# NUMERICAL SIMULATION OF VIOLENT SLOSHING BY CIP METHOD

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## INTRODUCTION

In the case of partially filled ship tanks, the sloshing-induced impact load may cause damage on tank structures when the excitation due to ship motions is near the natural frequency of the liquid cargo. It is thus necessary to investigate such violent surface waves in the tank and the resulting impact pressure to the walls to ensure the structural adequacy of the tank. The CFD simulation is considered as an efficient method for investigation of sloshing problems and many studies have been carried out in recent years. However, very limited amount of computation on violent sloshing has been done due to the difficulties in handling largely distorted free surface and its interaction with the tank structures. The recent development of SPH method has shown us a hopeful approach for computation of violent sloshing (e.g., Landrini et al, 2003). Such method, however, could be memory and computational time intensive and in general is difficult to apply to practical 3-D problems. In this study, we make use of a FDM in a Eulerian grid for the violent sloshing problems and we will demonstrate that this is a more efficient numerical approach.

The authors recently proposed a new CFD simulation approach for extremely nonlinear wave-body interactions such as slamming, water on deck, wave impact by the green water and capsizing due to large-amplitude waves (Hu and Kashiwagi, 2002). This method is a finite difference method based on the CIP (Constrained Interpolation Profile) algorithm. The important features of the method can be summarized by (1) multiphase computations in a stationary Cartesian grid while both the free surface and the body boundary are treated as inner interfaces, and (2) a pressure-based algorithm that can treat liquid, gas, and solid phases, irrespective of the flow being compressible or incompressible, by solving one set of equations.

The Eulerian approach to computing multiphase flows on a regular, stationary grid is pioneered by the MAC (marker and cell) method. This approach has been developed in the past decade to a number of methods with different interface/front capturing techniques, such as the VOF (volume of fluid) method and the level set method. The CIP method also belongs to the Eulerian approach in which the free surface and the body boundary are treated as inner interfaces that can be captured by solving the equation of density function

$$\frac{\partial \phi_m}{\partial t} + u_i \frac{\partial \phi_m}{\partial x_i} = 0 . \tag{1}$$

For a multi-phase problem, *m =1*, *2*, *3* denotes liquid, gas, and solid phases, respectively. As discussed in the authors' previous paper, owing to the subcell resolution feature of the CIP scheme, for many cases the gas-liquid interface can be successfully determined by solving Eq. (1) for the density function of liquid $\phi_1$. Therefore, CIP method is also considered as an interface capturing method. However, as the CIP scheme uses non-conservation form, the employment of CIP scheme as an interface capturing method may result in some disadvantages. For example, the mass conservation may become bad in the case of long-term computation, and at the phase interface in which breaking up occurs, numerical diffusion resulting from the calculation of Eq. (1) could be serious. These disadvantages, however, may become important for the violent sloshing computation and improvement of them should be a key issue of the research.

Recently, a conservative form of the CIP scheme, the CIP-CSL3 (Constrained Interpolation Profile – Conservative Semi-Lagrangian scheme with third-order polynomial function) scheme, was proposed by Xiao and Yabe (2001). This conservative scheme, which keeps most of the advantages of the original CIP scheme, is then extended to multidimensional hydrodynamic problems by Xiao and Ikebata (2003). The CIP-CSL3 scheme has a new feature that it provides a way to reduce the numerical diffusion at the phase interface, and is therefore considered as a solution to the numerical problems encountered in the original CIP scheme.

In the following, we will briefly summarize the numerical method and describe the conception of the CIP scheme and the CIP-CSL3 scheme. Then two-dimensional computations on sloshing inside a horizontally oscillating rectangular tank are carried out and compared to an experiment that is carried out at RIAM recently. Discussions will be made using the computed results of impact pressures on the wall and free surface profiles.

## NUMERICAL METHOD

### Governing Equations

For tank sloshing problems, the governing equations for fluids (gas and liquid) inside the tank can be written as

$$\frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} = -\rho \frac{\partial u_i}{\partial x_i} , \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i , \tag{3}$$

where $\sigma_{ij}$ is the total stress. For a Newtonian fluid, the total stress can be written as $\sigma_{ij} = -p\delta_{ij} + 2\mu S_{ij} - 2\mu\delta_{ij} S_{kk}/3$ , where $S_{ij} = \left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)/2$ . The second term on the right-hand side of Eq. (3) denotes a body force such as the gravity force. As we assume no temperature variation with the problem, the equation of state can be written as $p = f(\rho)$ . Then the equation for pressure is derived as

$$\frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} = -\rho C_S^2 \frac{\partial u_i}{\partial x_i} , \tag{4}$$

where $C_S$ is the sound speed. Eqs (2)-(4) are the governing equations for the problem, and can be solved numerically by a fractional step method in which the governing equations are divided into an advection phase and a non-advection phase. The coupling of velocities and pressure is treated by using the following Poisson equation in the non-advection phase calculation

$$\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_i}\right)=\frac{p-p^n}{\rho C_s^2\Delta t^2}+\frac{1}{\Delta t}\frac{\partial u_i^n}{\partial x_i}. \tag{5}$$

Note that this equation is valid for liquid and gas phases, and is computationally possible to apply to solid phase. By solving Eq. (5), the pressure in the whole computation domain can be obtained. The advection phase calculation is performed by CIP method, which is described as follows.

## CIP Scheme

Here by CIP we mean the scheme for advection calculation. For advection computation of a variable $\chi$, not only the transportation equation of $\chi$ but also the transportation equation of its spatial gradient, $\varphi_i=\partial\chi/\partial x_i$, is used. The value of $\chi$ may represent the density, the velocity and the pressure in the governing equations, respectively. We write a general transportation equation of $\chi$ in non-conservative form as

$$\frac{\partial\chi}{\partial t}+u_i\frac{\partial\chi}{\partial x_i}=H. \tag{6}$$

By differentiating Eq. (6) with respect to the spatial coordinates, we obtain the transportation equation of $\varphi_i$ as

$$\frac{\partial\varphi_i}{\partial t}+u_j\frac{\partial\varphi_i}{\partial x_j}=\frac{\partial H}{\partial x_i}-\varphi_j\frac{\partial u_j}{\partial x_i}. \tag{7}$$

Computation of (6) and (7) can be divided into two steps, an advection phase and a nonadvection phase. The advection phase calculation is carried out by a semi-Lagrangian procedure as

$$\chi^*(\mathbf{x})=\tilde{\chi}^n(\mathbf{x}-\mathbf{u}\Delta t), \tag{8}$$

$$\varphi_i^*(\mathbf{x})=\tilde{\varphi}_i^n(\mathbf{x}-\mathbf{u}\Delta t), \tag{9}$$

where $\tilde{\chi}^n$ is an interpolation approximation to $\chi^n$ and $\tilde{\varphi}_i^n=\partial\tilde{\chi}^n/\partial x_i$ .Therefore, the profile at the $n$+1 step is obtained by shifting the profile at the $n$ step by $\mathbf{u}\Delta t$ to the upwind direction. For one-dimensional case, at a grid point $x_1=x_{1_m}$ the cubic polynomial can be used to approximate $\tilde{\chi}^n$ as

$$\tilde{\chi}^n(x_1)=a_m\left(x_1-x_{1_m}\right)^3+b_m\left(x_1-x_{1_m}\right)^2+c_m\left(x_1-x_{1_m}\right)+d_m. \tag{10}$$

We can find an upwind computational cell for the grid point $x_1=x_{1_m}$, e.g., $[\,x_{1_{m-1}},x_{1_m}\,]$ for $u_1\ge 0$, then the 4 unknown coefficients of $\tilde{\chi}^n(x_1)$ can be determined using $\chi_m^n$, $\chi_{m-1}^n$, $\varphi_{1\,m}^n$ and $\varphi_{1\,m-1}^n$.

$$a_m=\left(\varphi_{1_m}^n+\varphi_{1_{m-1}}^n\right)\Big/\Delta x_1^2+2\left(\chi_m^n-\chi_{m-1}^n\right)\Big/\Delta x_1^3,$$

$$b_m=-\left(2\varphi_{1_m}^n+\varphi_{1_{m-1}}^n\right)\Big/\Delta x_1-3\left(\chi_m^n-\chi_{m-1}^n\right)\Big/\Delta x_1^2, \tag{11}$$

$$c_m=\varphi_{1_m}^n,\quad d_m=\chi_m^n.$$

As the advection of spatial gradients in each computation cell can be solved and only the information (value and its spatial gradients) at the grid points of one cell is needed for the interpolation function, the CIP scheme has both a subcell resolution feature and a compact structure. Therefore, for a multiphase computation in which there are discontinuities or large gradients for physical quantities at the interfaces, the CIP scheme can keep the sharpness better than other upwind schemes.

### CIP-CSL3 Scheme

The basic idea of conservative CIP scheme is similar to that of the original CIP scheme. Xiao and Yabe (2001) have proposed several types of the conservative scheme, among which the CIP-CSL3 scheme is applied to our numerical model. To describe the conservative scheme, we rewrite Eq. (6) into the conservative form as

$$\frac{\partial\chi}{\partial t}+\frac{\partial(u_i\chi)}{\partial x_i}=H'. \tag{12}$$

Similar to the way of construction of a semi-Lagrangian scheme by using the variable and its spatial gradient in the original CIP, the conservative CIP scheme uses another two values: the surface integrated average value ${}^l\overline{\chi}=\frac{1}{S^l}\oint_{S^l}\chi dS$ and the cell volume integrated average value $\overline{\overline{\chi}}=\frac{1}{V}\oint_V\chi dV$. Where $V$ denotes the volume of a computation cell, and $S^l$ denotes the area of the cell surface $l$. In a staggered grid, the surface integrated average value is the value defined at the cell surface center and we shall use $\chi$ instead of ${}^l\overline{\chi}$. Then by surface averaging and volume averaging of Eq. (12), we obtain the governing equations as

$$\frac{\partial\chi}{\partial t}+u_i\frac{\partial\chi}{\partial x_i}=H'', \tag{13}$$

$$\frac{\partial\overline{\overline{\chi}}}{\partial t}+\frac{1}{V}\sum_{l=1}^{3}\left({}^l\overline{u_i\chi}\,S^l-{}^{-l}\overline{u_i\chi}\,S^{-l}\right)=\frac{1}{V}\oint_V H'dV. \tag{14}$$

Computation using (13) and (14) can be performed by a fractional step method as used in the CIP method. The left hand side of Eqs (13) and (14) will be solved by a procedure as

$$\chi^*(\mathbf{x})=\tilde{\chi}^n(\mathbf{x}-\mathbf{u}\Delta t), \tag{15}$$

$$\overline{\overline{\chi}}^*=\overline{\overline{\chi}}^n-\frac{\Delta t}{V}\sum_{l=1}^{3}\left(\eta_l^+-\eta_l^-\right). \tag{16}$$

Equation (16) is derived from Eq. (14), where $\eta_l^+$ and $\eta_l^-$ are fluxes across the boundaries of the cell. For one-dimensional case, the CIP-CSL3 scheme also uses piece-wise cubic polynomial to approximate profiles in each cell as Eq. (10). Taking the case of $u_1\ge 0$ as an example, for the upwind computational cell $[\,x_{1_{m-1}},x_{1_m}\,]$, the 4 unknown coefficients can be determined by the following conditions.

$$\hat{\chi}^n(x_{1_m}) = \chi_m\,,\ \hat{\chi}^n(x_{1_{m-1}}) = \chi_{m-1}\,,$$
$$\int_{x_{1_{m-1}}}^{x_{1_m}} \hat{\chi}^n(x_1) = \bar{\bar{\chi}}_{m-1/2}\,,\ \frac{d\hat{\chi}^n(x_1)}{dx_1} = \lambda_{m-1/2}\,. \qquad (17)$$

In the last equation of Eq. (17), $\lambda_{m-1/2}$ is the slope (first-order derivative) of $\chi$ at the center of the cell. This slope is a free parameter to be determined. The four coefficients of Eq. (10) are then obtained as follows.

$$a_m = 4\left(\chi_m^n - \chi_{m-1}^n\right)/\Delta x_1^3 - 4\lambda_{m-1/2}^n/\Delta x_1^2\,,$$
$$b_m = -6\bar{\bar{\chi}}_{m-1/2}^n/\Delta x^3 + 3\left(3\chi_m^n - \chi_{m-1}^n\right)/\Delta x_1^2 - 6\lambda_{m-1/2}^n/\Delta x_1\,, \qquad (18)$$
$$c_m = -6\bar{\bar{\chi}}_{m-1/2}^n/\Delta x^2 - 6\chi_m^n/\Delta x_1 - 2d_{m-1/2}^n\,,\quad d_m = \chi_m^n\,.$$

Although the discussion in this section is about general transportation equations, in the present stage of research the CIP-CSL3 scheme is only used as an interface capturing method, i.e., in the computation of the density function of the liquid $\phi_1$.

In order to prevent the numerical diffusion at the phase interface, we can modify the interpolation function by designing a scheme for the slope $\lambda_{m-1/2}$. At first, the slope $\tilde{\lambda}_{m-1/2}$ can be obtained using an approximation method (Xiao and Yabe, 2001). Then a modification to the slope can be made by using a function $\beta$ (Xiao and Ikebata, 2003) as

$$\lambda_{m-1/2} = \beta\tilde{\lambda}_{m-1/2}\,. \qquad (19)$$

There are several choices of the function $\beta$, and that used for the present computation is

$$\beta = \alpha\left(1 - \left|\bar{\bar{\phi}}_{1m-1/2}^n - 0.5\right|\right). \qquad (20)$$

The parameter $\alpha$ is used to enhance the sharpness of the phase interface and $\alpha = 2.75$ is used for the computation example shown in this paper.

For multi-dimensional cases, a dimensional splitting method is applied.

## NUMERICAL RESULTS

We carried out a series of experiments with a horizontally oscillating rectangular tank to investigate violent sloshing phenomena. The schematic view of the tank is shown in Fig.1.

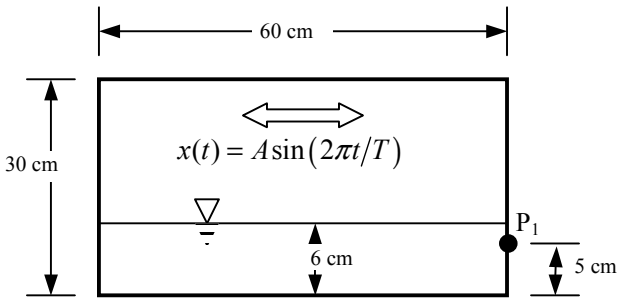



Fig.1 Schematic view of tank sloshing experiment

To demonstrate the capability of the present CIP approach to violent sloshing problems, one shallow water case is chosen for the computation. The computation details are shown in Table 1. Three oscillation periods, T=1.7, 1.3 and 0.8 *sec* are calculated for the reason that the first resonant period for this shallow water case is around 1.3 *sec* according to the analysis by Faltinsen and Timokha (2001).

Table 1 Computation condition

| Motion of tank | |
|---|---|
| amplitude (*m*) | $A$ = 0.06 |
| period (*sec*) | $T$ = 1.7, 1.3, 0.8 |
| Grid and Time Step | |
| grid spacing (*m*) | $\Delta x = \Delta z = 0.003$ |
| time step | $\Delta t / T = 5\times10^{-4}$ |

The details of the CIP based finite difference method are described in a previous paper (Hu and Kashiwagi, 2002). The interface capturing method implemented in the present model is CIP-CSL3. For comparison, results by using the original CIP as the interface capturing method are also shown. All the computations are two-dimensional.

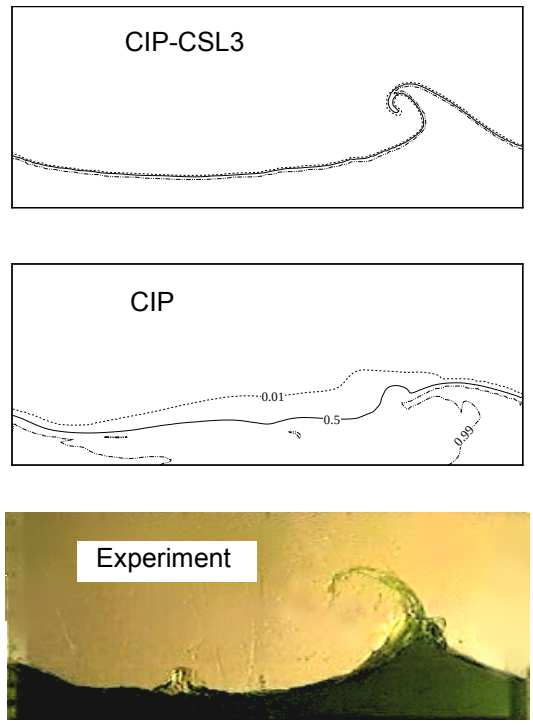
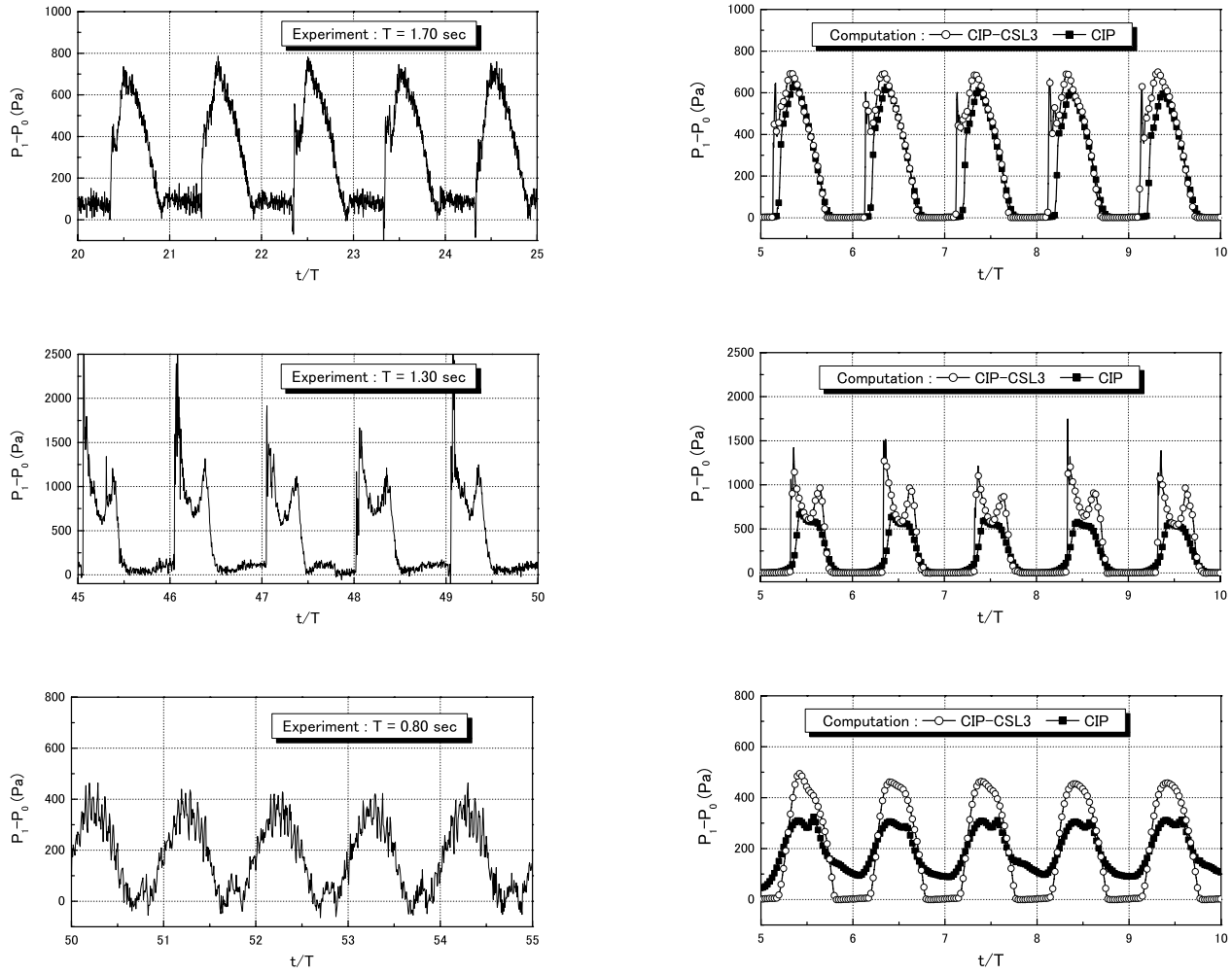



Fig.2 Comparison of computed free surface profile at t/T=5.8 using two schemes. T=0.8 *sec*. Three lines denote $\phi_1 = 0.01$, 0.50 and 0.99, respectively.

In Fig. 2, we compare the computed free surface profile for the case of T = 0.8 *sec* at t/T = 5.8 using two interface capturing methods. An experimental picture is also shown in the figure. The thickness of the computed air-water interface, which is the distance between lines of $\phi_1 = 0.01$ and $\phi_1 = 0.99$, is very different for the two computations. The diffusion of interface is successfully suppressed by the CIP-CSL3 scheme. We will find that such feature of keeping compact thickness of air-water interface for long-term computation is very essential for accurate prediction of free surface impact pressures.




(a) Experiment (b) Computation

Fig.3 Comparison of measured and computed pressures at point P1 for three oscillation periods.

Figure 3 is a comparison of experimental and computed pressure at point P1 for three oscillation periods. Good agreement between the experiment and the computation by CIP-CSL3 scheme is obtained. On the other hand, the results by CIP scheme show that only the case of T = 1.7 *sec* seems good. No impact pressure is obtained for the resonant case of T = 1.3 *sec*, and the computed pressure does not return to zero for the case of T = 0.8 *sec*. Detailed investigation of the computed free surface development indicates that the bad pressure predictions for the two violent cases in the original CIP computations are mainly caused by the severe interface smearing.

## CONCLUSIONS

We have applied a newly developed CIP scheme, the CIP-CSL3 scheme, as an interface capturing method for computation of violent tank sloshing problems. Two-dimensional numerical simulations are performed and the results of free surface and the pressure at the wall are compared to an experiment. The numerical results shown in this paper illustrated that sharp gas-liquid interfaces can be kept by the CIP-CSL3 scheme even for long-term computations. It also demonstrated the potential of the present CIP/ CIP-CSL3 based model as a powerful tool for the research of violent sloshing problems.

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