WAVE IMPACT ON ELASTIC BEAM CONNECTED WITH SPRING TO MAIN STRUCTURE

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ABSTRACT: The impact of an elastic plate of finite length that is dropped against a liquid free surface is analyzed. The liquid is assumed ideal and incompressible and its motion two-dimentional, symmetric and potential. The elastic plate is the bottom of a structure, which penetrates the liquid at a constant velocity. The plate deflection is governed by the Euler beam equation and the beam edges are assumed connected with a main structure by a spring. The problem is coupled because the liquid flow, the beam deflection and the geometry of the contact region between the body and the liquid must be determined simultaneously. The analysis is based on the normal mode method with hydroelastic behaviour of the plate being of the main interest. It is shown that, at the beginning of the impact stage, independently of the rigidity of the connecting springs, the edges of the plate move toward the liquid surface. This fact may lead to a separation of a ship shell from the basic construction. The constructed algorithm allows one to perform the analysis of elastic effects during the impact of a liquid with thin-walled plates of limited extent. In addition, free fall motion is considered within this model as a limiting case for which the junction-spring stiffness approaches zero.

INTRODUCTION

The hull surfaces of ships and other floating structures are covered by a special "shell" that absorbs impacts of surface waves and protects the structural elements of the vessel. This "shell" consists of thin elastic plates (of mild steel or other metal), which are connected along their edges to the main structure. The forces at the edges of the plate are of a special interest because hydrodynamics loads are transferred through these connections, which are the weakest points.

To model the reactions of a ship shell due to water-wave impacts, the problem of a symmetric wave impact on an elastic plate with edges connected to the rigid main structure by linear springs is considered. The main structure penetrates the water with constant velocity. The liquid flow is two-dimensional, symmetric, and potential. The cross section of the plate is constant, and its thickness is supposed to be much smaller than its length. Elastic vibrations of the plate are described by Euler's beam equation.

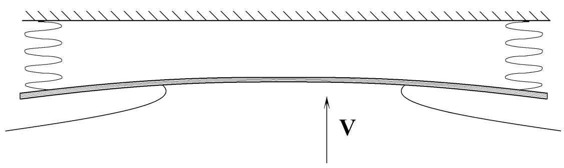

Figure 1.

The impact stage, during which the beam is only wetted partially, is considered here. During this stage, hydrodynamic loads are very large and depend on both the velocity of expansion of the contact region and the beam deflection. The analysis of the impact process is based on hydroelasticity, in which the coupled hydrodynamics and structural-dynamics problems are solved simultaneously. Even after all possible simplifications, the impact-stage problem remains nonlinear, because the dimension of the contact region is unknown in advance and must be determined together with the liquid flow and the beam deflection. The problem is analyzed within the framework of the Wagner theory, which takes into account changes of the form of a liquid free surface during an impact. The hydroelastic reaction of the beam is of main interest. Plate displacements and bending moments arising in the plate and on its edges are determined. The effects of the spring stiffness, the beam elasticity, and other impact parameters (notably, radius of curvature of the wave and impact velocity) are investigated.

The analysis is based on the normal mode method, which makes it possible to reduce the problem to an integration of a system of first-order differential equations for the principal coordinates of the plate displacement and the dimension of the contact region. A notable aspect of the analysis is that all the coefficients of this system of equations are obtained analytically.

This paper is an extension of [1]–[3]. However, the fact that the plate edges are allowed to move leads to the occurrence of new eigenvalues and consequently new normal modes, which bring essential difficulties in the analytical calculations of the elements of the added-mass matrix.

The numerical results demonstrate that the presence of spring connectors has important effects on bending stresses and displacements. It is shown that, at the beginning of the impact stage, independently of the rigidity of the connecting springs, the edges of the plate move toward the liquid surface. This fact may lead to a separation of ship shell from the basic construction. In the general case, the displacement of the plate grows, and the bending stresses in the plate decrease, with the reduction of rigidity of springs.

The free-fall motion of an elastic plate can be considered as a limiting case, for which the junction spring stiffness vanishes, of the model considered here.

STATEMENT OF THE PROBLEM

The unsteady plane problem of wave impact upon a elastic beam is considered. The edges of the beam are elastically connected to the bottom of the structure that penetrates the liquid, which is supposed to be ideal and incompressible. Initially ($t' = 0$) a wave crest touches the beam at its central point. Then the liquid starts to move up with constant velocity V. The initial contact point is taken as the origin of a Cartesian system of coordinates $x'Oy'$ (dimensional variables are identified by a prime). The curve $y' = -(x')^2/2R$ corresponds to the liquid free surface at $t' = 0$. This curve describes the shape of the wave crest with radius of curvature R. The flow caused by the plate impact is symmetric with respect to the line $x' = 0$, Figure 2.

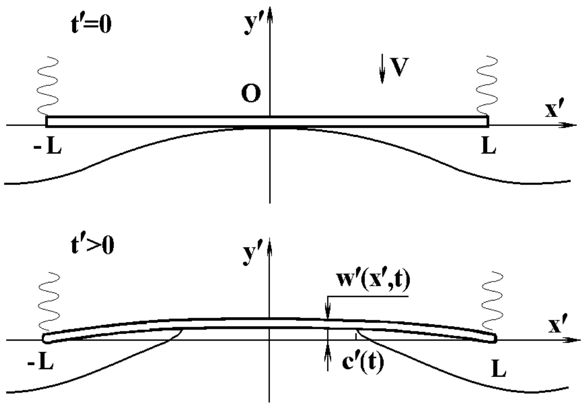

Figure 2.

Non-dimensional variables are used below. The beam length L is taken as the reference length and the impact velocity V as the reference velocity of liquid particles. $L^2/(RV)$ is taken as the reference time, L^2/R as the reference displacement, and $\rho V^2(R/L)$ as the reference pressure; here, ρ is the liquid density.

The plane potential flow generated by the plate penetration and the plate behaviour are described by the velocity potential $\varphi(x, y, t)$ and the beam deflection $w(x, t)$ which satisfy the following equations

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad (y < 0), \tag{1}$$

$$\varphi_y = -1 + w_t(x, t) \qquad (y = 0, \quad |x| < c(t)), \tag{2}$$

$$\varphi = 0 \qquad (y = 0, \quad |x| > c(t)), \tag{3}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty), \tag{4}$$

$$p(x, y, t) = -\varphi_t(x, y, t), \tag{5}$$

$$\alpha \frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial^4 w}{\partial x^4} = p(x, 0, t) \qquad (|x| < 1, \quad t > 0), \tag{6}$$

$$w = w_t = 0 \qquad (|x| < 1, \quad t = 0). \tag{7}$$

The presence of the elastic connectors is represented by the conditions

$$w_{xx} = 0 \qquad (x = \pm 1, \quad t > 0), \tag{8}$$

$$w_{xxx} = sign(x) k_l w \qquad (x = \pm 1, \quad t > 0), \tag{9}$$

The liquid-flow equation, the boundary and initial conditions, and the Euler beam equation, which are written in nondimensional variables, contain three parameters $\alpha = M_B/(\rho L)$, $\beta = (EJ)/(\rho L R^2 V^2)$ and $k_l = K_l L^3/EJ$. Here M_B is the beam mass per unit length, E is the elasticity modulus, $J = h^3/12$ is the inertia momentum of the beam cross-section, h is a thickness of the beam, and K_l is the stiffness of the elastic connectors. The bending stress distribution $\sigma(x, t)$ is given in the dimensionless variables as $\sigma(x, t) = w_{xx}(x, t)$, with its scale $Eh/(2R)$.

The boundary-value problem (1) - (9) is considered under the additional condition (Wagner condition [4]) that the elevation of the free surface is equal to the vertical position of the deformed plate at the the contact points. In a symmetrical case the positions of the contact points are described by the single function $c(t)$. The Wagner condition can be reduced to the equation suggested by Korobkin [5]. It has the form

$$\int_0^{\pi/2} y_b[c(t)\sin\theta, t] d\theta = 0, \tag{10}$$

where the function $y_b(x, t)$ describes the shape of the beam with respect to the initial position of the free surface. In the present case, $y_b(x, t) = x^2/2 - t + w(x, t)$, and equation (10) gives

$$t = \frac{1}{4}c^2 + \frac{2}{\pi}\int_0^{\pi/2} w[c(t)\sin\theta, t] d\theta. \tag{11}$$

The hydrodynamic part (1) - (5), the structural part (6) - (9), and the geometrical part (11) of the Wagner problem are closely connected and have to be treated simultaneously in general. It should be noticed that, although both the equations of motion and the boundary conditions are linearized, the problem remains nonlinear because $c(t)$ is unknown.

The Wagner problem (1) - (9), (11) is solved with the help of the normal mode method.

NORMAL MODE METHOD

Within this approach, the beam deflection $w(x, t)$ is sought in the form

$$w(x, t) = \sum_{n=1}^{\infty} a_n(t) \psi_n(x), \tag{12}$$

where the eigenfunctions $\psi_n(x)$ are the solution of the following problem

$$\frac{d^4 \psi_n}{dx^4} = \lambda_n^4 \psi_n \qquad (-1 < x < 1);$$

$$\frac{d^2 \psi_n}{dx^2} = 0, \quad \frac{d^3 \psi_n}{dx^3} = \text{sign}(x) k_l \psi_n \quad (x = \pm 1).$$

They represent the eigen modes of the beam - so called 'dry' modes. λ_n - are the corresponding eigenvalues, which are solution of the equation

$$\lambda_n^3 (\tan\lambda_n + \tanh\lambda_n) = -2k_l.$$

For $n = 1, 2, 3...$ the eigen modes are defined by

$$\psi_n = A_n(cos\lambda_n x + C_n \cosh \lambda_n x), \qquad C_n = \frac{\cos \lambda_n}{\cosh \lambda_n},$$

$$\frac{1}{A_n} = \sqrt{1 + C_n^2 + 3 \cos \lambda_n (\sin \lambda_n + \cos \lambda_n \tanh \lambda_n)/\lambda_n}.$$

In addition, for $n = 0$ we have eigen modes of other type

$$\psi_0 = A_0(\sin \mu x \; \sinh \mu x + C_0 \cos \mu x \; \cosh \mu x).$$

where $\lambda_0^4 = -4\mu^4$,

$$C_0 = \frac{\cos \mu \; \cosh \mu}{\sin \mu \; \sinh \mu},$$

$$A_0 = (4\sqrt{2\mu} \sin \mu \sinh \mu)/(\cos^2 \mu(8\mu \cos 2\mu + 6 \sin 2\mu) + +8\mu + 3 \sin 4\mu + 6 \cos 2\mu \sinh 2\mu + 3 \sinh 4\mu)^{1/2}.$$

μ is a solution of the equation

$$\frac{k_l}{2\mu^3} = \frac{1 + 2 \sin 2\mu \exp(-2\mu) - \exp(-4\mu)}{1 + 2 \cos 2\mu \exp(-2\mu) + \exp(-4\mu)}.$$

The eigenfunctions $\psi_n(x)$ satisfy the orthogonality condition

$$\int_{-1}^{1} \psi_n(x)\psi_m(x)\, dx = \delta_{nm},$$

where $\delta_{nm} = 0$ for $n \neq m$ and $\delta_{nn} = 1$.

It should be noticed that for a simply supported beam, the eigen modes and eigen functions have the forms $\psi_n(x) = \cos \lambda_n x$, $\lambda_n = \pi(n - 1/2)$, $n = 1, 2, \ldots$ [1].

Substitution of equations (12) into (6) - (9) and solution of the hydrodynamical part of the problem (1) - (5) provide the following system of ordinary differential equations with respect to the principal coordinates $\vec{a} = (a_1, a_2, a_3, ...)^T$ and auxiliary vector-function $\vec{d} = (d_1, d_2, d_3, \ldots)^T$:

$$\frac{d\vec{a}}{dt} = (\alpha I + \kappa S)^{-1}(\beta D\vec{d} + \vec{f}), \tag{13}$$

$$\frac{d\vec{d}}{dt} = -\vec{a}. \tag{14}$$

Here $d_n = (\beta\lambda_n^4)^{-1}(\alpha\dot{a}_n + b_n)$, $\vec{f} = (f_1(c), f_2(c), f_3(c), \ldots)^T$, I is the unit matrix, D is the diagonal matrix, $D = \text{diag}\{\lambda_1^4, \lambda_2^4, \lambda_3^4, \ldots\}$, S is the added mass matrix. The elements of these vectors and matrices are defined by

$$f_m(c) = \int_{-c}^{c} \sqrt{c^2 - x^2}\psi_m(x)\, dx, \tag{15}$$

$$S_{nm}(c) = \int_{-c}^{c} \varphi_n(x, 0, c)\psi_m(x)\, dx, \tag{16}$$

$$b_m(t) = -f_m(c) + \sum_{n=0}^{\infty} \dot{a}_n(t)S_{nm}(c). \tag{17}$$

Here, the function $\varphi_n(x, y, c)$ is harmonic in the lower half plane, and satisfies equations (1)-(4) with the right part of equation (2) being replaced by the function $\psi_n(x)$. It should be noticed, that all the elements (15)-(17) are defined analytically.

The right-hand side of the system (13)-(14) depends on $\vec{a}$, $\vec{d}$, c, but not on t. Therefore, it is convenient to take c as a new independent variable, $0 \leq c \leq 1$. Differential equation for $t = t(c)$ follows from (11) and has the form

$$\frac{dt}{dc} = Q(c, \mathbf{a}, \dot{\mathbf{a}}) = \frac{c/2 + (\mathbf{a}, \mathbf{\Gamma}_c(c))}{1 - (\dot{\mathbf{a}}, \mathbf{\Gamma}(c))}. \tag{18}$$

where

$$\Gamma_n(c) = \frac{2}{\pi} \int_0^{\pi/2} \psi_n(c \sin \theta) d\theta,$$

$$\Gamma_{nc}(c) = \frac{2}{\pi} \int_0^{\pi/2} \psi_n'(c \sin \theta) \sin \theta d\theta.$$

Multiplying the equations of the system (13), (14) by dt/dc and taking (18) into account, we get

$$\frac{d\vec{a}}{dc} = \vec{F}(c, \vec{d})Q(c, \vec{a}, \vec{F}(c, \vec{d})), \tag{19}$$

$$\frac{d\vec{d}}{dc} = -\vec{a}Q(c, \vec{a}, \vec{F}(c, \vec{d})), \tag{20}$$

where $\vec{F}(c, \vec{d}) = (\alpha I + \kappa S(c))^{-1}(\beta D\vec{d} + \vec{f}(c))$. The initial conditions are

$$\vec{a} = 0, \qquad \vec{d} = 0, \qquad t = 0 \qquad (c = 0). \tag{21}$$

The initial-value problem (18)-(21) is suitable for numerical simulations of the hydroelastic behavior of the wave impact on an elastic plate.

NUMERICAL RESULTS AND DISCUSSION

The initial-value problem (18)-(21) is solved numerically by the fourth-order Runge-Kutta method with uniform step Δc. The condition that the numerical scheme is stable was derived. The step Δc has to decrease as $O(N^{-2})$ if the number of modes N taken into account increases. In present investigation N is equal to 15.

Calculations were performed for the case $L = 0.5$m, $R = 10$m, $h = 2$cm, $E = 21 \cdot 10^{10}\text{H/m}^2$, $V = 3$m/s, $\varrho = 1000\text{kg/m}^3$, $\varrho_b = 7850\text{kg/m}^3$ where ϱ_b is the beam density. This gives following values nondimensional parameters $\alpha = 0.314$, $\beta = 0.311$. The displacement scale is equal to 2,5 cm, bending stiffness scale is equal to 420 N/mm^2 and the time scale is equal to 0,008s.

The numerical results for the elastic plate with edges connected to the rigid main structure by linear springs are compared with the results given in [1] for simply supported plate. The analysis of the results obtained for different values of spring rigidity gives:

• The evolution in time of the contact point velocity dc/dt are dependent of the spring rigidity. Figure 3 presents the time-dependent contact point velocity dc/dt. Here line 1 refers to simply supported plate, line 2 to the rigidity $k_l = 100$, line 3 to $k_l = 1$, and line 4 to $k_l = 0.001$. One can see that the duration of impact stage grows as the rigidity of springs decreases.

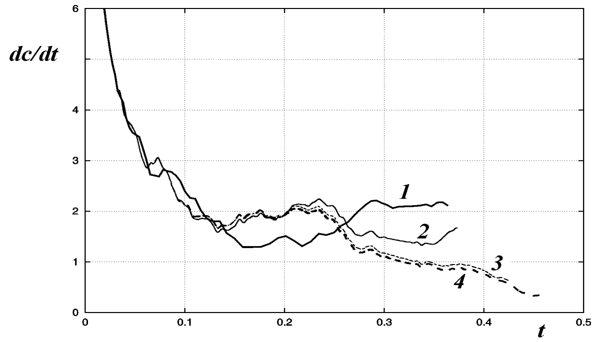

Figure 3.

• Figure 4 shows the displacement of the plate edge $w(1)$ as a function of time. Line 1 corresponds to the spring rigidity $k_l = 1000$, line 2 to $k_l = 10$, line 3 to $k_l = 1$, and line 4 to $k_l = 0.001$.

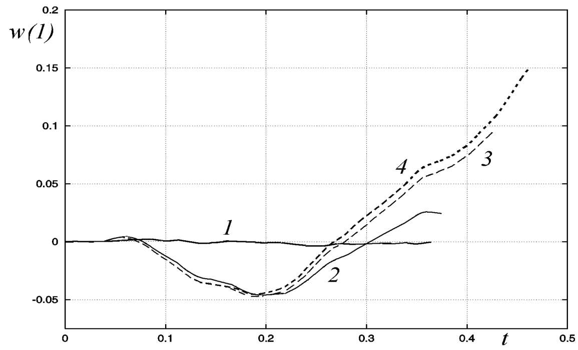

Figure 4.

• Figure 5 and Figure 6 show the distribution along the plate of the displacements and stresses correspondingly at the end of a impact stage. Curve 1 is for spring rigidity $k_l = 1000$, a curve 2 - is for $k_l = 10$ and a curve 3 is for $k_l = 0.001$. One can see, that rigidity of connectors has essentially influence on distribution of both displacements and stresses.

• It is interesting to note that at the end of the impact stage, the deflection of a plate at the central point is not significantl affected by the springs rigidity . At $k_l = 1000$, $w(0) = 0.24$, whereas at $k_l = 0.001$, $w(0) = 0.27$.

• For the case of free fall motion all results are identical with case $k_l = 0.001$.

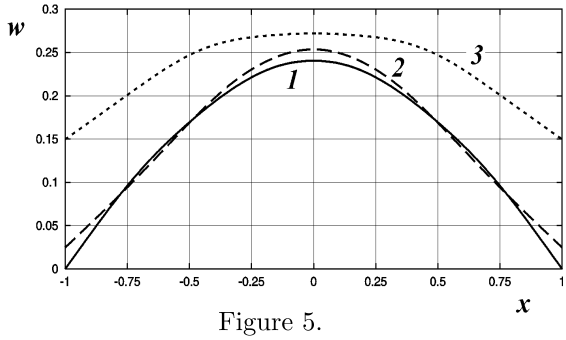

Figure 5.

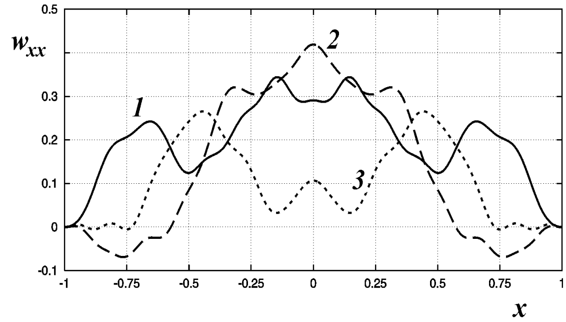

Figure 6.

CONCLUSION

The numerical results demonstrate, that at the beginning of the impact stage, independently of rigidity of the springs, the edges of a plate move toward the liquid surface. This result may lead to separation of a ship shell from the basic structure of the ship.

In general, the displacement of the plate grows as the rigidity of springs is reduced. At the same time the bending stresses in the plate decrease.

The constructed algorithm allows us to perform analysis of elastic effects during the impact of a liquid with thin-walled plates of limited extent. In addition, free fall motion can be considered within this model as a limit when the junction spring stiffness approaches zero.

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