A consistent strip-theory approach for wave loads and ship motions in rough seas

Jinzhu Xia
Australian Maritime College
P O Box 986, Launcestion, TAS 7250, Australia
Email: j.xia@amc.edu.au

1. Introduction

When dealing with wave-induced loads and ship motions in rough seas, seakeeping theories usually only accommodate nonlinear and extreme effects such as slamming in separate and empirical ways. An attempt is made here to formulate consistently the nonlinear hydromechanic forces, including the momentum slamming force, acting on a ship undergoing large-amplitude motions. The formulation is within the framework of potential flow theory. By satisfying the exact boundary condition on the time-varying body surface but assuming linearized free surface boundary condition, a 2D time-domain solution is presented, which extends Faltinsen's (1990) momentum approach for water entry analysis and reveals a consistent theoretical background for the nonlinear wave loading and ship motion calculations published in Xia *et al*. (1998). The present formulation provides a foundation for accurate and practical simulation of seakeeping in rough seas.

2. The momentum equation

The fluid momentum inside a volume Ω surrounding the ship hull can be written as

$$\boldsymbol{M}(t) = \iiint_{\Omega} \rho \boldsymbol{V} d\tau \qquad (1)$$

where $\boldsymbol{V}$ is the fluid velocity.

For an incompressible fluid, the equation for conservation of momentum can be written as (Eq (9.29), Faltinsen (1990)),

$$\frac{d\boldsymbol{M}}{dt} = -\rho \iint_{S} \left[\left(\frac{p}{\rho} + gz \right) \boldsymbol{n} + \boldsymbol{V}(V_n - U_n) \right] ds \qquad (2)$$

In this equation, S is the closed surface for the volume Ω and $\boldsymbol{n}$, the normal vector to S. The positive normal direction points outwards the fluid domain. U_n is the normal velocity component of the surface S. V_n is the normal component of the fluid velocity at the surface S. Assuming the existence of a velocity potential ϕ, we have $V_n = \dfrac{\partial \phi}{\partial n}$ and the Gauss theorem gives

$$\boldsymbol{M}(t) = \iint_{S} \rho \phi \boldsymbol{n} ds \qquad (3)$$

3. A time-domain strip theory formulation

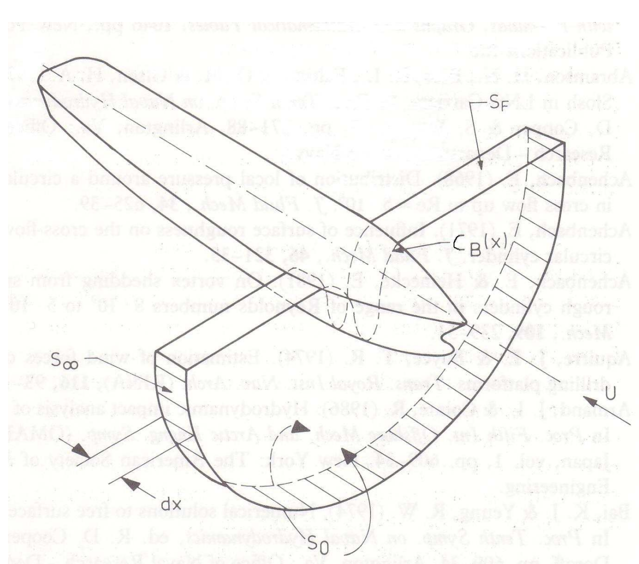

Figure 1 Definition of a control volume to calculate hydromechanic force on a ship section.

The equation for conservation of momentum (2) can be applied to derive the fluid forces on a strip or a section of a ship at forward speed. The reference frame is chosen as the moving equilibrium Cartesian coordinate system where the *oxy* plane is on the still water surface and traveling at the ship speed, *x*-axis is towards the ship bow and *z*-axis is upwards. The forward speed of the ship appears as an incident steady flow with velocity U in the opposite x-axis. In applying the momentum equation, the control surface S can be selected as shown in Figure 1. There are two lateral planes S_0 separated by a distance dx exterior to the ship. S_∞ is fixed in space and located a certain distance away from the ship. The intersection curve between the planes and the ship is called $C_B(x,t)$. We only consider the strip motions and fluid forces in the transverse plan.

We denote φ the 'oscillating' velocity potential due to unsteady ship motions. The velocity potential due to the steady forward speed effect is simplified as $-Ux$. The total velocity potential can be written as

$$\phi = -Ux + \varphi \tag{4}$$

The fluid velocity in the moving reference frame is therefore

$$\boldsymbol{V} = -U\boldsymbol{i} + \nabla\varphi \tag{5}$$

where $\boldsymbol{i}$ is the unit vector in x-direction.

If subscript j = 2,3 respectively defines the y- and z-component of any vector or vector equation, the equation for conservation of momentum can now be written as

$$\frac{d}{dt}\iint_S \rho\varphi n_j ds = -\rho\iint_S\left[\left(\frac{p}{\rho}+gz\right)n_j + \frac{\partial\varphi}{\partial x_j}(V_n - U_n)\right]ds \qquad j = 2,3 \tag{6}$$

Let's evaluate the integrals in (6) term by term. The left hand side of (6) may be rewritten

$$\frac{d}{dt}\iint_S \rho\varphi n_j ds = \rho\frac{d}{dt}\iint_{S_B+S_F}\varphi n_j ds + \rho\iint_{S_\infty}\frac{\partial\varphi}{\partial t}n_j ds \tag{7}$$

The contribution from S_0 is zero since n_j=0 on S_0. Because S_∞ is independent of time, the time derivative can be moved inside the integral over S_∞.

If, in addition to n_j = 0 on S_0, the Bernoulli's equation, $\frac{p}{\rho} = -\left(\frac{\partial\varphi}{\partial t} - U\frac{\partial\varphi}{\partial x} + \frac{1}{2}|\nabla\varphi|^2 + gz\right)$, is used to express the pressure over $S_F + S_\infty$, we have

$$\iint_S (p+\rho gz)n_j ds = \iint_{S_B}(p+\rho gz)n_j ds - \rho\iint_{S_F+S_\infty}\left(\frac{\partial\varphi}{\partial t} + \frac{1}{2}|\nabla\varphi|^2 - U\frac{\partial\varphi}{\partial x}\right)n_j ds \tag{8}$$

Finally, since $U_n = V_n$ on S_B and S_F and $U_n = 0$ on S_0 and S_∞, we have

$$\rho\iint_S \frac{\partial\varphi}{\partial x_j}(V_n - U_n)ds = \rho\iint_{S_0+S_\infty}\frac{\partial\varphi}{\partial x_j}V_n ds \tag{9}$$

Therefore, from (6), the transverse-plane hydromechanic force on a ship cross-section can be written as

$$\begin{aligned}\iint_{S_B} pn_j ds = &-\rho\frac{d}{dt}\iint_{S_B}\varphi n_j ds - \iint_{S_B}\rho gzn_j ds \\ &-\rho U\iint_{S_F+S_\infty}\frac{\partial\varphi}{\partial x}n_j ds - \rho\iint_{S_0}\frac{\partial\varphi}{\partial x_j}V_n ds \\ &-\rho\frac{d}{dt}\iint_{S_F}\varphi n_j ds + \rho\iint_{S_F}\left(\frac{\partial\varphi}{\partial t} + \frac{1}{2}\nabla\varphi\cdot\nabla\varphi\right)n_j ds \\ &+\rho\iint_{S_\infty}\left(\frac{1}{2}|\nabla\varphi|^2 n_j - \frac{\partial\varphi}{\partial x_j}V_n\right)ds\end{aligned} \tag{10}$$

A modified Stokes theorem may be used to convert the surface integral over $S_F + S_\infty$ in (10) into line integrals. Based on the approximation that $V_n = \pm U$ on the opposite sides of S_0 the surface integral over S_0 may also be converted into line integrals using the Green theorem. After cancellation, the second line of (10) will result in line integrals along the contour of the wetted body surface S_B. The waterline integral may further be neglected assuming small waterline angle. It may be demonstrated that the contribution from the third line of (10) can be replaced by a body surface integration, $\rho\iint_{S_B}\left[\frac{1}{2}(\nabla\varphi)^2\boldsymbol{n} - \varphi\frac{\partial}{\partial n}(\nabla\varphi)\right]ds$, and is of a higher order. If S_∞ is located infinitely far away from the body, the contribution over S_∞ in the last line of (10) is negligible. This finally leads to the following leading-order force representation

$$\begin{aligned}\iint_{S_B} pn_j ds = &-\rho\frac{d}{dt}\iint_{S_B}\varphi n_j ds - \iint_{S_B}\rho gzn_j ds \\ &+\rho Udx\frac{\partial}{\partial x}\int_{C_B}\varphi n_j dc\end{aligned} \tag{11}$$

The force distribution over unit ship length is therefore

$$\int_{C_B} pn_j dc = -\left(\frac{d}{dt} - U\frac{\partial}{\partial x}\right)\int_{C_B} \rho\varphi n_j dc - \int_{C_B} \rho g z n_j dc \quad (12)$$

The differentiation on the right hand side of the above equation cannot be interchanged with the contour integration when considering large-amplitude motion problems because the body contour is both time- and space-varying.

4. Vertical-plane force using 2D flow solution

Consider a vertical-plane motion problem with a 2-D solution of the velocity potential satisfying the following linearized free-surface condition and body boundary condition,

$$\frac{\partial^2 \varphi}{\partial t^2} + g\frac{\partial \varphi}{\partial z} = 0 \qquad on\ z = 0 \qquad (13)$$

$$\frac{\partial \varphi}{\partial n} = W\, n_3 \qquad on\ C_B(x,t) \qquad (14)$$

where W is the relative vertical velocity of a ship section.

The solution of the velocity potential may be written as

$$\varphi = W(x,t)\psi(y,z;t) + \int_0^t W(x,\tau)\chi(y,z;t-\tau)d\tau \quad (15)$$

where ψ is a normalized velocity potential for the impulsive flow due to the body boundary movement; χ defines a memory effect due to free-surface wave propagation;

$$\begin{cases} \nabla^2 \psi = 0 & in\, the\ fluid \\ \dfrac{\partial \psi}{\partial n} = n_3 & on\ C_B(x,t) \\ \psi = 0 & on\ z = 0 \end{cases} \qquad (16)$$

and

$$\begin{cases} \nabla^2 \chi = 0 & in\, the\ fluid \\ \dfrac{\partial^2 \chi}{\partial \tau^2} + g\dfrac{\partial \chi}{\partial z} = 0 & on\ z = 0 \\ \dfrac{\partial \chi}{\partial n} = 0 & on\ C_B(x,t) \\ \chi(0) = 0 & in\, the\ fluid \\ \dfrac{\partial \chi(0)}{\partial \tau} = -g\dfrac{\partial \psi}{\partial z} & on\ z = 0 \end{cases} \qquad (17)$$

We may verify that besides the Laplace equation φ given by (15) satisfies both the body boundary condition and the linearized free-surface condition. Note that $W(x,0) = 0$ and $\chi(y,z;0) = 0$ are necessary initial conditions to (15) - (17).

It is seen that the above boundary value problem is similar to the widely used linear time-domain potential flow model (e.g. Cummins, 1962) except that the body boundary condition is satisfied on the present instantaneous body surface $C_B(x,t)$. Thus, ψ and χ may be solved based on the solution techniques for the linear problem. In fact, the problem may be solved through the relation to a set of frequency-domain problems, avoiding solving for the time-domain velocity potential.

Inserting (15) in (12), the hydrodynamic force may now be written as

$$F(x,t) = -\frac{D}{Dt}\left[W(x,t)\mu(C_B) + \int_0^t W(x,\tau)K(C_B,t-\tau)d\tau\right] \qquad (18)$$

where $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} - U\dfrac{\partial}{\partial x}$;

$$\mu(C_B) = \int_{C_B} \rho\psi n_3 dc \qquad (19)$$

is the added mass of the instantaneous body section when the oscillating frequency tends to infinity; and

$$K(C_B,\tau) = \int_{C_B} \rho\chi(y,z;\tau)n_3 dc \qquad (20)$$

is the memory function of a unit impulse calculated for the present instantaneous body surface $C_B(x,t)$. It may be related to the frequency-domain damping coefficient for the same body surface through Fourier transform.

(18) can be rewritten as

$$F(x,t) = -\mu\frac{DW}{Dt} + U\frac{\partial \mu}{\partial x}W + \frac{\partial \mu}{\partial z}W^2 - \frac{D}{Dt}\int_0^t W(x,\tau)K(C_B,t-\tau)d\tau \qquad (21)$$

where the momentum slamming force, $\dfrac{\partial \mu}{\partial z}W^2$, is demonstrated.

5. Discussion

If the flow satisfies a free-surface condition $\varphi = 0$ on $z = 0$, the memory effect in (18) disappears. The result becomes identical to Faltinsen's (1990) approach discussed in his water entry problems.

Expression (18) is useful for not only transient water entry problems, but also seakeeping problems as it accounts for wave interactions. Based on (18) and (21), Xia *et al*. (1998) predicted heave, pitch, bow acceleration and mid-ship bending moment RAOs of the S175 Containership at different wave amplitudes, see Figure 2. The solid lines represent linear prediction. The nonlinear motion and structural loading predictions compared consistently well with the experimental results obtained by O'Dea *et al*. (1992) and Watanabe *et al*. (1989). It shows that if the concept of RAO is still to be used in motion assessment or design optimization, the RAOs should be calculated at different wave amplitudes for different sea states. The non-linear evaluation is particularly important for the bending moment because linear theories usually provide unsafe predictions.

The present analysis provides opportunities for more refined strip-theory simulations. This may be done by using a different velocity potential solution in (12) to represent a high-speed vessel problem, for example. One may also look back to Equation (10) to consider nonlinear effects beyond the leading order.

6. References

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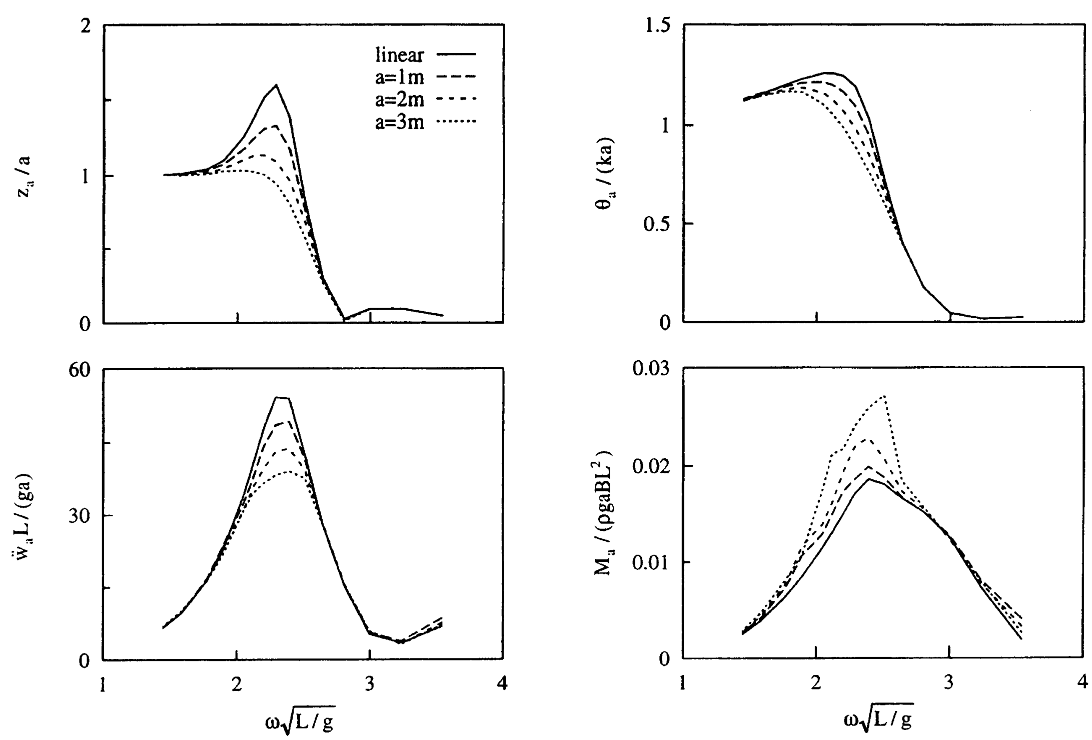

Figure 2 Calculated, based on (21), non-dimensional response amplitude operators (RAOs) of heave, pitch, bow acceleration (FP) and midship bending moment of the S175 Containership for different regular wave amplitudes, Fn=0.25 (Xia *et al*., 1998).