**JETTING BY FLOATING WEDGE IMPACT**

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The problem of sudden vertical motion of a floating wedge was studied by Iafrati & Korobkin (2003) within the potential theory of incompressible liquid flow. Combined initial asymptotics of the generated flow has been derived and analyzed in details. The flow region was divided into the main flow region and small vicinities of the intersection points between the wedge side walls and the liquid free surface. The inner flow close to the intersection points was obtained by means of combination of numerical and analytical methods. It was shown that the inner flow is non-linear and self-similar in the leading order during the initial stage of the impact. Close to the intersection points the free surface is turned over and the jets are originated. It should be noted that the free surface shape was numerically calculated as a part of the solution. The analysis of the non-linear boundary-value problem in the inner region revealed that this problem cannot describe the flow in the jet region and, in particular, cannot provide estimate of the jet length at the initial stage of the impact. Asymptotic analysis of the inner solution showed that the jet is predicted to be of infinite length with the jet thickness rapidly decreasing with the distance from the intersection points but with the flow velocity in the jet growing linearly with the distance. This implies that the first-order solution derived by Iafrati & Korobkin (2003) is not uniformly valid and should be improved in the jet region. The flow in the jet region caused by a floating body impact was not studied before. We do not expect that the details of the flow in thin jet region strongly affect either the pressure distribution in the main flow region or the free surface shape outside the jet region. Moreover, these thin jets are expected to be disintegrated into clouds of droplets owing to instability of the high-speed jets and, therefore, cannot be detected in experiments. However, formally speaking, the analysis by Iafrati & Korobkin (2003), which does not account for instability mechanisms, is incomplete because it does not contain explanations of the origin of the jets and gives no idea how the jet length can be estimated, which makes confusions in practical implications of the derived initial asymptotics. These subjects are covered in the present report.

It should be noted that infinite jets within the incompressible liquid model are well known in the theory of water entry problems (Wilson, 1989). Due to physical reasons - the liquid particles in the jets move inertially and independently, the feedback of the jet flow to the flow in the main region can be well neglected - the jet flow was not studied for long time. Analysis of such jets was initiated by attempts to calculate kinetic energy evacuated from the main flow region with these jets (Korobkin, 1994). Surprisingly, it was found that the jet energy is comparable with that in the main bulk of the liquid. Later on, the better method of estimating the jet energy was suggested (Molin *el al.*, 1996), which does not require details of the flow in the jet region. Nevertheless, in the problem of blunt body impact onto a liquid free surface Korobkin (1997) argued that the compressibility effects should be taken into account to obtain both the shape and the length of the spray jet. Calculations have been performed for the entry of a parabolic contour at constant velocity. This idea is used in the present report to estimate the length of the jet produced by impact of a floating wedge.

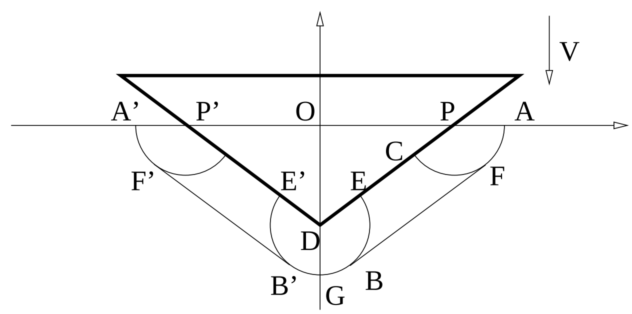


Figure 1.

The jetting by a floating wedge impact starts at the very early stage, when the compressibility effects must be taken into account and the disturbed part of the liquid is localized near the body surface. In Figure 1 the wave pattern at this stage of the impact is shown: $BF$ and $B'F'$ are the shock fronts, $CFA$, $C'F'A'$ and $E'B'GBE$ are the fronts of the relief waves. If the impact velocity $V$ is much smaller than the sound speed $c_0$ in the liquid at rest, the fronts of the relief waves are approximately circular with the radius being equal to $c_0t'$, where $t'$ is dimensional time. The distinguished stage lasts until points $C$ and $E$ meet each other, which happens at $t' = t_*$, $t_* = |DC|/(2c_0)$. During this stage the free surface is already deformed and the jet region is formed (these deformations are not shown in the figure). The liquid particles, which entered the jet region during this stage, will form the jet head at any following time instant. Important feature of the stage is that the flows in regions $BFCE$ and $B'F'C'E'$ are one-dimensional and the flows in regions $CFA$, $C'F'A'$ and $E'B'GBE$ are self-similar, which highly simplifies the analysis. Within the acoustic approximation the pressure in $BFCE$ is constant $p' = \rho_{\ell 0}c_0V_i$ and the flow velocity is equal to the normal velocity of the wedge surface $V_i = V\cos\gamma$, $\rho_{\ell 0}$ is the density of the liquid at rest and $\gamma$ is the wedge deadrise angle. The flow in $E'B'GBE$ is not considered here.

We consider the flow in $CFA$, where the jet is initiated, within the moving coordinate system $xPy$ attached to the wedge wall $DP$. Both the flow and wave patterns are shown in Figure 2. In the moving coordinates the flow is equivalent to that due to the liquid wedge impact onto the rigid plate and is self-similar. We shall determine the uniformly valid asymptotics of the liquid flow and the pressure distribution in the region $CFALP$ with the Mach number $M = V_i/c_0$ being a small parameter of the problem, and evaluate the jet length $L_{jet} = |PL|$.

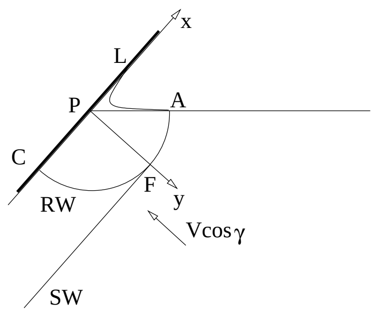


Figure 2: Sketch and notation used for the inner problem.

## Formulation of the problem

In the coordinate system $xPy$ the compressible flow in region $CFA$ is self-similar. The velocity component $u'(x', y', t')$ along the rigid surface, the component $v'$ normal to the surface, the hydrodynamic pressure $p'$ and the liquid density $\rho'_\ell(x', y', t')$ have the forms

$$u' = V_i U(\xi, \eta), \quad v' = V_i V(\xi, \eta), \quad p' = \rho_{\ell 0} V_i c_0 P(\xi, \eta),$$

$$\rho'_\ell = \rho_{\ell 0} R_\ell(\xi, \eta), \quad \xi = x'/(c_0 t'), \quad \eta = y'/(c_0 t').$$

Prime stands for dimensional variables. The new unknown functions $U(\xi,\eta)$, $V(\xi,\eta)$, $P(\xi,\eta)$ and $R_\ell(\xi,\eta)$ satisfy the following equations in the flow domain $CFALP$

$$\mathcal{L} < U >= R_\ell^{-1} P_\xi, \quad \mathcal{L} < V >= R_\ell^{-1} P_\eta, \tag{1}$$

$$\mathcal{L} < P >= (1+MnP)(U_\xi+V_\eta), \quad R_\ell = (1+MnP)^{\frac{1}{n}}, \tag{2}$$

$$\mathcal{L} = (\xi - MU)\partial/\partial\xi + (\eta - MV)\partial/\partial\eta, \tag{3}$$

the boundary conditions

$$P = P_{1D}(M), \quad U = 0, \quad V = 0 \quad (CF), \tag{4}$$

$$P = 0, \quad U = 0, \quad V = -1 \quad (FA), \tag{5}$$

$$V = 0 \quad (CL), \tag{6}$$

$$P = 0 \quad (AL), \tag{7}$$

$$V[1 + Mh'(\eta)] = U \tan\gamma - 1 - h(\eta) + \eta h'(\eta), \tag{8}$$

where $n$ is a characteristic of the liquid ($n = 7.15$ for pure water), $P_{1D}(M)$ is the one-dimensional pressure behind the shock front $FB$, the free surface shape is described by the equation

$$\xi \tan\gamma = \eta + Mh(\eta) + M. \tag{9}$$

The solution of the boundary-value problem (1) - (8) depends on the only parameter $M$, which is small in the present analysis. Assuming that the unknown functions and their first derivatives have finite limits as $M \to 0$, we arrive at the acoustic approximation. The acoustic solution has to be considered as the leading-order outer solution of the original problem and has to be verified against the basic assumption.

## Acoustic approximation

The limits of the unknown functions as $M \to 0$ are denoted with the subscript (0). It should be noted that $P_{1D}(M) \to 1$ and

$$(AL) \to \{0 < r < 1, \quad \hat\theta = \gamma\},$$

$$(AF) \to \{r = 1, \gamma < \hat\theta < \pi/2\},$$

$$(CF) \to \{r = 1, \pi/2 < \hat\theta < \pi\}$$

as $M \to 0$, where $\xi = r\cos\hat\theta$ and $\eta = r\sin\hat\theta$. In the leading order the outer flow is potential

$$U^{(0)} = \varphi_\xi, \quad V^{(0)} = \varphi_\eta, \quad P^{(0)} = \xi\varphi_\xi + \eta\varphi_\eta - \varphi \tag{10}$$

but the velocity potential $\varphi(\xi,\eta)$ is not an harmonic function, it satisfies the equation

$$(1 - \xi^2)\varphi_{\xi\xi} - 2\xi\eta\varphi_{\xi\eta} + (1 - \eta^2)\varphi_{\eta\eta} = 0.$$

However, the acoustic pressure $P^{(0)}(\xi,\eta)$ is harmonic function within the deformed coordinates $R$ and $\hat\theta$, where the new radial coordinate $R$ is connected with the original radial coordinate $r$ by Chaplygin's transform $r = 2R/(1 + R^2)$. This transform has been used by Dobrovol'skaya (1961) in the self-similar problem of wedge entering acoustic half-plane. Within the new coordinates $R$ and $\hat\theta$ the acoustic pressure $P^{(0)}(R,\hat\theta)$ is governed by the boundary-value problem

$$R^2 P^{(0)}_{RR} + RP^{(0)}_R + P^{(0)}_{\hat\theta\hat\theta} = 0 \quad (0 < R < 1, \gamma < \hat\theta < \pi),$$

$$P^{(0)} = 0 \quad (\hat\theta = \gamma), \quad P^{(0)}_{\hat\theta} = 0 \quad (\hat\theta = \pi),$$

$$P^{(0)} = 1 \quad (\pi/2 < \hat\theta < \pi), \quad P^{(0)} = 0 \quad (\gamma < \hat\theta < \pi/2),$$

solution of which has the form

$$P^{(0)} = \frac{2}{\beta}\sum_{n=0}^{\infty} \frac{R^{\sigma_n}}{\sigma_n}\cos[\sigma_n(\pi/2 - \gamma)]\sin[\sigma_n(\hat\theta - \gamma)], \tag{11}$$

where $\beta = \pi - \gamma$, $\sigma_n = \sigma_0(2n + 1)$ and $\sigma_0 = \pi/(2\beta)$.

The velocity potential is recovered with the help of equations (10), (11) and the boundary conditions (4) - (6). In a small vicinity of the intersection point, $r \ll 1$, the velocity potential behaves like

$$\varphi \simeq -Ar^{\sigma_0}\cos(\sigma_0\theta) \ , \tag{12}$$

where $\theta = \pi - \hat\theta$ and $A = 2^{2-\sigma_0}\sin[\pi^2/(4\beta)]/[\pi(1-\sigma_0)]$. It is important to notice that the asymptotic formula (12) is similar to the corresponding formula derived by Iafrati & Korobkin (2003) within the incompressible liquid model. The peculiarity of the present case affects only the formula for the factor $A$. Here $\frac{1}{2} < \sigma_0 < 1$, which implies that the acoustic solution predicts unbounded velocity of the flow close to the intersection point. In order to obtain uniformly valid description of the flow during the stage under consideration, an 'inner' solution must be considered within stretched variables. Since there is the only small parameter, $M$, in the problem, the stretching should be dependent on this parameter.

### Non-linear inner flow

Asymptotic analysis provides that the stretched variables in a small vicinity of the intersection point $P$ have to be introduced as

$$r = (MA)^{\frac{1}{2-\sigma_0}}\rho, \quad U = M^{-\frac{1-\sigma_0}{2-\sigma_0}}A^{\frac{1}{2-\sigma_0}}\tilde{u}(\rho,\theta),$$
$$V = M^{-\frac{1-\sigma_0}{2-\sigma_0}}A^{\frac{1}{2-\sigma_0}}\tilde{v}(\rho,\theta), \tag{13}$$
$$P = M^{\frac{\sigma_0}{2-\sigma_0}}A^{\frac{2}{2-\sigma_0}}\tilde{p}(\rho,\theta),$$
$$h = M^{-\frac{1-\sigma_0}{2-\sigma_0}}A^{\frac{1}{2-\sigma_0}}\zeta(\mu) - 1,$$
$$\xi = (MA)^{\frac{1}{2-\sigma_0}}\lambda, \quad \eta = (MA)^{\frac{1}{2-\sigma_0}}\mu,$$

where the new unknown functions $\tilde{u}(\rho,\theta)$, $\tilde{v}(\rho,\theta)$, $\tilde{p}(\rho,\theta)$ and $\zeta(\mu)$ are assumed bounded together with their first derivatives. The second equation in (2) shows that $R_\ell = 1 + O[M^{\frac{2}{2-\sigma_0}}]$ as $M \to 0$ close to the intersection point, which implies that in the leading order the inner flow can be treated as incompressible. The left-hand side of the first equation in (2) is of the order of $O[M^{-1}]$ but the left-hand side is of the order $O[M^{\frac{\sigma_0}{2-\sigma_0}}]$ and tends to zero as $M \to 0$. Therefore,

$$\tilde{u}_\lambda + \tilde{v}_\mu = O[M^{\frac{2}{2-\sigma_0}}] \tag{14}$$

and equations (1) provide

$$\tilde{\mathcal{L}} < \tilde{u} >= \tilde{p}_\lambda + O[M^{\frac{2}{2-\sigma_0}}], \quad \tilde{\mathcal{L}} < \tilde{v} >= \tilde{p}_\mu + O[M^{\frac{2}{2-\sigma_0}}],$$
$$\tilde{\mathcal{L}} = (\lambda - \tilde{u})\partial/\partial\lambda + (\mu - \tilde{v})\partial/\partial\mu. \tag{15}$$

Equations (14) and (15) show that in the leading order the inner flow is potential with accuracy up to $O[M^{\frac{2}{2-\sigma_0}}]$ as $M \to 0$. The inner velocity potential $\phi(\lambda,\mu)$ satisfies the Laplace equation

$$\Delta\phi = 0 \quad \text{(in the flow region)}, \tag{16}$$
$$\tilde{u} = \phi_\lambda, \quad \tilde{v} = \phi_\mu,$$
$$\tilde{p} = \lambda\phi_\lambda + \mu\phi_\mu - \phi - \frac{1}{2}(\phi_\lambda^2 + \phi_\mu^2).$$

The boundary conditions for equation (16) are

$$\phi_\mu = 0 \quad (\mu = 0, \lambda < \lambda_j), \tag{17}$$

$$\lambda\phi_\lambda + \mu\phi_\mu - \phi = \frac{1}{2}(\phi_\lambda^2 + \phi_\mu^2) \quad (AL), \tag{18}$$

$$\phi_\mu(1+\zeta') = \phi_\lambda \tan\gamma - \zeta(\mu) + \mu\zeta'(\mu) \quad (AL), \tag{19}$$

where the free surface position is described by the equation

$$(AL) \qquad \lambda\tan\gamma = \mu + \zeta(\mu) \tag{20}$$

and $\lambda_j$ is the jet length in the stretched coordinates. In the original coordinates the jet length $L_{jet}(t')$ grows linearly with time as

$$L_{jet}(t') = M^{-\frac{1-\sigma_0}{2-\sigma_0}}A^{\frac{1}{2-\sigma_0}}\lambda_j(V_i t'). \tag{21}$$

The quantity $\lambda_j$ has to be determined as a part of the solution of the problem for the inner velocity potential $\phi(\lambda,\mu)$. The inner flow has to be matched to the acoustic flow in the main flow region. Asymptotic formula (12) provides the far field condition for unknown velocity potential $\phi(\lambda,\mu)$

$$\phi \sim -\rho^{\sigma_0}\cos(\sigma_0\theta) \quad (\rho \to \infty). \tag{22}$$

It is convenient to introduce the modified velocity potential $S = \phi - \frac{1}{2}\rho^2$, which satisfies the following equations

$$\Delta S = -2 \qquad \text{in the flow region}, \tag{23}$$

$$S_n = 0 \qquad (AL \quad \text{and} \quad \mu = 0), \tag{24}$$

$$S_\tau^2 + 2S = 0 \qquad (AL), \tag{25}$$

$$S \simeq -\frac{1}{2}\rho^2 - \rho^{\sigma_0}\cos(\sigma_0\theta) \qquad (\rho \to \infty), \tag{26}$$

where $S_n$ and $S_\tau$ are the normal and tangential derivatives of the unknown function $S(\lambda,\mu)$ on the liquid boundary. Equations (23) - (26) follow from (16) - (19) and (22), respectively. The boundary-value problem (23) - (26) is similar to the corresponding inner problem within the incompressible liquid model except of the dynamic boundary condition (25), which is much simpler for compressible liquid. This condition can be integrated along the free surface with the result

$$S = -\frac{1}{2}(\tau + C)^2, \tag{27}$$

where $\tau$ is the curvilinear coordinate along the free surface and $C$ is an arbitrary constant of integration. The local analysis of the flow close to the jet tip shows that the distribution of the modified potential (27) matches the boundary condition (24) at the jet tip with finite jet angle if and only if $C = 0$ and $\tau$ is measured from the point $L$ towards the far field. Equation (27) implies that the inner velocity potential along the free surface is known

$$\phi = \frac{1}{2}[\rho^2 - \tau^2] \tag{28}$$

once the free surface shape has been obtained.

The inner problem (23) - (26) is non-linear and still complicated. Moreover, the shape of the free surface is unknown in advance and has to be determined together with the liquid flow. This can be only done by numerical methods. In order to reduce the size of the computational domain required to solve the inner problem (23) - (26), the asymptotic behavior of the potential in the far field is estimated. The free-surface shape in the far field is described by the equation $\theta = \tilde{\theta}(\rho)$, where $\tilde{\theta}(\rho) \to \beta$ as $\rho \to \infty$. It is useful to introduce the new angular variable $\alpha = \theta\beta/\tilde{\theta}(\rho)$ so that the fluid domain in the far field corresponds to $0 \le \alpha \le \beta$. Three cases are distinguished: (i) $\gamma > \pi/4$,

(ii) $\gamma = \pi/4$ and (iii) $\gamma < \pi/4$. The case $\gamma > \pi/4$ is only considered here. We obtain

$$\tilde{\theta}(\rho) = \beta + \frac{\sigma_0}{2-\sigma_0}\rho^{\sigma_0 - 2} + \theta_2(\rho), \qquad (29)$$

$$\bar{S}(\rho,\alpha) = -\frac{1}{2}\rho^2 - \rho^{\sigma_0}\cos(\sigma_0\alpha) + \frac{\sigma_0^2}{(2-\sigma_0)}\times$$
$$\left\{\frac{\alpha}{\beta}\sin(\sigma_0\alpha) + \left[\frac{4-3\sigma_0}{3-2\sigma_0}\right]\frac{\cos[2(1-\sigma_0)\alpha]}{2\cos(2\gamma)}\right\}\rho^{2\sigma_0-2} \qquad (30)$$
$$+\bar{S}_2(\rho,\alpha),$$

$$\bar{S}(\rho,\beta) = -\frac{1}{2}\rho^2 + \frac{\sigma_0^2}{2(3-2\sigma_0)}\rho^{2\sigma_0-2} + \bar{S}_2(\rho,\beta), \qquad (31)$$

where the products $\bar{S}_2(\rho,\alpha)\rho^{2(1-\sigma_0)}$ and $\theta_2(\rho)\rho^{2-\sigma_0}$ tend to zero as $\rho \to \infty$.

For the wedge deadrise angle of 60 degrees, $\gamma = \pi/3$, the modified velocity potential in the far field is given in parametric form as

$$\theta = \alpha\left\{1 + \frac{9}{10\pi}\rho^{-\frac{5}{4}} + ...\right\} \qquad (0 < \alpha < \frac{2}{3}\pi),$$

$$S(\rho,\theta) = -\frac{1}{2}\rho^2 - \rho^{\frac{3}{4}}\cos(3\alpha/4)$$
$$+\frac{9}{20}\left\{\frac{3\alpha}{2\pi}\sin(3\alpha/4) - \frac{7}{6}\cos(\alpha/2)\right\}\rho^{-\frac{1}{2}} + ......,$$
$$S(\rho,\tilde{\theta}(\rho)) = -\frac{1}{2}\rho^2 + \frac{3}{16}\rho^{-\frac{1}{2}} + ......,$$

**Numerical solution**

The boundary value problem (23) - (26) is solved with the help of a pseudo-time stepping iterative procedure similar to that developed in Iafrati & Korobkin (2003). At each iteration a boundary integral representation for the velocity potential $\phi$ is used. The velocity potential is assigned along the far field boundary by (29) and (30) and its normal derivative is assigned along the wetted portion of the body contour. Along the free surface the velocity potential is assigned according to equation (27) where the curvilinear coordinate $\tau$ is evaluated from the free surface shape obtained at the end of the previous iteration.

Taking the limit of the boundary integral representation on the fluid domain boundary, a boundary integral equation of mixed kind is obtained solution of which, achieved by using a zero order panel method, provides the velocity potential along the free surface and its normal derivative along the far field boundary and the free surface. In this way $\partial S/\partial n$ is derived as $\partial\phi/\partial n - \rho\rho_n$ and then is verified to which extent the kinematic constraint (24) is satisfied. Then, the free surface is moved in a pseudo-time stepping fashion by using $(\partial\phi/\partial\tau)\boldsymbol{\tau} + (\partial S/\partial n)\boldsymbol{n}$ as a velocity field. This velocity field ensures that once convergence is achieved, that is $\partial S/\partial n = 0$ along the free surface, a further update only shifts the free surface panels along it while left unchanged the free surface profile. Once the free surface shape has been updated, the curvilinear coordinate along it is reinitialized thus allowing to recompute the velocity potential along it through equation (27).

The iterative procedure is started by using equations (29) and (31) to assign an initial guess for the free surface and for the velocity potential along it, respectively.

In order to reduce the computational effort needed to accurately describe the flow in the thin jet layer, a shallow water model similar to that adopted in Battistin & Iafrati (2004) has been adopted. In this model the velocity potential within the jet layer is written as an expansion in terms of the local jet thickness and is matched with the solution provided by the boundary element solution. The matching is enforced in a region where the jet thickness is still large compared with the local panel size. This expansion of the velocity potential allows to derive the velocity field for the free surface panels lying within the modeled part of the jet.

In Figure 3 the free surface shape about the jet root for $\gamma = \pi/3$. From this result the jet length can be also evaluated as $\lambda_j \simeq 1.68$.

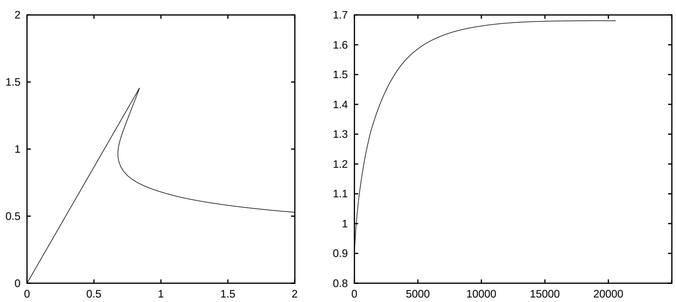


Figure 3: Solution for $\gamma = \pi/3$. On left the final free surface shape is shown. On right the convergence history of $\lambda_j$ is drawn.

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