Resonances of a heaving structure with a moonpool

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SUMMARY

A freely-floating structure with a moonpool constrained to move in heave can display two types of resonance. One is associated with the fluid motion in the moonpool and the other with the hydrostatic restoring force on the structure. Here the interaction between these resonances is investigated and time-domain calculations are used to illustrate the main results.

1 INTRODUCTION

This work is concerned with floating structures, constrained to move in heave, that enclose a portion of the free surface to give a "moonpool". Such a structure in two-dimensions has two separated elements that both intersect the free surface, and in three dimensions there is a single element with a "hole" that contains an isolated portion of the free surface. Structures with a moonpool have been investigated by many authors; the main features are discussed, for example, in [1] within the context of a floating torus. In the linearised water-wave problem for such structures two types of resonant behaviour can be identified. The first, referred to here as type I, is the motion of the internal free surface as might, for example, be excited by a wave incident on the fixed structure. The second, or type II, resonance arises from the motion of the structure and, in particular, from the hydrostatic restoring force.

In the present work these resonances, and the interaction between them, are investigated by examination of the frequency-domain singularity structures of the velocity potential and hydrodynamic coefficients. These results from the frequency domain are then interpreted in terms of time-domain motions. At the time of writing the work is incomplete and partial results only are given that mainly, but not exclusively, focus on radiation problems in which there is no incident wave. The main result is that the type II resonance is usually dominant.

Structures of the type discussed here include some special geometries that support a "trapped mode" [2]. In the presence of a fixed structure, such a mode is a free oscillation of the fluid that has finite energy, does not radiate waves to infinity, and in the absence of viscosity will persist for all time. The forced oscillation of trapping structures has been investigated elsewhere [3] and it was shown that trapped modes are excited by almost any forcing. Here it is shown that trapped modes cannot be excited by any motion of a freely-floating trapping structure.

The plan of this abstract is as follows. The problem is first formulated in the time domain and the link between resonant motions in the time domain and singularities of frequency-domain quantities is described. Numerical simulations are used to illustrate the main points.

2 THE INITIAL-VALUE PROBLEM

An inviscid and incompressible fluid with a free surface is contained within a horizontal layer of depth h that is bounded below by a rigid bed and extends to infinity in all horizontal directions. Cartesian coordinates (x, y, z) are chosen with z measured vertically upwards and the origin in the mean free surface. The fluid layer contains a floating structure constrained to move in heave with displacement $z = Z(t)$. The submerged surface of the structure is denoted by Γ, a normal coordinate to Γ is denoted by n, and n_z is the z component of the unit normal to Γ.

The fluid motion is assumed to be irrotational so that it may be described by a velocity potential $\Phi(\mathbf{x}, z, t)$, $\mathbf{x} = (x, y)$, that satisfies

$$\nabla^2\Phi = 0 \tag{1}$$

within the fluid, the bed condition

$$\frac{\partial\Phi}{\partial z} = 0 \quad \text{on} \quad z = -h, \tag{2}$$

the free-surface condition

$$\frac{\partial^2\Phi}{\partial t^2} = -g\frac{\partial\Phi}{\partial z} \quad \text{on} \quad z = 0, \tag{3}$$

where g is the acceleration due to gravity,

$$\frac{\partial\Phi}{\partial n} = \dot{Z}(t)n_z \quad \text{on} \quad \Gamma, \tag{4}$$

and for all time

$$\nabla\Phi \to 0 \quad \text{as} \quad |x| \to \infty. \tag{5}$$

The motion is subject to the initial conditions

$$\Phi(\mathbf{x}, 0, 0) = P(\mathbf{x}), \quad \frac{\partial\Phi}{\partial t}(\mathbf{x}, 0, 0) = Q(\mathbf{x}), \tag{6}$$

where $P(\mathbf{x})$ and $Q(\mathbf{x})$ correspond to a prescribed incident wave. It is assumed that this incident wave is initially localised within a region away from the structure so that the fluid around the structure is initially at rest.

The equation of motion of the structure is

$$m\ddot{Z}(t) = -\rho g W Z(t) \\ - \rho \iint_\Gamma \frac{\partial \Phi}{\partial t}(\mathbf{x}, z, t) n_z \, \mathrm{d}S + F(t). \quad (7)$$

By Archimedes principle the mass of the structure is the fluid density ρ times the submerged volume of the structure. The first term on the right-hand side of (7) is the hydrostatic restoring force and W is the waterplane area of the structure. The second term is the hydrodynamic force arising from the fluid motion, and $F(t)$ is an applied force (when $F(t) \equiv 0$ the structure is called "freely floating"). The initial displacement $Z(0)$ and velocity $\dot{Z}(0)$ of the structure are prescribed.

3 LONG-TIME ASYMPTOTICS

Information about the initial-value problem described in §2 can be obtained using a Fourier transform in time. The transforms of the potential $\Phi(\mathbf{x}, z, t)$ and displacement $Z(t)$ are respectively

$$\phi(\mathbf{x}, z, \omega) = \int_0^\infty \Phi(\mathbf{x}, z, t) \, \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}t \quad (8)$$

and

$$\zeta(\omega) = \int_0^\infty Z(t) \, \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}t. \quad (9)$$

The inversion formula for the potential is

$$\Phi(\mathbf{x}, z, t) = \frac{1}{\pi} \operatorname{Re} \int_0^\infty \phi(\mathbf{x}, z, \omega) \, \mathrm{e}^{-\mathrm{i}\omega t} \, \mathrm{d}\omega. \quad (10)$$

Resonances correspond to poles of the frequency-domain potential $\phi(\mathbf{x}, z, \omega)$ that lie on or close to the real ω axis. In the case of a pole on the axis the path of integration in (10) must pass above the pole.

A simple pole of ϕ at $\omega = \omega_0 - \mathrm{i}\epsilon$, with $\omega_0 > 0$ and $\epsilon \geq 0$ (from causality ϵ must be non-negative), gives

$$\phi(\mathbf{x}, z, \omega) \sim \frac{\phi_0(\mathbf{x}, z)}{\omega - (\omega_0 - \mathrm{i}\epsilon)} \quad \text{as} \quad \omega \to \omega_0 - \mathrm{i}\epsilon \quad (11)$$

and it may be shown from (10) that as $t \to \infty$

$$\Phi(\mathbf{x}, z, t) \sim -2 \operatorname{Re} \left\{ \mathrm{i}\phi_0(\mathbf{x}, z) \, \mathrm{e}^{-\mathrm{i}\omega_0 t} \right\} \mathrm{e}^{-\epsilon t}. \quad (12)$$

In general this is a damped oscillation of the fluid but for $\epsilon = 0$, which corresponds to a trapped mode, the oscillation persists for all time. Each pole of ϕ will contribute a similar term to the large-time asymptotics of Φ.

For a structure in infinite depth that is released from rest the decay of the motion is ultimately algebraic [4] and not a damped oscillation of the form given by (12). However, for the finite-depth case considered here this algebraic decay does not occur and the long-time asymptotic behaviour is indeed a damped oscillation.

4 THE FREQUENCY DOMAIN

The frequency-domain potential ϕ can be decomposed as

$$\phi(\mathbf{x}, z, \omega) = \phi_\mathrm{S}(\mathbf{x}, z, \omega) + v(\omega)\phi_\mathrm{R}(\mathbf{x}, z, \omega). \quad (13)$$

where ϕ_S is the scattering potential that satisfies

$$\frac{\partial \phi_\mathrm{S}}{\partial n} = 0 \quad \text{on} \quad \Gamma, \quad (14)$$

ϕ_R is the radiation potential that satisfies

$$\frac{\partial \phi_\mathrm{R}}{\partial n} = n_z \quad \text{on} \quad \Gamma, \quad (15)$$

and

$$v(\omega) = -\mathrm{i}\omega\zeta(\omega) - Z(0). \quad (16)$$

From the Fourier transform of the equation of motion (7), the frequency domain displacement

$$\zeta(\omega) = \frac{X(\omega) + f(\omega) - \mathrm{i}\omega[m + q(\omega)]Z(0) + m\dot{Z}(0)}{\rho g W - \omega^2[m + q(\omega)]}. \quad (17)$$

Here $X(\omega)$ is the exciting force corresponding to ϕ_S, $f(\omega)$ is the Fourier transform of $F(t)$, and the complex force coefficient

$$q(\omega) = a(\omega) + \mathrm{i}b(\omega)/\omega \quad (18)$$

where a and b are respectively the added mass and damping coefficients.

5 RESONANCES

Although some numerical results for motions resulting from an incident wave will be presented later, the discussion in this section focuses on radiation problems. In the time domain the structure is either forced to move by the application of an applied force $F(t)$, or given an initial displacement or velocity and allowed to move freely thereafter.

A type I resonance (see §1) is associated with a simple pole of the radiation potential ϕ_R. For $\epsilon \neq 0$, fluid oscillations in the time domain described by (12) can arise from forced oscillations of the structure [3]. In the special case $\epsilon = 0$, corresponding to a trapped mode of frequency ω_0, the situation is more complicated and the excitation of both steady and growing oscillations of the fluid by the forced oscillations of the structure is possible [3].

A type II resonance (see §1) is associated with the motion of the structure and corresponds to a pole of the displacement ζ given in (17); the location of the pole in the complex ω plane is a solution of

$$\rho g W - \omega^2[m + q(\omega)] = 0. \quad (19)$$

For real ω the peak response is in the vicinity of a solution for ω of the real part of this equation.

For a floating structure with an internal free surface both types of resonance can occur and there will be a mutual influence through the last term in (13). A pole in the radiation potential leads to a corresponding singularity in the complex force coefficient q so that as $\omega \to \omega_0 - \mathrm{i}\epsilon$

$$q(\omega) = \frac{q_{-1}}{\omega - (\omega_0 - \mathrm{i}\epsilon)} + q_0 + O\big(\omega - (\omega_0 - \mathrm{i}\epsilon)\big). \quad (20)$$

Here q_{-1} and q_0 are constants and, for small ϵ at least, q_{-1} is real and negative to ensure that the damping coefficient is non-negative [5]. In the absence of an incident wave so that $X = 0$, and if f has no singularities in the vicinity of the pole in q, then for real frequencies around $\omega = \omega_0$ the peak displacement (17) occurs at ω_0 and

$$\zeta(\omega_0) = \frac{\mathrm{i}Z(0)}{\omega_0} + O(\epsilon) \quad \text{as} \quad \epsilon \to 0 \qquad (21)$$

so that from (16)

$$v(\omega_0) = O\big(\epsilon\big) \quad \text{as} \quad \epsilon \to 0. \qquad (22)$$

Thus, for a type I resonance with small damping ϵ, $v(\omega_0)$ is also small and will much reduce the effects of the pole in ϕ_{R}. Thus, both the fluid and structural motions will be dominated by the type II resonance.

In the case of a trapping structure, for which $\epsilon = 0$, then $v(\omega) = O\big(\omega - \omega_0\big)$ as $\omega \to \omega_0$ and the pole in ϕ_{R} is annulled completely so that a trapped mode cannot be excited by the motions of a freely-floating body. In addition there is no pole in the scattering potential at $\omega = \omega_0$ and a trapped mode cannot be excited by an incident wave [6]. Thus, on the basis of linear theory, there is no mechanism for the excitation of trapped modes by wave interaction with a freely-floating trapping structure. Each case of trapped-mode excitation discussed in [3] involves the prescription of a structural velocity $\dot{Z}(t)$ that is equivalent to the application of an external force $F(t)$ whose Fourier transform $f(\omega)$ has a pole at the trapped-mode frequency $\omega = \omega_0$. With such a pole in f, equation (21) no longer holds, the pole in ϕ_{R} at $\omega = \omega_0$ is not annulled, and hence a trapped mode is excited.

6 NUMERICAL SIMULATIONS

Presented here are the results of numerical simulations in two dimensions performed using the time-domain method described in [3]. Times and frequencies are made non-dimensional by scaling appropriately by $\sqrt{h/g}$.

The structural geometry corresponding to figures 1–3 is a pair of closely-spaced half-immersed cylinders each of radius $0.28h$ and with centres at $\pm 0.31h$; the forced oscillations of this structure are discussed in [3]. The location of the pole corresponding to the type I resonance is given by $\omega_0 = 2.17$ and $\epsilon = 0.012$ (see [3]), while for the type II resonance solution of the real part of (19) gives the estimate $\omega_0 = 2.50$.

Figures 1 and 2 correspond to the free-motion of the structure (i.e. $F(t) = 0$) that results from the initial conditions $Z(0) = 1$, $\dot{Z}(0) = 0$; that is the structure is displaced and released from rest. Figure 1 shows the free-surface elevation η at the mid point of the internal free surface and the structural displacement Z as functions of time. After about time $t = 15$ both motions have settled to a decaying oscillation that appears to be dominated by a single frequency. This is confirmed by figure 2 which shows the discrete Fourier transform of the free-surface elevation shown in figure 1 ($|u_n|$ is proportional to the amplitude of the Fourier component with index n). The peak response is at the type II resonance while there is no discernible effect of the type I resonance as is to be expected from the discussion in §5.

The type I resonance can be excited by an incident wave on the fixed structure and this is illustrated in figure 3. The incident wave elevation is a Gaussian packet of plane waves with a peak frequency at $\omega \approx 2$. The Fourier transform of the signal (not shown) confirms that the oscillation is at the type I resonant frequency $\omega_0 = 2.17$.

The last set of results presented in figure 4 are for a freely-floating trapping structure subject to the same Gaussian wave packet referred to above. The peak frequency of the wave packet coincides exactly with the trapped-mode frequency. The structure is set in motion by the wave and a slowly decaying oscillation of both the free-surface and structure remains after the wave has passed (these oscillations are of small amplitude and are barely discernible on the scale of figure 4). The discrete Fourier transform of these time signals (not shown) reveals that the oscillations result from a type II resonance and there are no measurable components of the motion at the trapped-mode frequency as predicted by the arguments of §5.

7 CONCLUSION

A floating structure with a moonpool that is constrained to move in heave can support two types of resonant motion. One is associated with oscillations of the internal free surface and the other with the motion of the structure. It has been demonstrated that when the structure is released from rest the structural resonance dominates the response of both the fluid and structure. For the special case of a trapping structure the free-surface resonance cannot be excited at all by the motion of a freely-floating structure.

8 REFERENCES

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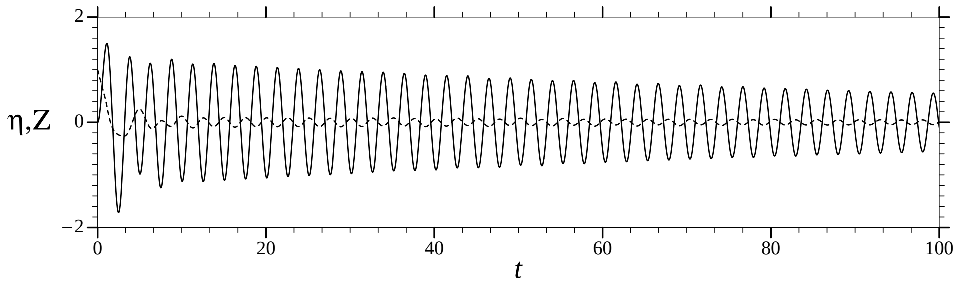

Figure 1: Release of two freely-floating half-immersed cylinders from rest: free-surface elevation η at the mid point of the internal free surface (——) and displacement Z of the structure (– – –).

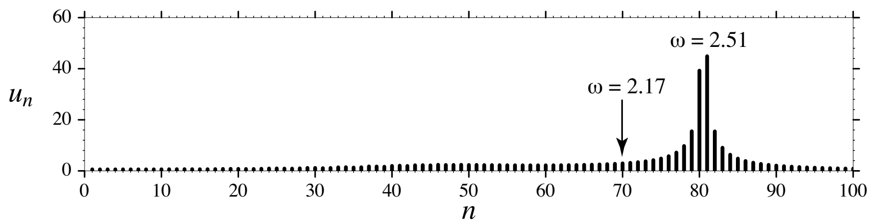

Figure 2: Discrete Fourier transform of the free-surface elevation given in figure 1.

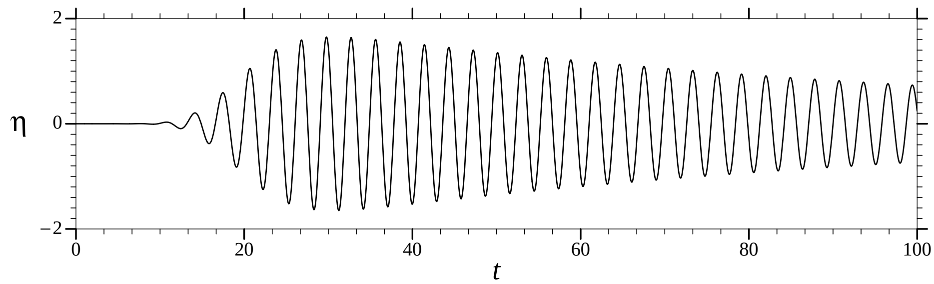

Figure 3: Two fixed half-immersed cylinders subject to an incident wave: free-surface elevation η at the mid point of the internal free surface (——).

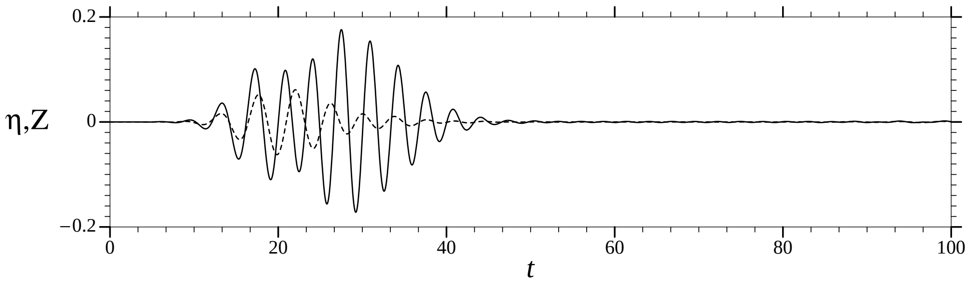

Figure 4: A freely-floating trapping structure subject to an incident wave: free-surface elevation η at the mid point of the internal free surface (——) and displacement Z of the structure (– – –).