# Wave-maker ramp functions in numerical tanks

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**Introduction**
In order to avoid possible numerical problems associated with rapid transients when a wavemaker is started from rest, it is not unusual to multiply the early stage of the wavemaker input signal by a starting ramp function. The aim of the present work is to investigate the effects of such ramp functions, particularly in the region of the wavefront. They have implications concerning the effects of reflections by structures, walls or beaches, and in relation to the initial direction of motion of floating or submerged untethered bodies when they undergo non-linear responses to an incoming wave front (e.g. in which direction do they initially drift?)

A study of the non-linear behaviour of the leading waves was given by Clamond & Grue (2000). The preliminary study described here is largely based on classical linear analysis, complemented by some results from fully non-linear simulations using a boundary element code. The linear analysis follows the approach described by Joo et al (1990), though their formulation also allows for surface tension which is ignored here.

**Derivation of the free surface elevation**
We consider long-crested waves generated by a piston wave-maker, driven with a velocity $u(t)$. The depth of the tank and gravitational acceleration are taken to be unity. The classical linear boundary conditions are invoked on the free surface, and the tank is initially considered to be infinitely long. Following Joo et al, we write the velocity potential as the sum of an impulsive term and a memory term: the former satisfies the boundary conditions on the wavemaker and the bottom of the tank, and a Dirichlet condition on the free surface; and the latter is a correction such that the total solution satisfies the free surface condition. Thus we have

$$\phi(x,z,t) = u(t)\bar{\phi}(x,z) + \phi^*(x,z,t). \tag{1}$$

$\bar{\phi}$ is a simple series, and $\phi^*$ is written as a Fourier cosine integral

$$\phi^*(x,z,t) = \int_0^\infty A(k,t)\cosh\left[k(z+1)\right]\cos kx\, dk. \tag{2}$$

The origin of coordinates is at the intersection of the wave maker and the mean free surface. Substitution of equation (2) into the free surface conditions leads to

$$A_{tt}\cosh k + Ak\sinh k = -\frac{2}{\pi}u(t)\frac{\tanh k}{k}. \tag{3}$$

The general solution of (3) is

$$A(k,t) = -\frac{2}{\pi}u_p(t)\frac{\tanh k}{k\cosh k} + c_1(k)\sin\beta t + c_2(k)\cos\beta t, \tag{4}$$

where $\beta^2 = k\tanh k$ and $u_p$ is the particular integral satisfying:

$$u_{ptt} + \beta^2 u_p = u(t). \tag{5}$$

We now impose initial conditions $A(0) = A_t(0) = 0$, and after some algebra obtain a solution for the free surface elevation:

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$$\eta(x,t) = \frac{2}{\pi}\int_0^\infty \left[\beta \sin\beta t\, u_p(0) - \cos\beta t\, u_{pt}(0) + u_{pt}(t)\right]\frac{\tanh k}{k}\cos kx\, dk. \qquad (6)$$

As discussed by Joo et al, the contribution from the impulsive term $\bar{\phi}$ is cancelled by a singular term arising in the expression for $\phi^*$.

**Elevations with and without ramp functions**

Joo et al give the result for an unramped sinusoidal piston wavemaker with imposed horizontal velocity $u(t) = \sin\omega t$, namely

$$\eta_0(x,t) = \frac{2\omega}{\pi}\int_0^\infty \frac{\cos\beta t - \cos\omega t}{\omega^2 - \beta^2}\frac{\tanh k}{k}\cos kx\, dk. \qquad (7)$$

We consider here three ramp functions, one of which tends to unity asymptotically as $t \to \infty$; and the others increase to unity during one cycle $(T = 2\pi/\omega)$. The second ramp provides continuity at $t = T$ with the subsequent sinusoidal velocity signal and its first two derivatives; while the third ramp leads to a discontinuity in acceleration at $t = T$.

Ramp 1

The piston velocity is defined by

$$u(t) = (1 - e^{-\gamma\omega t})\sin\omega t. \qquad (8)$$

The resulting wave elevation is (for $t>T$):

$$\begin{aligned}\eta_1(x,t) = \frac{2\omega}{\pi}\int_0^\infty &\{-2\gamma\omega\beta\sin\beta t + (\beta^2 - \omega^2(\gamma^2+1))\cos\beta t \\ &+ e^{-\gamma\omega t}[\gamma(\omega^2(\gamma^2+1)+\beta^2)\sin\omega t + (\omega^2(\gamma^2+1)-\beta^2)\cos\omega t]\} \\ &\cdot\frac{1}{[\omega^2(\gamma^2-1)+\beta^2]^2 + 4\gamma^2\omega^4}\frac{\tanh k}{k}\cos kx\, dk. \qquad (9)\end{aligned}$$

Ramp 2

The piston velocity is defined by

$$u(t) = \begin{cases} 2\pi\tau^2(-2\tau^2 + 5\tau - 3), & t \le T \\ \sin 2\pi\tau, & t > T \end{cases}, \qquad (10)$$

where $\tau = t/T$. It may be verified that $u(T^-) = u_{tt}(T^-) = 0$, and $u_t(T^-) = 2\pi$. The corresponding wave elevation can be written as two expressions valid for $t < T$ and $t > T$ respectively, after solving equation (3) with appropriate initial conditions in each case.

Ramp 3

The piston displacement has a cosine ramp function applied over the first cycle:

$$s(t) = \begin{cases} \frac{1}{2}(1 - \cos\frac{\omega t}{2})\sin\omega t, & t \le T \\ \sin\omega t, & t > T \end{cases}, \qquad (11)$$

The resulting wave elevation may be written (for $t>T$):

$$\begin{aligned}\eta_3(x,t) &= \eta_0(x,t-T) \\ &+ \frac{2}{\pi}\sum_i^3 a_i\int_0^\infty \frac{\omega_i\cos\beta t - \omega_i\cos\omega_i T\cos\beta(t-T) + \beta\sin\omega_i T\sin\beta(t-T)}{\omega_i^2 - \beta^2}\frac{\tanh k}{k}\cos kx\, dk, \qquad (12)\end{aligned}$$

where $\omega_1 = \omega$, $\omega_2 = \omega/2$, $\omega_3 = 3\omega/2$ and $a_1$=1/2, $a_2$=-1/8, $a_3$=-3/8, and $\eta_0$ is given in eq. (7).

It is straightforward also to obtain expressions for the propagating disturbances due to one or a few cycles of sinusoidal wavemaker motion, or one "cycle" of ramps 2 or 3 with $u(t) = 0$ for $t > T$. The results can be compared in various regions with the stationary phase approximation (the trailing waves), the Airy approximation (near the front) and the Fresnel envelope (at the front) – as described, for example, by Mei (1983, pages 26, 30 and 55 respectively). Here we restrict our attention to the continuously oscillating wavemaker, with and without a ramp.

**Reflected waves**

It is instructive to consider reflection by a vertical wall at the end of a tank. Dissipation by a beach can be considered subsequently. For a tank of length *L*, the incident plus reflected wave is simply

$$\eta_{total}(x,t) = \eta(x,t) + \eta(2L - x, t). \qquad (13)$$

**Results**

The following are for $\omega = 1$. Figure 1 shows time histories of the wave at (a): $x$ = 0; and (b): $x$ = 10, for the pure sinusoidal wavemaker, and two ramped inputs. To yield unit steady state amplitudes, the elevations are scaled by $c_g\, k_0 / \tanh k_0$, where $c_g$ and $k_0$ are the group velocity and wavenumber corresponding to a finite water-depth wave of frequency $\omega$. In the case of ramp 1, results are obtained here for $\gamma = 0.2$. Figures 2a – 2c show the wave profiles for these 3 cases at $t$ = 99, and figures 2d – 2f the equivalent results at $t = 100$. Results are given for both an infinitely long tank, and one of length $L$ = 60. The incident, reflected and total waves are shown separately. Figure 3 shows wave profiles corresponding to ramp3, and compares the results from the linear theory (eq. (12)) with those from a nonlinear quadratic Boundary Element (BE) analysis, calculated with a cosine ramp (ramp3) for two piston amplitudes: 0.01 and 0.043. Profiles at $t$=7$T$ and $t$=15$T$ are given in figures 3a and 3b respectively, for a tank of length 14. For the BE simulations, a numerical beach is operational over one wavelength at the right hand end of the tank. The elevations are scaled by the piston amplitude in these latter figures. Further results, including the implications of ramp functions on wave diffraction by a circular cylinder, will be discussed in the presentation.

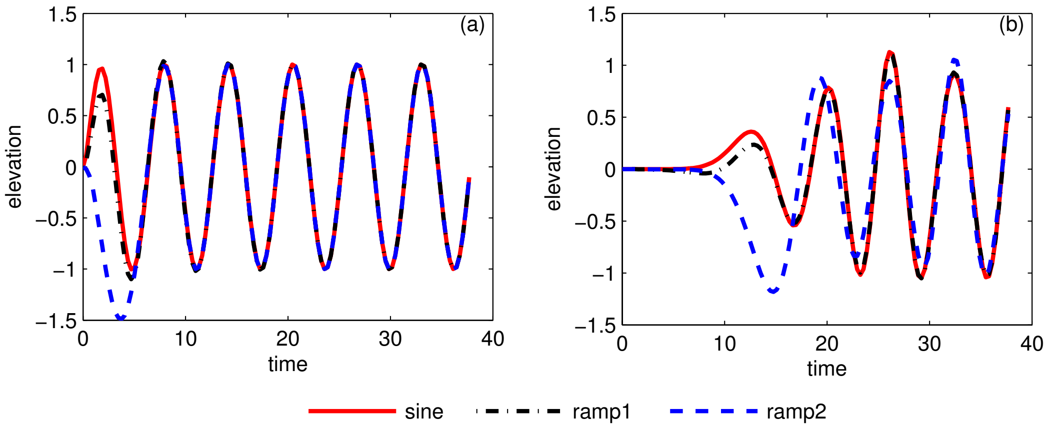


Figure 1. Time histories of elevation: a) $x$=0; b) $x$=10

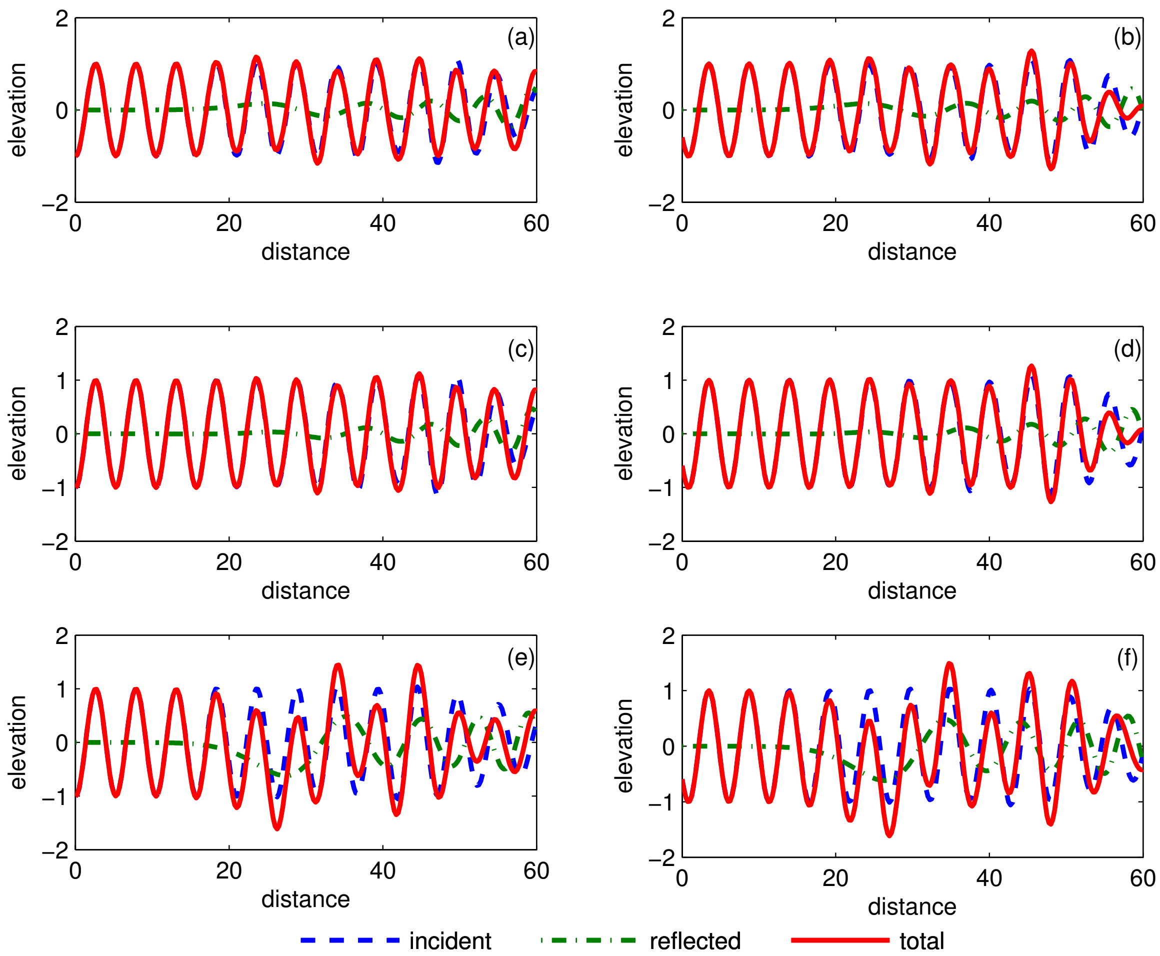


Figure 2. Wave profiles, and reflections, for sinusoidal input ((a) and (b)); ramp1 ((c) and (d)); ramp2 ((e) and (f)). Left figures: $t$=99; right figures: $t$=100

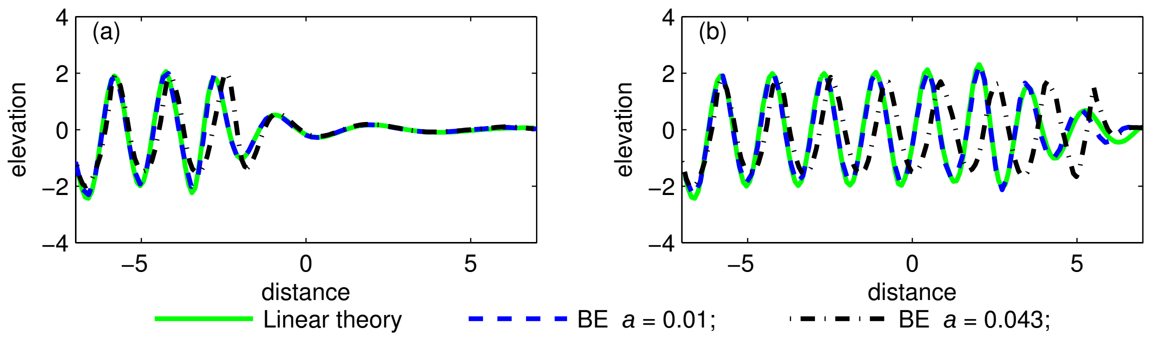


Figure 3. Wave profiles from linear theory and BE calculations: a) $t$=7$T$; b) $t$=15$T$

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**References**

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