MODIFIED LOGVINOVICH MODEL FOR HYDRODYNAMIC LOADS ON ASYMMETRIC CONTOURS ENTERING WATER

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SUMMARY

A new model of water impact is introduced and applied to two-dimensional problem of asymmetric body entering water vertically. The model is based on Wagner theory of impact and the flat-disc approximation but approximately accounts for both the body shape and the nonlinear terms in the Bernoulli equation for the hydrodynamic pressure. The model which is referred to as Modified Logvinovich Model (MLM), was originally designed to evaluate the hydrodynamic loads acting on a symmetric ship sections during ship slamming. In the case of roll motion of the ship or oblique waves the entry problem becomes asymmetric, which affects the hydrodynamic loads. General formula for the vertical component of the nonlinear hydrodynamic force is derived. Analytical calculations have been performed for an inclined wedge entering water vertically. Simple formula for evaluation of maximal acceleration magnitude of the inclined wedge dropped onto the liquid free surface has been derived and compared with experimental data. It is shown that the new model much better predicts the maximal acceleration of a free-dropped wedge than Wagner theory.

1. INTRODUCTION

Plane unsteady problem of asymmetric body entering water vertically is considered with special attention given to the vertical component of the hydrodynamic force acting on the body. Accurate prediction of the force is important in analysis of global elastic response of a ship in the case of slamming. If a body is thin and enters water almost vertically, one expects small values of the hydrodynamic force, which can be obtained by using linear theory without account for free surface deformations. Such a force is still important once the entering body trajectory should be determined. If a body is blunt with small deadrise angle in the impact area, the hydrodynamic force is very large at initial stage of the impact and may be responsible for elastic vibration of the body. Amplitude of the hydrodynamic force acting on a blunt body is strongly dependent on the "piled-up" effect, this is an additional increase of the impact area due to deformation of the liquid free surface. It is well known that Wagner theory of water impact correctly describes this effect and can be used for estimation of hydrodynamic loads on blunt bodies entering liquid. It is also known that this theory overpredicts the loads for bodies with moderate deadrise angles. Several models were developed in the past to improve the loads predictions. These models generalize the Wagner ideas in different way taking into account real shape of the body (Generalized Wagner Approach), nonlinear terms in the free surface boundary conditions (Vorus Model), fine structure of the flow close to the spray jet region (Matched Asymptotic Approach) and some global corrections of the Wagner solution by adding extra terms (Original Logvinovich Model). It is important to note that all models employed nonlinear Bernoulli equation for the hydrodynamic pressure in contrast to the Wagner model, which is based on linearized Bernoulli equation. It was demonstrated that the models essentially improve the classical Wagner approach with respect to prediction of the hydrodynamic force on entering bodies both in two-dimensional and three-dimensional cases. The Original Logvinovich Model is the simplest in this group. In some cases this model provides useful formulae for the loads without numerical analysis of boundary-value problems.

Recently the Original Logvinovich Model was analyzed with the help of asymptotic methods [1] and generalized with the aim to account for three-dimensional effects, elasticity of the body and its asymmetry. The new model of water impact is referred to as Modified Logvinovich Model (MLM). The model is based on Wagner theory of impact and the flat-disc approximation but approximately accounts for both the body shape and the nonlinear terms in the Bernoulli equation for the hydrodynamic pressure. In two-dimensional symmetric case the MLM was tested against both numerical (for wedge) and experimental (for circular cylinder) results. A fairly good agreement was reported.

In this paper the MLM is generalized to asymmetric problem and applied to shapes of ship sections, which are not blunt. It is shown that the MLM essentially improve prediction of the hydrodynamic loads. The obtained results are compared with the experimental results by Xu [2] for inclined wedge entering water vertically. It is shown that asymmetry gives an important contribution to the hydrodynamic force. Approximate approach based on a combination of symmetric solutions within MLM was tested. It is shown that this approach can be used for estimation of the loads only for small inclination angles.

2. FORMULATION OF THE PROBLEM

In this section general formulae for the two-dimensional pressure distribution along the wetted part of a blunt body entering liquid vertically and the hydrodynamic force acting on the body are derived. The liquid is assumed ideal and incompressible. The body is asymmetric and blunt. The liquid flow is assumed plane and potential. The liquid region is of infinite depth. Initially the liquid is at rest, its free surface is horizontal ($y = 0$) and the body touches the free surface at a single point ($x = 0$) taken as the origin of the Cartesian coordinate system Oxy (see Figure 1). The position of the body is described by the equation $y = f(x) - h(t)$, where $h(t)$ is the prescribed penetration depth and the function $f(x)$ describes the body shape, $f(0) = 0$. The velocity potential $\varphi(x, y, t)$ of the flow originated by the entering contour satisfies the Laplace equation in the flow domain, the kinematic and dynamic boundary conditions on the liquid free surface and the body boundary condition on the entering contour. Gravity effects and surface tension are neglected in the present analysis.

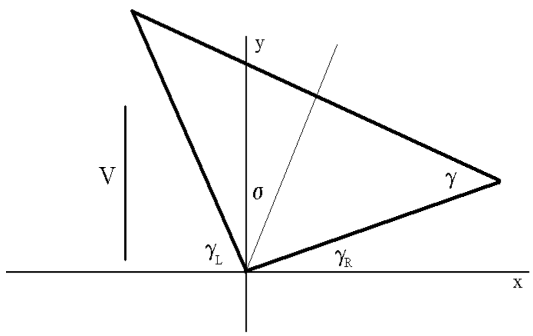

Figure 1: Initial position of wedge and free surface

To evaluate the pressure distribution $P(x, t)$ along the wetted part $D(t)$ of the blunt body during the initial stage, we assume that both the velocity potential $\phi(x, t)$ in the contact region, where $\phi(x, t) = \varphi(x, f(x) - h(t), t)$, and the dimension of this region are known. The hydrodynamic pressure $p(x, y, t)$ in the flow domain is given by the Cauchy-Lagrange integral

$$p(x, y, t) = -\rho \left(\varphi_t + \frac{1}{2}|\nabla\varphi|^2 \right), \qquad (1)$$

where ρ is the liquid density. By using the definition

$$P(x, t) = p(x, f(x) - h(t), t) \qquad (2)$$

and the boundary condition on the body surface

$$\varphi_y = \varphi_x f'(x) - \dot{h}(t), \qquad (3)$$

we obtain

$$P(x, t) = -\rho \Big[\phi_t + \frac{f'(x)\dot{h}}{1 + f_x^2}\phi_x + \frac{1}{2}\frac{\phi_x^2 - \dot{h}^2}{1 + f_x^2} \Big]. \qquad (4)$$

Dot stands for the time derivative. The hydrodynamic force $F(t)$ acting on the entering contour is given as

$$F(t) = \int_{D(t)} P(x, t) dx. \qquad (5)$$

Equations (4) and (5) are exact within the potential flow theory. The equations require the function $\phi(x, t)$, which can be numerically evaluated at each time step. It is suggested to obtain approximately the distribution $\phi(x, t)$ and to derive approximate formulae for both the pressure $P(x, t)$ and the force $F(t)$ suitable for practical use.

3. MODIFIED LOGVINOVICH MODEL

The present approximation is based on the Taylor expansion

$$\phi(x, t) \approx \varphi^{(w)}(x, 0, t) - \dot{h}(t)[f(x) - h(t)], \qquad (6)$$

where $\varphi^{(w)}(x, y, t)$ is the solution of the classical Wagner problem

$$\Delta\varphi^{(w)} = 0 \qquad (y < 0), \qquad (7)$$

$$\varphi^{(w)} = 0 \qquad (y = 0, x > a(t) \text{ and } x < -b(t)), \qquad (8)$$

$$\varphi_y^{(w)} = -\dot{h}(t) \qquad (y = 0, -b(t) < x < a(t)), \qquad (9)$$

$$\varphi^{(w)} \to 0 \qquad (x^2 + y^2 \to \infty). \qquad (10)$$

Within the Wagner approach the contact region $D(t)$ between asymmetric contour and liquid corresponds to the interval $-b(t) < x < a(t)$, $y = 0$, where the functions $b(t)$ and $a(t)$ should be determined with the help of the so called Wagner condition. In asymmetric case the Wagner condition, which is the condition that the elevation of the free surface is bounded, can be written as system of two transcendental equations

$$\int_{-1}^{1} G(\tau) \Big[\frac{1+\tau}{1-\tau}\Big]^{\frac{1}{2}} d\tau = 0, \qquad (11)$$

$$\int_{-1}^{1} G(\tau) \Big[\frac{1-\tau}{1+\tau}\Big]^{\frac{1}{2}} d\tau = 0, \qquad (12)$$

$$G(\tau) = f[x(\tau)] - h(t), \quad x(\tau) = A(t)\tau + B(t),$$

$$A(t) = \frac{1}{2}(a + b), \quad B(t) = \frac{1}{2}(a - b),$$

where $A(t)$ is the half-width of the contact region and $B(t)$ characterizes the region asymmetry.

In the problem of inclined wedge entry we introduce the deadrise angle of the wedge γ, the inclination angle σ, actual deadrise angle γ_R on the right-hand side of the body and actual deadrise angle γ_L on the left-hand side of the body, where $\gamma_R = \gamma - \sigma$ and $\gamma_L = \gamma + \sigma$ (see Figure 1). At the initial stage, when the wedge is only partly wetted, equations (11) and (12) provide

$$a(t) = a_0 h(t), \quad b(t) = b_0 h(t), \qquad (13)$$

$$a_0 = \frac{\pi}{2\tan\gamma_R} \frac{1-\epsilon}{(1-\mu)\sqrt{1-\mu^2}}, \qquad b_0 = a_0 \frac{1-\mu}{1+\mu},$$

where $\epsilon = \sin(2\sigma)/\sin(2\gamma)$ and $\mu(\epsilon)$ is the solution of the equation

$$\mu\sqrt{1-\mu^2} + \arcsin\mu = \pi\epsilon/2.$$

The latter equation is identical to that derived by Toyama [3] for asymmetric wedge.

In the case of arbitrary shape of the body equations (11) and (12) are solved numerically with respect to the functions $A(t)$ and $B(t)$.

The solution of the boundary-value problem (7) - (10) provides the velocity potential in the contact region, $-b(t) < x < a(t)$, as

$$\varphi^{(w)}(x,0,t) = -\dot{h}(t)\sqrt{(a-x)(b+x)}. \qquad (14)$$

By substituting (14) into (6) and (4), we obtain the pressure distribution in the contact region

$$P(x,t) = \frac{1}{2}\rho\dot{h}\dot{a}\sqrt{\frac{b+x}{a-x}} + \frac{1}{2}\rho\dot{h}\dot{b}\sqrt{\frac{a-x}{b+x}} - \qquad (15)$$
$$\frac{1}{8}\rho\dot{h}^2\frac{(a-b-2x)^2}{(a-x)(b+x)(1+f_x^2)} - \frac{1}{2}\rho\dot{h}^2 +$$
$$\rho\ddot{h}[\sqrt{(a-x)(b+x)} + f(x) - h(t)].$$

Equation (15) predicts negative pressures close to the contact points $x = a(t)$ and $x = -b(t)$. It is suggested to consider the pressure only in a part of the contact region, where the pressure is positive. This is a common point in almost any approximate models of water impact.

We introduce two positive functions $\tilde{a}(t)$ and $\tilde{b}(t)$ as solutions of the equations

$$P[\tilde{a}(t),t] = 0, \quad P[-\tilde{b}(t),t] = 0.$$

These equations are solved numerically by bisection method. Once the equations have been solved, we introduce quantities

$$\mu_a = \tilde{a}/a, \quad \mu_b = \tilde{b}/a, \quad b_a = b/a,$$
$$\lambda_1 = \frac{1}{2}(1+b_a), \quad \lambda_2 = \frac{1}{2}(1-b_a),$$
$$\nu_a = 1 - 2\frac{1-\mu_a}{1+b_a}, \quad \nu_b = 1 - 2\frac{b_a-\mu_b}{1+b_a}.$$

The vertical component of the hydrodynamic force $F(t)$ is given within the MLM by the formula

$$F(t) = \int_{-\tilde{b}(t)}^{\tilde{a}(t)} P(x,t)dx. \qquad (16)$$

The integral in (16) is calculated analytically for first, second, fourth and seventh terms in (15) and by panel method for other terms.

In the case of wedge the force is calculated analytically and presented in the form

$$F(t) = \rho\dot{h}^2 hF_v(\gamma,\sigma) + \rho\ddot{h}h^2F_w(\gamma,\sigma), \qquad (17)$$

$$F_w = \frac{1}{2}a_0^2[\mu_a^2\tan\gamma_R + \mu_b^2\tan\gamma_L] - a_0\lambda_1(\nu_a+\nu_b) + a_0^2\lambda_1^2 \times$$
$$[\arcsin\nu_a + \nu_a\sqrt{1-\nu_a^2} + \arcsin\nu_b + \nu_b\sqrt{1-\nu_b^2}]/2.$$

The expression for F_v is more complicated and is not reproduced here.

Equation (17) can be used to find the body acceleration after the impact in drop tests. By using the second Newton law and (17), we can integrate the body dynamic equation and find the acceleration of the entering wedge as

$$\ddot{h} = -2mkV_0\frac{h}{(1+mh^2)^{2k+1}}, \qquad (18)$$

where

$$m = \frac{\rho F_w}{M}, \quad k = \frac{F_v}{2F_w}, \quad V_0 = \sqrt{2gH_d},$$

M is the mass of the entering body per unit length, H_d is the drop height and g is the gravity acceleration. Note that in Wagner theory the body acceleration is given by the same formula with $k = 1$. The maximum of the acceleration is obtained as

$$\frac{1}{g}\max|\ddot{h}| = 4\sqrt{\frac{\rho H_d^2 F_w}{M}} \times L(k) \qquad (19)$$
$$L(k) = \frac{k}{\sqrt{4k+1}}\left[\frac{4k+1}{4k+2}\right]^{2k+1}.$$

Within Wagner approach the functions $a(t)$ and $b(t)$ are given by formulae (13), decomposition (17) is still valid but now

$$F_w(\gamma,\sigma) = \frac{1}{8}\pi a_0^2(1+b_a)^2, \quad F_v(\gamma,\sigma) = 2F_w(\gamma,\sigma).$$

There is another approximate method to evaluate the force for inclined contour. In this method one considers an inclined contour as intersection of two symmetric contours and takes the average value of the forces calculated for each symmetric contour. In the case of the inclined wedge the corresponding formula is

$$F_{approx}(t,\gamma,\sigma) = \frac{1}{2}[F(t,\gamma_R,0) + F(t,\gamma_L,0)]. \qquad (20)$$

A similar idea was used in the past for analysis of inclined cone entry.

4. NUMERICAL RESULTS

The presented model was used to study the effect of inclination angle on the vertical component of the hydrodynamic force in the problem of wedge entry. The entry velocity was constant in calculations. This implies that only the first term in (17) matters. The coefficient F_v as function of the inclination angle σ for $\gamma = 20^o$ is shown in Figure 2 by solid line together with the corresponding coefficients from the Wagner approach (dashed line) and from the approximate formula (20)(dotted line).

Analysis demonstrates that the relative difference between the MLM predictions and those by Wagner approach always vanish with increase of the inclination angle. The maxima of the difference occur at $\sigma = 0$ (9% for $\gamma = 5^o$, 27% for $\gamma = 20^o$, 35% for $\gamma = 30^o$ and

42% for $\gamma = 40^o$). For $\sigma = \gamma - 1^o$ the relative difference is about 4% for any deadrise angle.

In contrast, the relative difference between the MLM predictions and those by approximate formula (20) always grow with increase of the inclination angle. The difference is less than 5% if $\sigma < 0.1\gamma + 1^o$ within the interval $5^o < \gamma < 45^o$.

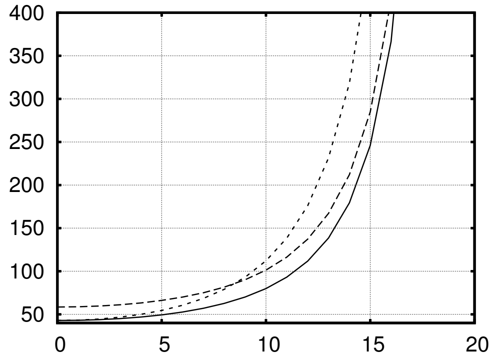

Figure 2: The coefficient F_v as function of the inclination angle σ for wedge with deadrise angle of 20 degrees.

Predictions of maximal acceleration of inclined wedge by formula (19) were compared with the experimental results by Xu [2]. In experiments the wedge with dimensions $2ft \times 8ft$, deadrise angle of 20^o and different mass was dropped from different heights at zero inclination angle and at the angle of 5 degrees. Experimental results and theoretical predictions of the maximal acceleration of the wedge are summarized in Table 1 for $\sigma = 5^o$ and in Table 2 for symmetric impact with $\sigma = 0^o$. It is seen that the theoretical results by MLM rather accurately correspond to the experimental data.

Mass	Drop height	MLM	Experiment by Xu [2]	Wagner theory
274 lb	0.61 m	13.44	13.0	16.1
274 lb	1.22 m	26.88	27.0	32.18
646 lb	0.61 m	8.75	8.0	10.48

Table 1: Maximal acceleration of the wedge entering water with inclination angle of 5^o.

Mass	Drop height	MLM	Experiment by Xu [2]	Wagner theory
269 lb	0.61 m	12.6	12.0	15.27
269 lb	1.22 m	25.21	24.0	30.54
269 lb	1.83 m	37.82	34.0	45.81
641 lb	0.61 m	8.16	8.0	9.9
641 lb	1.22 m	16.33	16.0	19.78
641 lb	1.83 m	24.5	23.0	29.67
1007 lb	0.61 m	6.51	6.0	7.89
1007 lb	1.22 m	13.03	13.0	15.78
1007 lb	1.83 m	19.55	18.0	23.67

Table 2: Maximal acceleration of the wedge entering water with zero inclination angle.

5. MLM FOR SHIP SECTIONS

To demonstrate the practical importance of the MLM method, the comparisons of the results obtained using the Modified Logvinovich Model with those obtained using the Generalized Wagner Model are presented for general ship sections. The ship lines are shown in Figure 3 (chosen section is indicated with bullets) together with the time history of the vertical force during the water entry of this section at vertical velocity of $8m/s$. The force obtained by using the MLM is shown with dotted line.

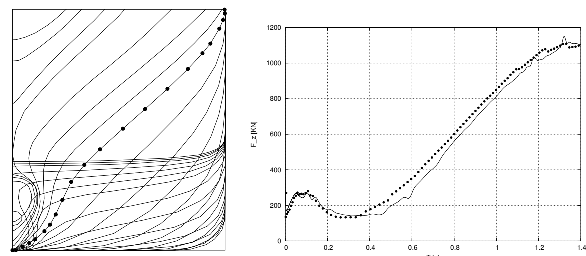

Figure 3: Ship lines with distinguished section and the vertical force acting on this section.

One can observe very good agreement between the hydrodynamic force time evolutions. Taking into account the enormous difference in the CPU time between two methods, one can easily understand the practical importance of the MLM method.

6. CONCLUDING REMARKS

It was demonstrated that the MLM can be used to evaluate the vertical hydrodynamic force acting on asymmetric bodies such as ship sections in oblique sea and/or in roll motion. It was shown that inclination of a ship section leads to increase of the force and may be responsible for larger elastic deflections of the ship hull and larger bending stresses in the ship girder than in the head sea.

7. ACKNOWLEDGMENTS

KAA acknowledges the supports from the grant NS-902.2003.1

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