Application of QALE-FEM to the interaction between nonlinear water waves and periodic bars on the bottom

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Introduction

As has been known, one of efficient methods to simulate the nonlinear water waves is the finite element method based on a fully nonlinear potential theory. A drawback of the FEM, however, is that a complex unstructured mesh is generally required and may need to be remeshed at every time step to follow the motion of waves and/or structures. Repeatedly regenerating such a mesh can make the required CPU time prohibitive in a simulation of several thousands steps on a normal workstation. In order to reduce the time spent on the remeshing, simple structured mesh has been used in [1] and [2]. For the same purpose, Wu et al [3] have recently employed a hybrid mesh. In their approach, a 2D mesh in a horizontal plane (say, the free surface at t=0) is first generated and then vertical lines are drawn to construct a 3D mesh. The 2D mesh is formed by combining an unstructured mesh in a region near structures with a simple structured mesh in other regions. This is a good approach but restricted only for cylindrical structures without roll and pitch motions.

Recently the authors of this paper have developed a new method called QALE-FEM [4]. The main difference of this method from the conventional finite element method, such as in [1] – [3] is that the complex mesh is generated only once at the beginning and is moved at all other time steps in order to conform to the motion of the free surface and structures. This feature allows one to use an unstructured mesh with any degree of complexity without the need of regenerating it every time step. Due to this feature, the QALE-FEM has high potential in enhancing the computational efficiency when is applied to problems associated with the complex interaction between large steep waves and structures since the use of an unstructured mesh in such a case is likely necessary. To achieve overall high efficiency, the numerical techniques involved in the QALE-FEM have been developed, including the method to move inner nodes, the technique to re-distribute the nodes on the free surface, the calculation of velocities and so on. The method has been validated by comparing its numerical results with those in publications.

Since the experimental demonstration by Heathershaw [5], the problem about the periodic bars has been studied by many researchers using various mathematical models with particular attention paid to the Bragg resonance that leads to large reflecting waves. These models were developed by making various approximations, including linear perturbation approach [6], multiple scale analysis [7], mild-slope approach [8], fully linear analysis [9] and so on. The results obtained from these models agreed satisfactorily with experiments carried out by Heathershaw [10] and Davies & Heathershaw [11] in cases with small surface waves and bar wave steepness. Only the paper by Liu and Yue [12] performed fully nonlinear analysis using a spectral method. In this paper, the recently-developed QALE-FEM method will be used to investigate the problems associated with the interaction between the water waves and periodic bars on the bottom, with particular attention paid to the nonlinear effects on the reflection at the Bragg resonance.

Description of the QALE-FEM

Details about the QALE-FEM have been given in [4] and a brief description about it will be presented herein. As indicated above, the finite element formulation is similar to those in [1] and [3]. The main difference is that the complex mesh is generated only once at the beginning and is moved at other time steps in order to conform to the motions of the free and structure surfaces. In this approach, the mesh can be generated by any generator and can have any complexity, any structure and any favourable distribution. Because the mesh generator is used only once in a simulation of several thousands time steps, the CPU time spent on the mesh generation is not an important matter since it may be only a small proportion of total computational time even it is as long as, say, several ten minutes. In addition, the generator is not necessarily included in the main code. However, it is obvious that the technique for moving mesh is vital in order to achieve a good quality mesh at all time steps and to avoid a large CPU requirement. To achieve this, the following strategies are adopted

- ensuring that there is no nearly flat elements in the initial mesh;

- considering the interior nodes and boundary nodes separately;
- considering the nodes on the free surface and on rigid boundaries separately;
- using relatively stiffer springs near the moving boundaries, such as the free surface.

The high quality of initial mesh is achieved by using the mesh generators based on the available technologies, such as the mixed Delaunay triangulation and advancing front technique. The interior nodes are moved by using the linear spring analogy method that has been well developed in computational aerodynamics. In the linear spring method, nodes are considered to be connected by springs. The whole mesh is then deformed like a spring system. At each time step, the equation for the spring system is solved to find the new positions of all inner nodes and nodes on rigid boundaries. The difference of the spring system in the QALE-FEM from that in computational aerodynamics is that the spring stiffness used here is determined by

$$k_{ij} = \frac{1}{l_{ij}} e^{\gamma\left[1+\left(z_i+z_j\right)/2d\right]}$$

where k_{ij} is the spring stiffness, l_{ij} is the distance between Nodes I and J; z_i and z_j are the vertical coordinates of Nodes I and J; d is the water depth; and γ is an coefficient that should be assigned a larger value if the springs are required to be stiffer at the free surface. In computational aerodynamics, the stiffness is taken as the inverse of the distance between two nodes.

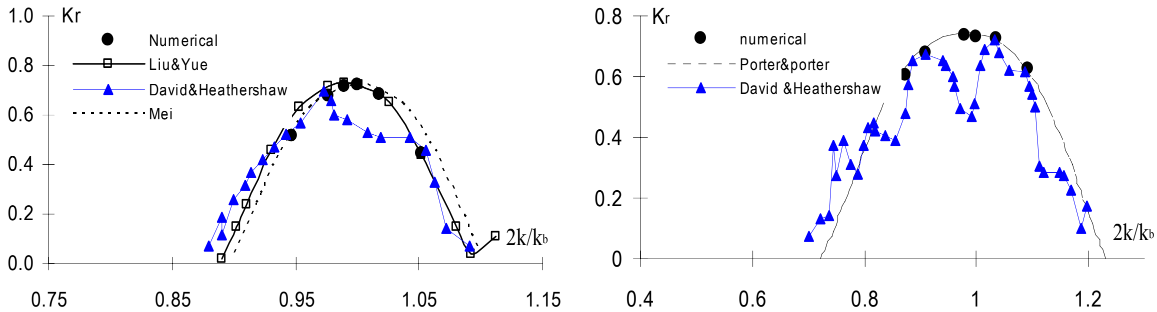
The nodal positions on the surface are determined by physical boundary conditions, i.e., following the fluid particles in the most of time steps. However, to prevent these nodes becoming too close to or too far from each other, these nodes need to be relocated every several time steps. In order to do so, the nodes on the free surface are grouped into those on curved waterlines and those that do not lie on the waterlines. The nodes in two groups are treated separately. Those on the waterlines is re-distributed based on a principle for self-adaptive mesh while those on the free surface but not on the waterlines is moved also using the spring method but with spring stiffness taking into account of the free surface gradient.

It is crucial in the simulation of water waves to evaluate the fluid velocities on the free surface because they are used to update the information on the surface every time step. The velocity at a node may be estimated by using a finite difference technique from the velocity potentials at this node and nodes connected to it. The approach is quite efficient. However, since the neighbours of a node on the free surface distribute either on or below the surface, the normal (or nearly vertical) component of the velocity estimated by the approach generally possesses relatively low accuracy, which is understandable from the fact that backward or forward finite difference schemes approximating a derivative have a lower order of accuracy than a central scheme. In order to enhance the overall accuracy, Ma, Wu & Eatock Taylor [1] suggested that the horizontal components of the velocities at nodes on the free surface are evaluated separately from their vertical components. For estimating the vertical component, they developed a three-point formula that needs the velocity potentials at the node considered and at other two nodes on the same vertical line as the previous node, which is next but below the free surface. After the vertical component is found, the horizontal components are computed by averaging those given by the difference of the velocity potentials at all neighbour nodes on the free surface. This approach is very efficient and accurate. However, it is limited to structural meshes with vertical grid lines. In the QALE-FEM, the above approach is extended to unstructured meshes generally without vertical grid lines. The basic idea of the new approach is similar to the above approach. The main differences are that (1) the vertical line is replaced by a normal line perpendicular to the free surface at the node considered; (2) the two nodes on the vertical line are replaced by two points that are not necessarily coincided with nodes; and (3) the normal (instead of vertical) component of the velocity is found before computing the components in tangential directions.

Numerical Results

In this section, the numerical results obtained by using the QALE-FEM will be presented and will be compared with the experimental data and analytical solutions from some of publications with particular attention paid to the reflecting wave properties near the Bragg resonance and to the nonlinear effects. The two cases to be considered are the same as those in [6], i.e., bar patches with 4 and 10 sinusoidal bars on the bottom of the wave tank, respectively. The wave will be generated by the wavemaker with the motion specified by $S(\tau) = -a\cos(\omega\tau)$, where $S(\tau)$ is the displacement of the wavemaker, $U(\tau)$ is its velocity, a and ω are respectively its amplitude and frequency.

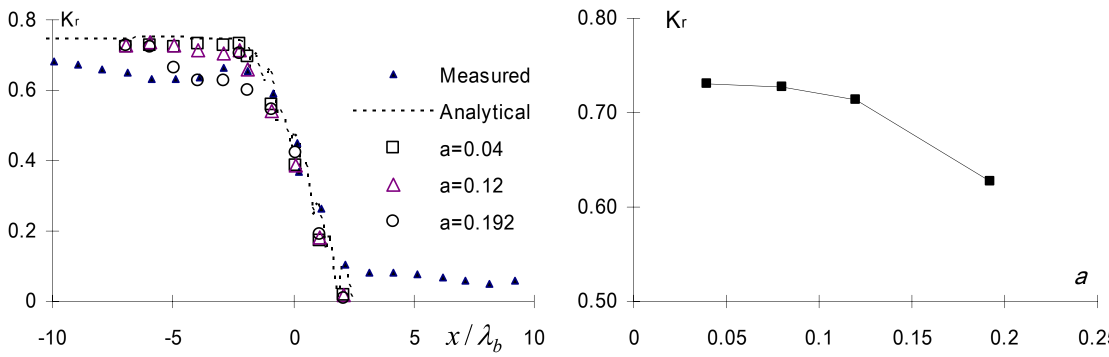
First considered are the cases with the small wave amplitudes. For these cases, the water waves are generated by using small amplitudes of the wavemaker and the resulting (incident) wave steepness is less than 0.01. In order to compare our results with experimental data in [6], the dimensionless bar wave number ($k_b d$) is assigned to a value of $\pi/10$, the ratios of the bar amplitude (a_b) to the water depth are taken respectively as a_b/d = 0.32 for 4 bars and a_b/d = 0.16 for 10 bars. The wave histories recorded at two points about 5 bar-lengths before the front side of the bar patch are used to compute the reflection coefficients. The reflection coefficients near the resonant condition ($2k/k_b$= 1, where k is the water wave number) are depicted in Figure 1 together with experimental data from [6]. For the case with 10 bars, the nonlinear results from [10] and analytical results from simplified models [7] are also included. For the case with 4 bars, the results from [11] are plotted apart from the experimental and our numerical results. Figure 1a for 10 bars indicated that our numerical results are almost coincided with those from [10] and closer to the experimental data than the analytical solution based on the simplified model [7] on the side of $2k/k_b$ >1. On the side of $2k/k_b$ <1, our results have some difference from [10] but closer to experimental data and the analytical results from the simplified model [7]. From Figure 10b, it can be seen that the numerical results obtained by using the QALE-FEM method agree well with the analytical results given in [11] and satisfactorily with experimental data.



(a) 10 bars, $k_b d$ =π/10, a_b/d =0.16 (b) 4 bars $k_b d$ =π/10, a_b/d =0.32

Figure 1 Reflection coefficient as a function of $2k/k_b$.

In order to investigate the nonlinear effects, the case with 4 bars is simulated with different amplitudes and results are presented in Figure 2. In Figure 2a, the coefficients at different positions are plotted together with experimental results from [6] and their analytical solution. It can be seen that the reflection coefficients before the bar patch tends to decrease with the increase of the amplitudes. To further show this trend, the reflection coefficients at a point x/λ_b= -4 are plotted in Figure 2b.

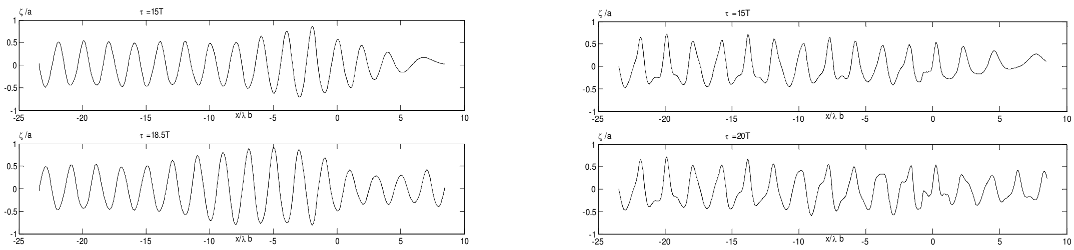


(a) Spatial variation (b) The variation with the ka

Figure 2 Reflection coefficients with 4 bars in a range of

$-2<x/\lambda_b<2$ ($k_b d$ =π/10, a_b/d = 0.32)

Apart from the effects on the reflection coefficients, the shapes of wave profiles are also different for different amplitudes. This is illustrated in Figure 3, where the profiles for different amplitudes are depicted. As can be seen, the wave profiles on the left of the bar patch for the smaller amplitude seems to be superimposed (so the wave become higher) by two harmonic waves traveling in opposite directions but the shape is still similar to the shape of harmonic waves. For the larger amplitude, the wave amplitude on the left of the bar batch seems not to be changed dramatically by the reflection waves, instead, the shape of the waves is significantly modified.



(a) a=0.04 (b) a=0.192

Figure 4 Wave profiles corresponding to different amplitudes for 4 bars

Summary

In this paper, the QALE-FEM recently developed by the authors are described and applied to simulate the interaction between water waves and periodic bars on the bed. Good agreement of numerical results from the QALE-FEM with those in publications is shown. The nonlinear effects on the reflection properties are investigated and it seems that the effects tend to reduce the reflection coefficients and make the wave profiles more complex. More results will be presented in the workshop.

Acknowledgement

This work is sponsored by EPSRC, UK (GR/R78701), for which the authors are most grateful. The authors also thank Dr Ping Dong, Dundee University, UK, for him to bring the problem about periodic bars to our attention.

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