Slender Body Theory Approach to Nonlinear Ship Motions

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1 Introduction

The accurate prediction of large amplitude ship motions in severe seas represents still a major challenge to naval architects. While three-dimensional panel methods have reached the state of maturity in linear seakeeping analysis, the original nonlinear problem, governed by strongly nonlinear boundary conditions, is far from being solved satisfactory. These nonlinearities are associated with the instantaneously wetted surface of the ship and the nonlinearities in the free surface conditions.

Over a period of years, the problem of solving an instantaneous nonlinear boundary value problem has been circumvented by accounting for the Froude–Krylov force integrated over the actual wetted surface while treating a linear radiation/diffraction problem. The negligence of higher order hydrodynamic effects has been justified by the different orders of magnitude of the Froude–Krylov and the linear radiation/diffraction forces. However, Huang and Sclavounos (1998) demonstrated in a study of heave and pitch motions in steep head seas, that the nonlinear hydrodynamic effects can attain the same order of magnitude as the nonlinear geometric corrections to the Froude–Krylov force.

Another scenario where nonlinear effects become essentially important is the parametric excitation of large amplitude roll motions in head and following seas. In a recent study, Hashimoto and Umeda (2004) demonstrated that the calculation of the roll restoring moment in waves based on the Froude–Krylov assumption leads to an overestimation of the roll response. The experimental tests confirmed that the reduction of the initial metracentric hight in waves of length comparable to the ship length is overestimated by the Froude–Krylov assumption.

In consequence, a consistent investigation of nonlinear ship motions must consider both, geometric and hydrodynamic nonlinearities. A promising approach, with regard to efficiency, is provided by the so-called $2D+t$ theory which has been successfully applied to the prediction of high-speed craft wave resistance and deck wetness problems, for a comprehensive review see Fontaine and Tulin (2001). Following the lines of slender body theory, the three-dimensional flow problem is reduced to a number of two-dimensional problems for the free surface flow perpendicular to the ship forward velocity U, see Fig. 1. Traditionally, the nonlinear two-dimensional free surface flow is computed assuming potential flow theory by the Mixed Eulerian–Lagrangian method (Longuet-Higgins and Cokelet, 1976). Recently, however, Andrillon and Alessandrini (2004) have employed a VOF–scheme for viscous flow computations in combination with the $2D+t$ theory.

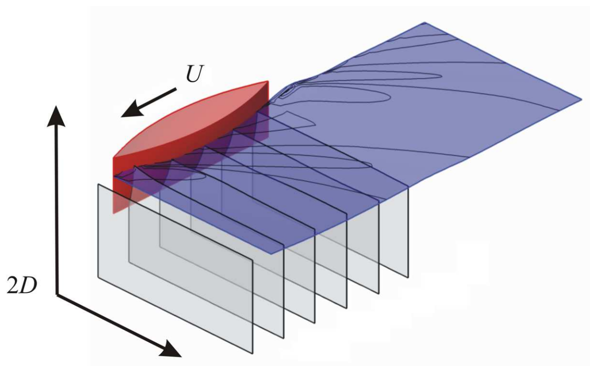

Figure 1: Principle of the $2D+t$ theory.

Until present, most authors have applied the $2D+t$ theory to the forced motion problem with focus on the wave built-up and jet generation in the bow region of the ship, cf. Fontaine et al. (2000). We will consider the free motions of a ship traveling with an average forward velocity U. With the investigation of the nonlinear hydrodynamic effects on the coupled heave–pitch–roll motions, we intent to improve the current mathematical models for the prediction of large amplitude roll motions in irregular seas.

In our derivation of the boundary conditions, we assume potential flow, but all kinematic conditions and the description of the rigid body motions of the ship are independent of the underlying flow theory, so that a viscous flow solver can be equally employed. For the scaling assumptions according to the slender body theory, we refer to the work of Wu et al. (2000).

2 Problem Formulation

The motion of the ship and the wave flow are described with respect to the inertial frame of reference $\mathrm{O}(x, y, z)$. The (x, y) plane defines the free water surface at rest, and the vertical z-axis is pointing upwards out of the fluid domain. The x–axis coincides with the average forward direction of the ship, the vector $\mathbf{x}_\mathrm{G}$ denotes the position of the center of gravity G, and $\boldsymbol{\omega}$ denotes the vector of angular velocity.

The two-dimensional boundary value problems are solved in cross sections parallel to the (y, z) plane. The total flow potential Φ is defined by the sum of ϕ^w, the potential of the incident wave, and ϕ, the disturbance potential induced by the ship.

2.1 Boundary value problem

According to the slender body theory, the disturbance potential ϕ must satisfy the two-dimensional Laplace equation

$$\frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0. \tag{1}$$

The total flow potential Φ must satisfy a zero-flux condition on the body surface. The corresponding boundary condition for the disturbance potential ϕ yields

$$\phi_n = (\dot{\mathbf{x}}_\mathrm{G} + \boldsymbol{\omega} \times \mathbf{r} - \nabla\phi^w) \cdot \mathbf{n}, \tag{2}$$

where $\mathbf{n}$ is the outward surface unit normal vector of the ship, and $\mathbf{r}$ is the relative position with respect to G.

The total potential also has to match the kinematic and dynamic boundary conditions on the free surface defined by the contour line $z = \eta_w + \eta$. Discarding quadratic terms of the incident wave steepness, the boundary conditions for the disturbance potential are obtained

$$\eta_t + \eta_y\phi_y - \phi_z = 0, \tag{3}$$
$$\phi_t + \tfrac{1}{2}\left(\phi_y^2 + \phi_z^2\right) + \dot{\eta}_w\phi_z + g\eta = 0. \tag{4}$$

The boundary value is complete, when additional conditions are applied on the control planes sufficiently far away form the ship ($|y| \to \infty$) and at the sea bottom. The complete boundary value problem is illustrated in Figure 2.

The instantaneous two-dimensional boundary value problem is further decomposed into an initial value problem for the evolution of the free surface quantities (ϕ, η) and a boundary value problem for (ϕ, ϕ_n) fixed in time. The boundary integral equation is obtained through the use of Green's theorem with a

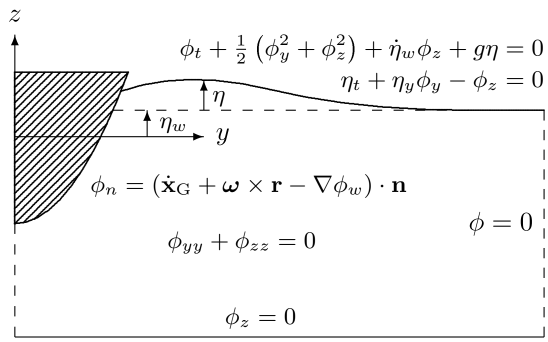

Figure 2: Definition of boundary conditions.

simple source Green function, $G(\mathrm{M}, \mathrm{P}) = \ln \|\overrightarrow{\mathrm{MP}}\|$,

$$\begin{aligned} \pi\phi(\mathrm{M}) &= \oint \phi(\mathrm{P}) \frac{\partial G}{\partial n_\mathrm{P}}(\mathrm{M}, \mathrm{P}) \mathrm{d}\Sigma_\mathrm{P} \\ &\quad - \oint \frac{\partial \phi}{\partial n_\mathrm{P}}(\mathrm{P}) G(\mathrm{M}, \mathrm{P}) \mathrm{d}\Sigma_\mathrm{P}, \end{aligned} \tag{5}$$

which can be solved for the unknown potential ϕ on the hull and the unknown flux ϕ_n on the free surface by a standard boundary element method.

The free surface conditions (3) and (4) are recast in the Lagrangian sense making use of the substantial derivative $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \nabla\phi \cdot \nabla$. We obtain

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \phi_z, \tag{6}$$
$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \tfrac{1}{2}\left(\phi_y^2 + \phi_z^2\right) - \dot{\eta}_w\phi_z - g\eta. \tag{7}$$

With the solution of (5) we may compute all required right hand side terms of the free surface conditions (6) and (7). The initial value is complete, when corresponding initial conditions are specified for η and ϕ.

2.2 Hydrodynamic forces

The hydrodynamic forces are determined by integration of the total hydrodynamic pressure in each cross-section, given by Bernoulli's equation

$$p = -\rho\left(\Phi_t + \tfrac{1}{2}\nabla\Phi \cdot \nabla\Phi + gz\right). \tag{8}$$

The total hydrodynamic forces contain components proportional to the acceleration of the ship, which gives rise to numerical instabilities during the integration of the ship motions (Kring and Sclavounos, 1995). Instead of determining ϕ_t by a numerical backwards difference scheme from the potential values ϕ at the present and previous time step, we solve an auxiliary boundary value problem for the unknown ϕ_t on the ship hull. The Neumann boundary condition for the auxiliary problem on the ship hull is obtained by taking the local derivative with respect to time of the kinematic boundary condition on the ship hull (2)

$$\begin{aligned}\phi_{t,n} \quad &= \quad (\dot{\mathbf{x}}_{\mathrm{G}} + \boldsymbol{\omega} \times \mathbf{r} - \nabla\phi^w - \nabla\phi) \cdot \mathbf{n}_t - \phi^w_{t,n} \\ &\quad + [\ddot{\mathbf{x}}_{\mathrm{G}} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})]\, \mathbf{n}. \end{aligned} \tag{9}$$

Reordering the dynamic free surface condition (4) yields the Dirichlet condition on the free surface for the auxiliary problem

$$\phi_t = -\tfrac{1}{2}\left(\phi_y^2 + \phi_z^2\right) - \dot{\eta}_w \phi_z - g\eta. \tag{10}$$

The boundary condittions applied on the control planes and at the bottom line for the auxiliary problem are similar to those of the original boundary value problem. Note, that the solution for ϕ_t is obtained at moderate cost, since the influence matrices for the discretization of the integral equation (5) are the same for the auxiliary problem and have to be assembled only once per time step.

2.3 Rigid body motions

The Newton-Euler equations of motion with respect to the generalized coordinates describing the translations and rotations of the ship

$$\boldsymbol{\xi} = [\boldsymbol{\xi}_{\mathrm{T}}\ \boldsymbol{\xi}_{\mathrm{R}}]^T = [x_{\mathrm{G}}\ y_{\mathrm{G}}\ z_{\mathrm{G}}\ \varphi\ \theta\ \psi]^T, \tag{11}$$

are given by

$$\mathbf{M}\ddot{\boldsymbol{\xi}} + \mathbf{k}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, t) = \mathbf{q}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \ddot{\boldsymbol{\xi}}, t), \tag{12}$$

with the mass matrix

$$\mathbf{M} = \begin{bmatrix} m\mathbf{I}_{3\times3} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Theta}\mathbf{J}_R \end{bmatrix}, \tag{13}$$

where m denotes the total mass and $\boldsymbol{\Theta}$ the inertia tensor of the ship. The Jacobian $\mathbf{J}_{\mathrm{R}}$ is defined by

$$\boldsymbol{\omega} = \mathbf{J}_R \dot{\boldsymbol{\xi}}_{\mathrm{R}} \tag{14}$$

and

$$\mathbf{k} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Theta}\dot{\mathbf{J}}_{\mathrm{R}}\dot{\boldsymbol{\xi}}_R + (\mathbf{J}_{\mathrm{R}}\dot{\boldsymbol{\xi}}) \times (\boldsymbol{\Theta}\mathbf{J}_{\mathrm{R}}\dot{\boldsymbol{\xi}}_{\mathrm{R}}) \end{bmatrix}. \tag{15}$$

The vector of generalized forces $\mathbf{q}$ contains gravitational and hydrodynamic forces. From the boundary condition (9) and the relation

$$\dot{\boldsymbol{\omega}} = \mathbf{J}_{\mathrm{R}}\ddot{\boldsymbol{\xi}}_{\mathrm{R}} + \dot{\mathbf{J}}_{\mathrm{R}}\dot{\boldsymbol{\xi}}_{\mathrm{R}}, \tag{16}$$

it is obvious that the component depending on the second derivative of the generalized coordinates can be extracted from the force vector

$$\mathbf{q}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \ddot{\boldsymbol{\xi}}, t) = \hat{\mathbf{q}}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, t) - \mathbf{A}\ddot{\boldsymbol{\xi}}. \tag{17}$$

Hence, the equations of motion (11) can be recast in the general form where the force vector is no longer dependent on the acceleration

$$(\mathbf{M} + \mathbf{A})\,\ddot{\boldsymbol{\xi}} + \mathbf{k}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, t) = \hat{\mathbf{q}}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, t). \tag{18}$$

This guarantees the stability of the numerical integration. The state variables of the rigid body motions and the free surface quantities can be integrated satisfactorily by an explicit method, e.g. by a fourth-order Runge–Kutta or Adam–Bashford–Moulton scheme.

3 Summary

The application of the $2D + t$ theory for the prediction of nonlinear ship motions has proved to be reliable for slender ship hulls like the Wigley hull in head seas, Fig. 3. For the stability of explicit numerical integration schemes, it is necessary to extract the acceleration-dependent components from the hydrodynamic forces.

The work subject to presentation at the workshop focusses on the application of the theory to a more complicated hull shape of a fast Ro–Pax ferry. The analysis is further extended to the roll and sway modes in order study the parametric rolling behavior.

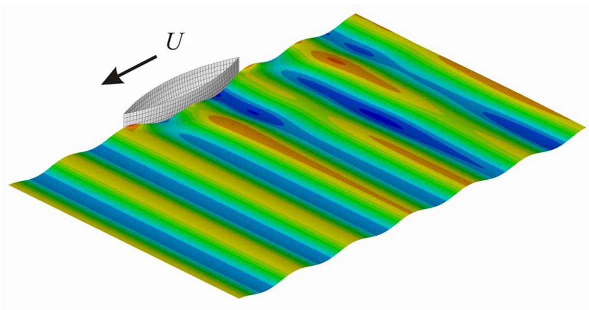

Figure 3: Wave pattern generated by Wigley hull advancing in head seas, Froude number 0.3, wave steepness 0.18

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