catamaran when traveling in water of finite depth and restricted width. The predictions are applied to the case of a small model towed in a tank. Different water depths, demihull spacings, and speeds are considered.

There is excellent correlation between the traditional inviscid predictions for the wave elevation and the experimental measurements, but but not for Froude numbers in the vicinity of the critical depth Froude number. It is also demonstrated that the surface-tension plays a vital rôle in the predictions at low Froude numbers, for such small models.

1 Introduction

In a recent paper, Doctors (2003) investigated the matter of the influence of the viscosity of the water on the wave generation of a small model vessel towed in a ship-model test basin. It is well known that viscosity has been ignored in classical ship hydrodynamics. It was shown that the use of large values of the viscosity (corresponding to typical values of the eddy viscosity), would provide excellent correlation with the experimental data, particularly at low speeds.

However, it was suggested during the discussion following the presentation of that work, that eddy viscosity would not be a significant factor outside the viscous wake region and that a more convincing explanation for the less-than-expected measured wave amplitudes might be the influence of either surface tension or surfactants, or both.

2 Mathematical Formulation

The coordinate system and principal parameters defining the problem are shown in Figure 1(a). The vessel has a waterline length L. The experiments were, in fact, conducted on one demihull, which was offset a distance $s/2$ from the near wall of the towing tank. As a consequence, the effective width of the channel is w and the effective spacing between the demihulls is s. The depth of the water is d. The acceleration due to gravity is g, while U is the speed of the vessel. The water possesses a density ρ, a surface tension τ, and a molecular viscosity μ (considered to be small in the analysis).

The free-surface condition may be expressed in terms of the perturbation velocity potential ϕ as:

$$U^2\phi_{xx} + g\phi_z + (\tau/\rho)\phi_{zzz} - (4\mu U/\rho)\phi_{xzz} = 0\,. \quad (1)$$

Equation (1) is equivalent to that given by Wehausen and Laitone (1960, Equation (24.3) on p. 632), except for the additional last term which accounts for the effect of viscosity (assumed to be small), as explained by Tuck, Scullen, and Lazauskas (2002). This term introduces an imaginary component to the wave number of the free-surface waves and results in a spatial damping factor. This equation corresponds to Equation (61) of Zilman and Miloh (2001), if their terms due to the presence of a surfactant are ignored.

We now consider the potential for a finite-depth wave of the form:

$$\phi = \phi_0 \frac{\cosh[(k(z+d)]}{\cosh(kd)} \exp[\mathrm{i}k(x\cos\theta + y\sin\theta)]\,, \quad (2)$$

where k is the wave number and θ is the wave angle.

Substitution of Equation (2) into Equation (1) yields the dispersion relationship:

$$\begin{aligned} f &= k^2 - [k_0 k + (\tau/\rho U^2)k^3]\tanh(kd) - k_y^2 \\ &\quad + (4\mu\mathrm{i}/\rho U)k^2 k_x \qquad (3)\\ &= 0 \qquad (4)\end{aligned}$$

and its gradient:

$$\begin{aligned} \frac{\mathrm{d}f}{\mathrm{d}k} &= 2k - [k_0 + 3(\tau/\rho U^2)k^2]\tanh(kd) \\ &\quad - [k_0 k + (\tau/\rho U^2)k^3] d\,\mathrm{sech}^2(kd)\,. \qquad (5)\end{aligned}$$

We may now define the complex wave number:

$$k^* = k + \mathrm{i}\delta\,, \quad (6)$$

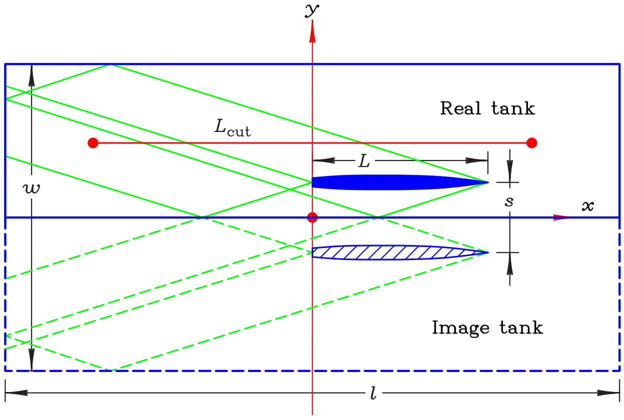

Figure 1: Definition of the Problem
(a) Principal Features

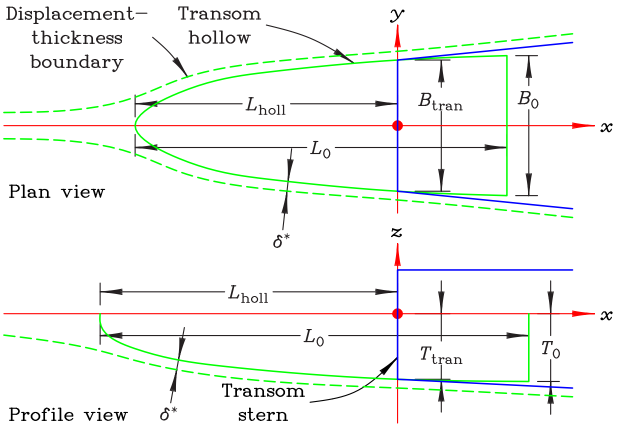

Figure 1: Definition of the Problem
(b) Transom Hollow

where k is the standard (inviscid) solution of Equation (4). Substituting this definition into Equation (4) and retaining the leading-order terms in δ yields the imaginary component of the wave number:

$$\delta = -4\mu k_x k^2 \Big/ \rho U \left[\frac{\mathrm{d}f(k,k_y)}{\mathrm{d}k} - \frac{k_y^2}{k} \right] . \tag{7}$$

In the case of deep water, $d \to \infty$, the result of Tuck, Scullen, and Lazauskas (2002) is recovered.

The viscous damping factor that is to be included for each component of the wave spectrum is then

$$V = \exp[-|\delta|(|x^*|\cos\theta + |y^*|\sin\theta)] . \tag{8}$$

The offsets x^* and y^* are the distances from the source point (approximated by the center of buoyancy) to the field point.

3 Model Vessel

A modified version of the Series 64 hull defined by Yeh (1965) was the subject of the experiments. Only one such demihull was needed for the current towing-tank tests, as noted above. The offset from the near tank wall could be set as required. The hull is characterized by its high-speed form with a transom stern. A pictorial view was published by Doctors (2003).

The waterline length of the model is 1.500 m and the three effective demihull spacings that were tested were 0.300 m, 0.400 m, and 0.500 m. The effective tank width was 7.100 m and the five water depths employed were 0.300 m, 0.450 m, 0.600 m, 0.900 m, and 1.500 m.

4 Influence of Transom-Stern-Hollow Model

The two parts of Figure 2 present a comparison of the experimental and theoretical wave elevation ζ for two values of the Froude number F. The depth Froude numbers F_d are also indicated. In each case, the experimental wave elevation is compared with that predicted by two idealized models for the transom-stern hollow. The first model is that proposed by Doctors and Day (1997), while the second model is based on the concept that there is a minimum reattachment length for the flow behind a backward-facing step, even at very low speeds, as noted by Hall (2001). The approximate influence of the "angle of run" of the stern has also been incorporated in this technique. This is illustrated in Figure 1(b).

It can be seen that the newer model is considerably superior, because it essentially eliminates the undesired large-amplitude high-frequency predictions of the older model. There is still a visible (but smaller) improvement at the higher Froude number of 0.3021. At higher speeds still, the significance of the precise transom-stern hollow model is lost.

5 Effect of Surface Tension and Viscosity

Figure 3 shows a further comparison of the experiment and theory. It is noted, for the low Froude number of 0.2028 in Figure 3(a), that the inclusion of the effect of surface tension τ is to reduce the amplitude of the waves (a desirable effect). However, unwanted high-frequency wave components are also introduced.

It is observed that the most sophisticated theory, which also includes the molecular viscosity μ (*not the much larger eddy viscosity*), partly eliminates these wave components, providing a slightly better correlation between theory and experiment.

These same phenomena are seen to occur, but on a smaller scale, for the higher Froude number of 0.3021 in Figure 3(b).

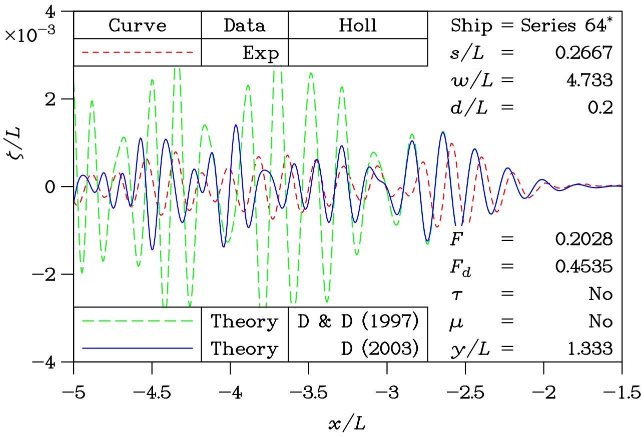

Figure 2: Effect of Hollow Length on Wave Profiles (a) $d/L = 0.2$ and $F = 0.2028$

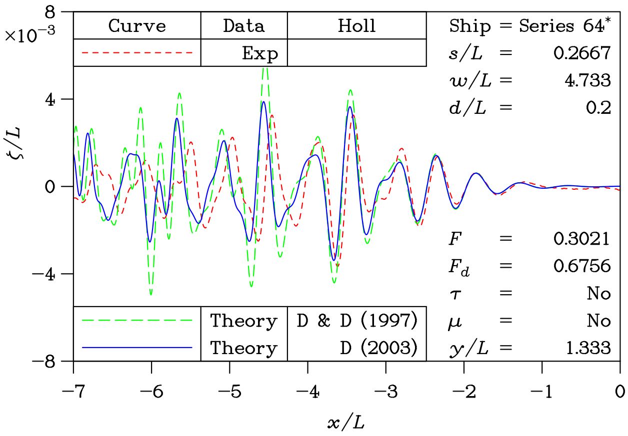

Figure 2: Effect of Hollow Length on Wave Profiles (b) $d/L = 0.2$ and $F = 0.3021$

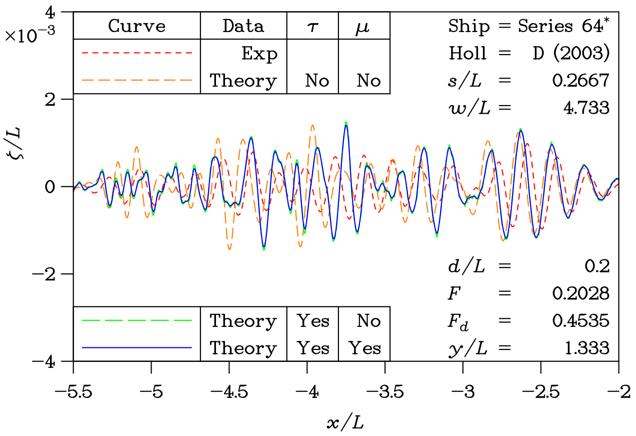

Figure 3: Effect of τ and μ on Wave Profiles (a) $d/L = 0.2$ and $F = 0.2028$

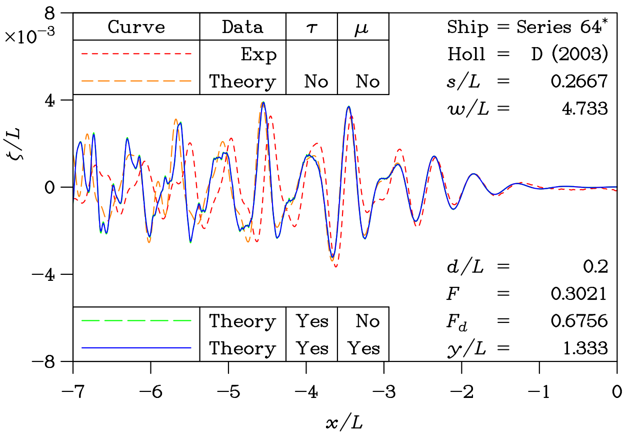

Figure 3: Effect of τ and μ on Wave Profiles (b) $d/L = 0.2$ and $F = 0.3021$

6 Root-Mean-Square Wave Elevation

The root-mean-square wave elevation ζ_{RMS} over all five longitudinal wave cuts is plotted in Figure 4 for two demihull spacings and for the deepest $d/L = 1.0$ condition. The low-speed range of the data only is shown. This allows one to see more clearly that the influence of surface tension and of viscosity is to reduce the numerical value of the predictions. However, the effects are relatively unimportant with regard to the root-mean-square wave elevation.

7 Demihull Spacing

Finally, Figure 5 presents the effect of demihull spacing s on the root-mean-square wave elevation for two different water depths. It is seen that increasing the spacing leads to a lower wave generation.

The experimental effects are predicted accurately by the current enhanced theory, with the exception already noted by Doctors (2003). This is the problem of vessel speeds corresponding to a depth Froude number of unity, where the theory behaves poorly, if the width of the waterway w is also finite.

8 Conclusions

This work has demonstrated that the inclusion of surface tension in the theory enhances the accuracy of the predictions. Further work should involve a more detailed study of the transom-stern hollow in order to include the boundary-layer displacement thickness on the effective wave-generating hull.

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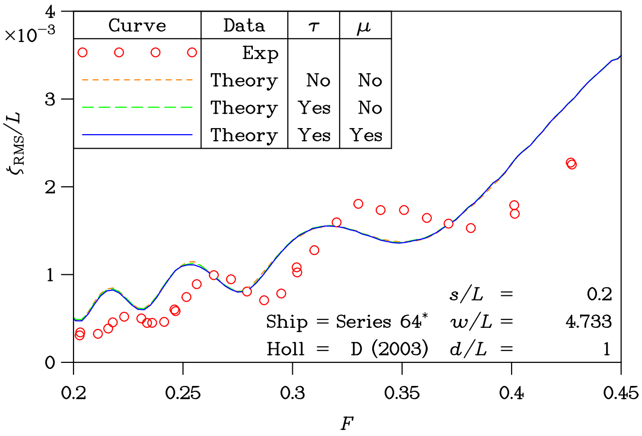

Figure 4: Effect of τ and μ on RMS Wave Elevation (a) $s/L = 0.2000$ and $d/L = 1.0$

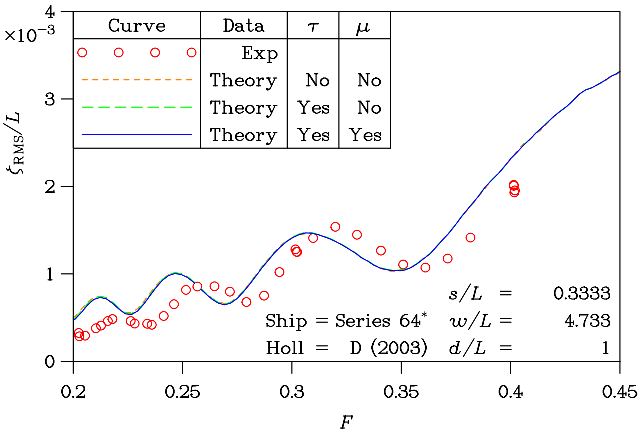

Figure 4: Effect of τ and μ on RMS Wave Elevation (b) $s/L = 0.3333$ and $d/L = 1.0$

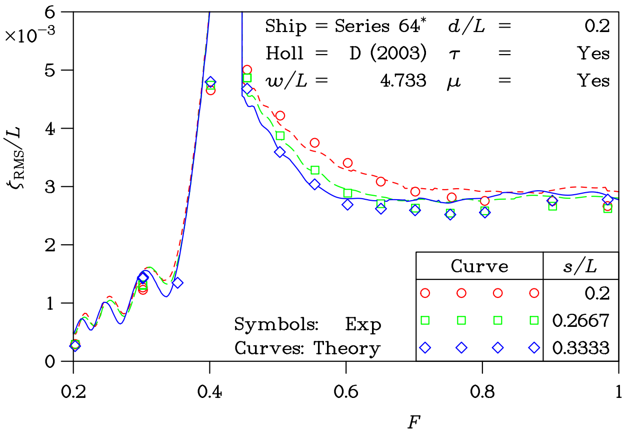

Figure 5: Effect of Demihull Separation on RMS Wave Elevation (a) $d/L = 0.2$

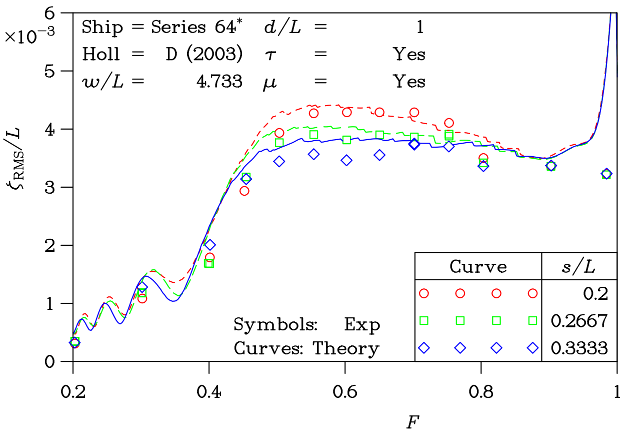

Figure 5: Effect of Demihull Separation on RMS Wave Elevation (b) $d/L = 1.0$

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