A new numerical method to compute nonlinear hydrodynamic forces on a submerged cylinder

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1 Introduction

A circular cylinder submerged under the free surface in an ideal fluid is considered when it is steadily moving or undergoing periodic heaving motions. A new fast, accurate numerical method is developed to compute nonlinear hydrodynamic forces on the cylinder.

The method presented here is based on a generalization of the formulation proposed by Choi [2]. The original third-order nonlinear equations for the evolution of the free-surface of an inviscid fluid in the absence of a body are modified for singularities representing the cylinder below the free-surface. The resulting evolution equations for the free surface are solved by using an efficient pseudo-spectral method. This numerical method requires a distribution of singularities only along the body boundary and therefore the computation is run more quickly than the fully-nonlinear boundary integral methods, hopefully without losing the essential effects of the nonlinearity. Our numerical solutions are also validated against available analytical, numerical, and experimental results.

2 Nonlinear Evolution Equations

The free surface evolution equations are derived by modifying the method for free waves outlined by Choi [2]. The free surface velocity potential is modified to be the summation of the velocity potential due to the singularities on the body and the velocity potential due to wave effects. By formally solving the Laplace equation with systematic asymptotic expansion, the kinematic and dynamic boundary conditions yield a system of two equations for the evolution of the one-dimensional free surface:

$$\begin{aligned}\zeta_t = -H[\Phi_x + (\zeta(H[\Phi_x] - S))_x + (\zeta H[\zeta(H[\Phi_x] - S)]_x)_x + \tfrac{1}{2}(\zeta^2\Phi_{xx})_x] \\ -[\zeta\Phi_x + \tfrac{1}{2}(\zeta^2(H[\Phi_x] - S))_x]_x + S, \qquad (2.1)\end{aligned}$$

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$$\Phi_t = \tfrac{1}{2}(H[\Phi_x] - S)^2 + \zeta\Phi_{xx}(H[\Phi_x] - S) + (H[\Phi_x] - S)H[\zeta(H[\Phi_x] - S)]_x - g\zeta - \tfrac{1}{2}\Phi_x^2 - \frac{P}{\rho}, \quad (2.2)$$

where $\Phi(x,t)$ is the total velocity potential on the free surface, $\zeta(x,t)$ is the free surface elevation, g is acceleration due to gravity, ρ is the density of the fluid. In (2.1)–(2.2), $S(x,t)$ is a term connected with the velocity potential for the singularities representing the body, $\phi^s(x,z,t)$ given by

$$S(x,t) = \phi_z^s(x,0,t) + H[\phi_x^s(x,0,t)]\,. \quad (2.3)$$

These equations are correct to the 3^{rd} order in wave steepness, a/λ, where a and λ are wave amplitude and wavelength, respectively.

With given Φ and ζ, the body boundary condition is imposed at the body's instantaneous position, using a desingularized boundary integral method, described by Beck [1]. This gives the strength of singularities and hence ϕ^s can be computed at a given time step. Then equations (2.1)–(2.2) are evaluated to update Φ and ζ by using the pseudo-spectral method with which the Hilbert transform operator H defined by

$$H[f(x)] = \frac{1}{\pi}\fint_{-\infty}^{\infty} \frac{f(x')}{x - x'}dx'. \quad (2.4)$$

has a simple form in Fourier space.

3 Results

3.1 Steadily translating dipole

A translating unit dipole with depth Froude number $\frac{1}{\sqrt{5}}$ is chosen as the first test problem, which was solved linearly by Havelock [3] and others. The solution for various orders can be seen in figure 1 and notice that the first-order solution is quite different from the higher-order solutions. The difference between our 1^{st} order solution and the Havelock solution is on the order of 10^{-6}.

3.2 Submerged translating cylinder

Scullen and Tuck [4] investigated the inviscid fully-nonlinear problem for a radius/submergence ratio of 0.2 at three different depth Froude numbers. These same Froude number calculations were run using our pseudo spectral method, with 40 points on the body and approximately 15 points per wavelength. The 1^{st} and 3^{rd} order solutions are compared to the fully nonlinear Scullen and Tuck results in figures 1 and 2.

The agreement with the results of Scullen and Tuck is very good for the 3^{rd} order calculations, especially considering the theoretical accuracy of the method is only 3^{rd} order in wave steepness (a/λ). There is poor comparison between the 1^{st} order solutions and the fully nonlinear calculation, except in the high froude number case which is consistent with the the observation made by Tuck [6]. Tuck showed several issues with the linear solution. Overall the method quite accurately reproduces the fully nonlinear solution, and the results show that the linear solution has significant errors for this problem.

3.3 Submerged heaving cylinder

Results for a cylinder heaving below the free surface are shown in figure 3. Again these results were calculated for 40 points on the body and approximately 15 points per wavelength. At present these have not been compared with any previous results, but comparison with either experiment or theoretical results will be attempted.

References

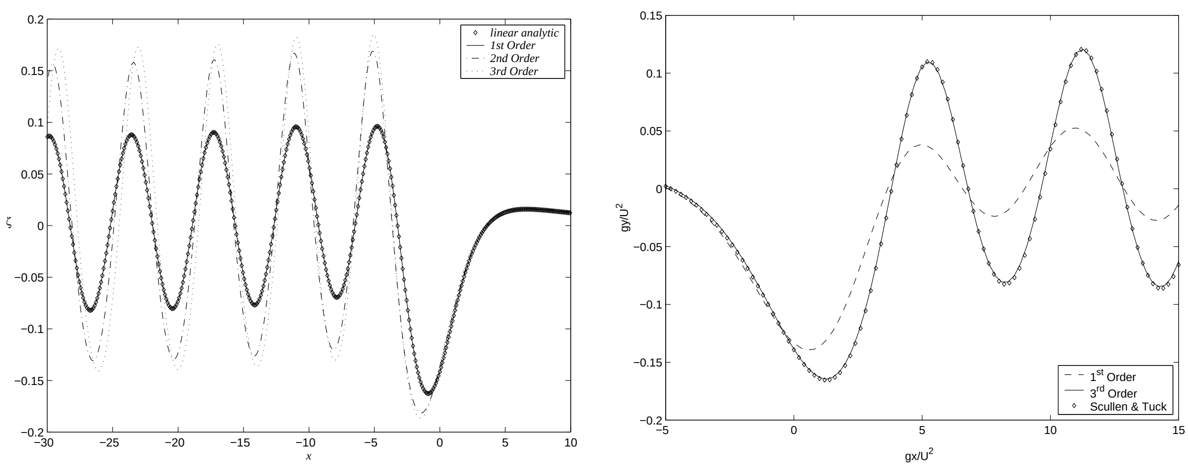
[1] Beck, R. F. 1999, A fully nonlinear water wave computations using a desingularized Euler-Lagrange time-domain approach, Nonlinear Water Wave Interaction, WIT Press, pp. 1-58.

[2] Choi, W. 1995 Nonlinear evolution equations for two-dimensional waves in a fluid of finite depth. J. Fluid Mech. 295, 381-394

[3] Havelock, T. 1926, The Method of Images in Some Problems of Surface Waves. Proc. Royal Soc. A. 115, 265-277

[4] Scullen, D. & Tuck, E. O. 1995, Nonlinear Free-surface Flow Computations for Submerged Cylinders J. Ship Res. 39(1995), 185-193

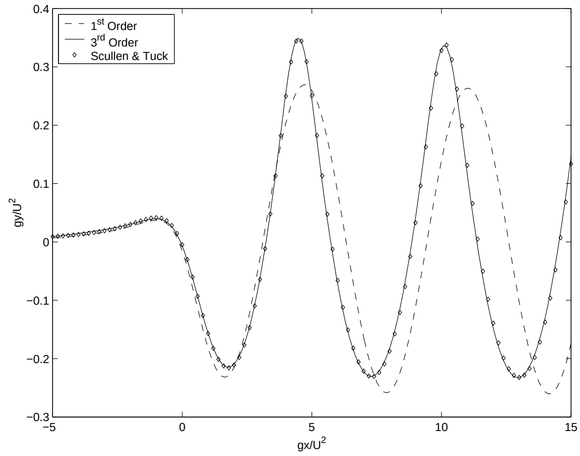
[5] Tuck, E. O. 1965, The effect of non-lineaity at the free surface on flow past a submerged cylinder J. Fluid Mech. 22, part 2, 401-414



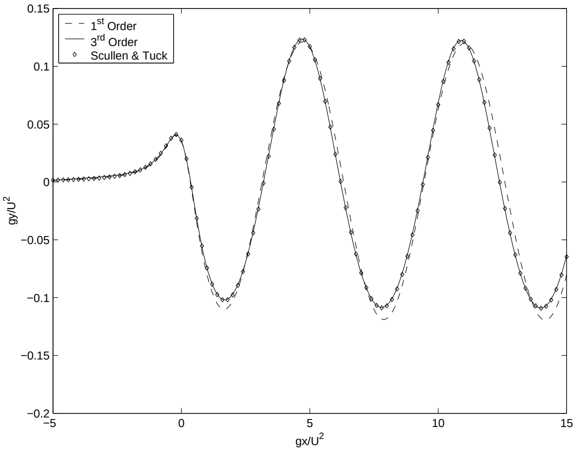
(a) Translating unit dipole $\frac{U}{\sqrt{gh}} = \frac{1}{\sqrt{5}}$

(b) Translating cylinder $\frac{U}{\sqrt{gh}} = 0.4$

Figure 1: Plot of ζ for calculations for translating dipole and for a translating submerged cylinder

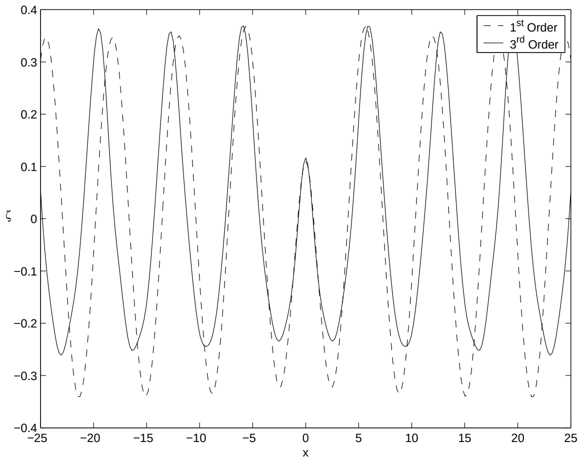


(a) Translating cylinder $\frac{U}{\sqrt{gh}} = 0.8$

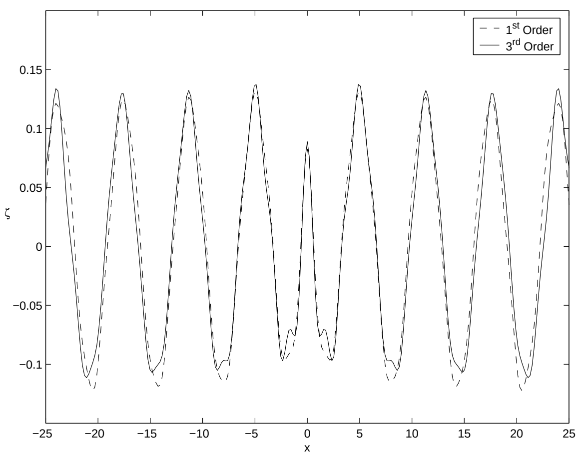


(c) Translating cylinder $\frac{U}{\sqrt{gh}} = 1.2$

Figure 2: Plot of ζ for calculations for translating submerged cylinders



(a) Deep submergence and large amplitude
$\frac{amp.}{R} = 1,\ \frac{h}{R} = 3,\ \omega\sqrt{\frac{R}{g}} = 1$



(b) Shallow submergence and small amplitude
$\frac{amp.}{R} = .1,\ \frac{h}{R} = 1.25,\ \omega\sqrt{\frac{R}{g}} = 1$

Figure 3: Plot of ζ for preliminary calculations for cylinder heaving below the free surface

Discusser: H. Bingham

Explicit free surface perturbation/Taylor expansion schemes such as the one you use here tend not to converge for nonlinearity about 80% of the highest stable wave. This is true in all water depths and the nonlinearity can be measured in either H/λ or H/h. Can you confirm this or are you able to exceed this limit?

Author's reply:

We cannot confirm this convergence issue with our formulation to third order in wave steepness, although we have successfully completed runs for wave of 85 % of the highest wave with good preservation of the wave shape. For a run at $H/\lambda = 0.12$ energy was conserved to better than 0.005 % and the phase speed error was approximately 0.5 %. This was achieved with the maximum wave-number of interest adjusted in the same manner as detailed in the paper. Attempts at running an $H/\lambda = 0.13$ wave were made but the wave shape was preserved poorly though the simulation remained stable. It is possible that this is the limit of wave amplitude that our 3^{rd} order formulation can handle and higher order terms are required to describe higher amplitude waves.