Numerical Analysis of Three-Dimensional Slamming Forces in Waves

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1. Introduction

The occurrence of ship slamming depends on the relative motion and velocity between the ship and water waves. This implies that the instantaneous profile of free surface at slamming occurrence is important, as figure 1 shows. When the free-surface profile is involved, the three-dimensional effects become more significant. Despite significant past efforts for slamming analysis (e.g. Faltinsen, 1997, Korobkin, 2000), very limited information is known for the three-dimensional slamming problem. Moreover, the effects of instantaneous free-surface profile are little known. In the present study, the three-dimensional impact problem in the presence of non-flat free-surface profile is considered.

Figure 1. Slamming on a containership in waves (courtesy of American Bureau of Shipping)

2. Formulation

Let's consider the fluid flow around a three-dimensional body falling vertically with speed V (see Figure 2). An incident wave profile is also taken into account. In the realm of ideal flow, let's decompose the velocity potential $\Phi(\vec{x},t)$ and wave elevation $h(x,y,t)$ into two components; incident wave component and slamming component, s.t.

$$\Phi(\vec{x},t) = f_o(\vec{x},t) + j(\vec{x},t) \text{ and } h(\vec{x},t) = h_o(\vec{x},t) + V(\vec{x},t). \quad (1)$$

where the subscript indicates the incident wave component. Then the corresponding free-surface boundary conditions are written as follows:

$$\frac{\partial V}{\partial t} + \nabla f_o \cdot \nabla V + \nabla j \cdot \nabla h_o + V\nabla j \cdot \nabla V - \frac{\partial j}{\partial z} = 0 \quad (2)$$

$$\frac{\partial j}{\partial t} + \nabla f_o \cdot \nabla j + \frac{1}{2}\nabla j \cdot \nabla j + gV = 0 \quad (3)$$

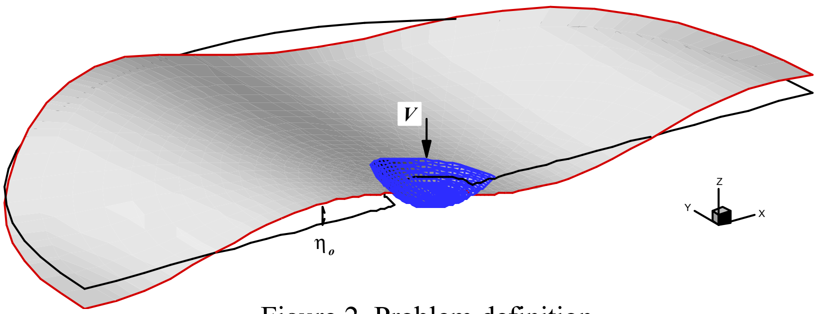

Figure 2. Problem definition

Two different scales can be considered for two components. For the incident waves, the velocity potential and wave elevation are assume that

$$f_o = V_o A_o \tilde{f}, \quad h_o = A_o \tilde{h}_o, \quad \nabla = \frac{1}{L_o}\tilde{\nabla}_o \tag{4}$$

where V_o, A_o, L_o are the velocity, amplitude, and characteristic length of incident wave. Another possible scaling factor for the incident wave potential is wave steepness. However, either factor will lead the same formulation that will be scribed later. For the disturbance due to impact, a different scaling can be considered as follows:

$$j = VL\tilde{j}, \quad V = VT\tilde{V}, \quad \nabla = \frac{1}{L}\tilde{\nabla}, \quad t = T\tilde{t} \tag{5}$$

where L and T are the characteristic length and impact time. These scalings lead equation (2) and (3) into the following forms:

$$\frac{\partial \tilde{V}}{\partial \tilde{t}} + kd\left(\tilde{\nabla}_o \tilde{f}_o \cdot \tilde{\nabla}\tilde{V}\right) + k\left(\tilde{\nabla}\tilde{j} \cdot \tilde{\nabla}_o \tilde{h}_o\right) + e\left(\tilde{\nabla}\tilde{j} \cdot \tilde{\nabla}\tilde{V}\right) - \frac{\partial \tilde{j}}{\partial \tilde{z}} = 0 \tag{6}$$

$$\frac{\partial \tilde{j}}{\partial \tilde{t}} + kd\left(\tilde{\nabla}_o \tilde{f}_o \cdot \tilde{\nabla}\tilde{j}\right) + \frac{1}{2}e\left(\tilde{\nabla}\tilde{j} \cdot \tilde{\nabla}\tilde{j}\right) + \frac{e^2}{F_r^2}\tilde{V} = 0 \tag{7}$$

where

$$e = \frac{VT}{L}, \quad d = \frac{V_o T}{L}, \quad k = \frac{A_o}{L_o}, \quad F_r = \frac{V}{\sqrt{gL}} \tag{8}$$

For impact occurrence in a very short time, let's consider the case that $e << 1$ and $F_r \geq O(e^{1/2})$. Furthermore, we assume that $V >> V_o$, so that $d << e$. Then the leading-order terms of equation (6) and (7) take the following forms:

$$\frac{\partial \tilde{V}}{\partial \tilde{t}} + k\left(\tilde{\nabla}\tilde{j} \cdot \tilde{\nabla}_o \tilde{h}_o\right) - \frac{\partial \tilde{j}}{\partial \tilde{z}} = 0 \tag{9}$$

$$\frac{\partial \tilde{j}}{\partial \tilde{t}} = 0 \tag{10}$$

These conditions are valid on the leading-order term of wave elevation, i.e. $z = h_o + V$.

Adopting the same scaling, the non-dimensional leading-order elevation becomes $\tilde{z} = (A_o / L)\tilde{h}_o$. It should be noted that there is no assumption for the order of wave slope, k, and L / L_o yet.

In the particular case when the incident wave is not very steep, i.e. $k << 1$, the above scale observation provides two approaches comparable to the well-known von Karman's and Wagner's methods.

Von Karman Type Approach

$$j = 0 \quad \text{on} \quad z = h_o \tag{11}$$

Wagner Type Approach

$$j = 0 \quad \text{on} \quad z = h_o + V, \quad \text{where} \quad \frac{\partial V}{\partial t} = \frac{\partial j}{\partial z} \tag{12}$$

In fact, these results are not much different from the generalized von Karman's and Wagner's methods. However, solving the boundary value problem in a three-dimensional domain is not an easy task since the zero potential condition should be satisfied on the incident wave surface. Furthermore, the Wagner's method (that assumes the zero-potential condition on a plane elevated to intersection between the body and free-surface pile-up) is very hard to be achieved in a three-dimensional case, especially when an arbitrary body shape is involved. In the present case, the zero-potential condition is applied on actual free surface.

3. Numerical Computation

The solution algorithm of this study is based on a three-dimensional Rankine panel method that adopts a higher-order B-spline basis function (Sclavounos & Nakos, 1988). The boundary of the fluid domain is discretized into quadrilateral panels, and the physical variables are represented with a higher-order B-spline basis function. The velocity potential adopts the representation,

$$f(\vec{x}_i, t) \approx \sum_j f_j(t) B_j(\vec{x}_i) = \sum_j f_j(t) b^{(p)}(x_1; \vec{x}_i) b^{(q)}(x_2, \vec{x}_i) \tag{13}$$

where $B_j(\vec{x}_i)$ is the B-spline basis function of order (p, q), defined relative to the local panel coordinates (x_1, x_2).

Figure 3 shows an example of instantaneous solution grids distributed on the wetted body surface and incident wave surface. Since the zero-potential condition is valid on the incident wave surface which is not fixed in time, a complete regrinding of wave surface as well as the wetted body surface is essential at each time step.

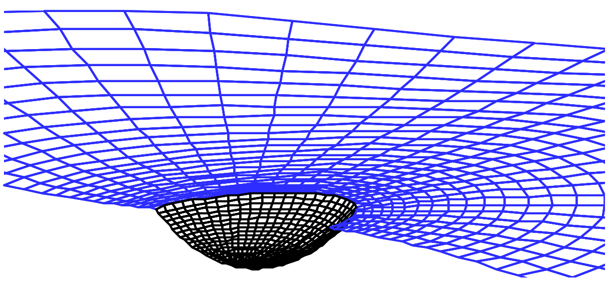

Figure 3. Examples of instantaneous grids on a falling sphere and incident wave surface.

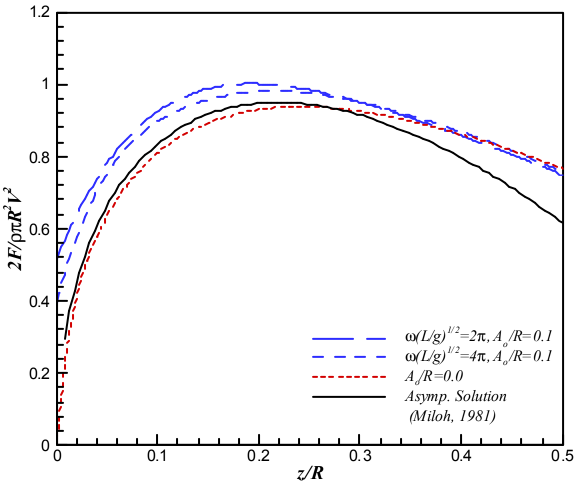

Figure 4. Heave lamming coefficients at different L/L_o: 90-deg. phase

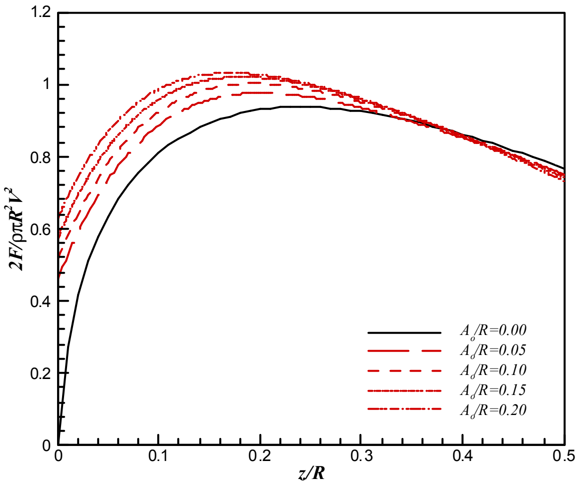

Figure 5. Effects of wave slopes: 90-deg. phase, wavelength/R= $(2p)^2$.

Fig.4~6 shows heave slamming coefficients for a falling sphere. Slamming forces are obtained by added-mass method based on the von Karman type approach. The significant dependency on wave steepness and initial body location relative wave profile is obvious. Since the splash-up effect is not included in these results, the slamming coefficients are close to the asymptotic solution (Miloh,1981) when wave profile becomes flat. In particular, the initial drop position relative to wave profile shows great importance.

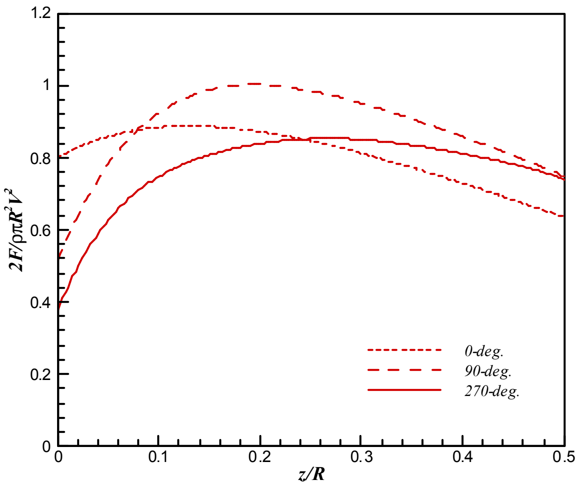

Figure 6. Effects of body location: wavelength $/ R = (2p)^2$

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Acknowledgement

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