Non-linear Interaction of Directionally Spread Waves with FPSO

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Introduction

Fixed and floating offshore production units are very much prone to the effects of the weather, in particular as a result of their requirement to remain in the same position. For ships it is often assumed that there is an equal probability of encountering waves from any direction and thus long-term calculations are made on the assumption of a uniform distribution of directions. FPSOs have a ship-like form with one axis of symmetry and with the longitudinal dimension much larger than the transverse one, which make them much more sensitive to the direction of the loading. Furthermore, as FPSOs are very susceptible to combined loading from more than one direction, wave directionality is much more important than for ship structures or other types of platform. However, there are difficulties with standard computational tools in predicting the effects of wave directionality in the prediction of second order mooring forces. The conventional method is very demanding on computer and human resources – in particular adequate resolution of wave spreading is hard. Therefore, advanced methods for the design and planning of operation of these vessels require more sophisticated models of the directionality of the environment [1] as well as models of hydrodynamic loads and structural response that are able to cope with the additional directional information.

As part of the FP5 REBASDO Programme, we are examining the effects of directional wave spreading on the non-linear hydrodynamic loads and the wave run-up in the region of the bow of a ship shaped FPSO. This paper presents a novel approach to deal with directionally spread seas in the numerical simulation of non-linear wave interaction with the FPSO. Numerical solutions of the problem are obtained for various types of directional spreading. The results are compared with the corresponding results for a unidirectional wave. From the comparison, it is shown that uni-directional waves don't always lead to the maximum wave forces on a ship-shaped FPSO, which contrasts to the effect of spreading seas on a circular cylinder [2]. This finding indicates that the conventional calculation for uni-directional waves may underestimate the wave forces on the vessel in a directionally spread sea.

Numerical approach

For an FPSO, with very complex body geometry, numerical simulation is required to perform the non-linear diffraction analysis. The computation for conventional second-order diffraction in directional spreading seas needs quadratic transfer function (QTF) matrices for a large number of bi-directional and bi-chromatic waves. The normal procedure to obtain QTFs in unidirectional waves requires the integration of free surface integrals, involving first order scattered wave results at pairs of frequencies [3,4]. The most obvious way of performing the analysis in a spread sea would be to do these integrals for each pair of frequencies at each pair of directions. Thus if there are N frequencies and M directions in the discretisation of a directional spectrum, $N^2 \times M^2$ pairs of second order diffraction calculations would be required, which leads potentially to an extremely large computation.

However, examination of the structure of the second order wave diffraction theory and its presentation in numerical analysis using BEM shows that a major reduction of computational demand may be obtained if we perform the analysis based on non-planar waves. The idea is to group all the waves from different directions but with same frequency together as a single incoming wave, then input this non-planar wave as a component for each pair of diffraction calculations in bi-chromatic waves. Using this new approach, one only needs to evaluate N^2 pairs of second order solutions, rather than $N^2 \times M^2$. This will reduce the calculation for spread waves to around the same amount of the computational effort as for unidirectional waves. The first order analysis is in effect performed for a non-planar wave of the form shown in Figure 1.

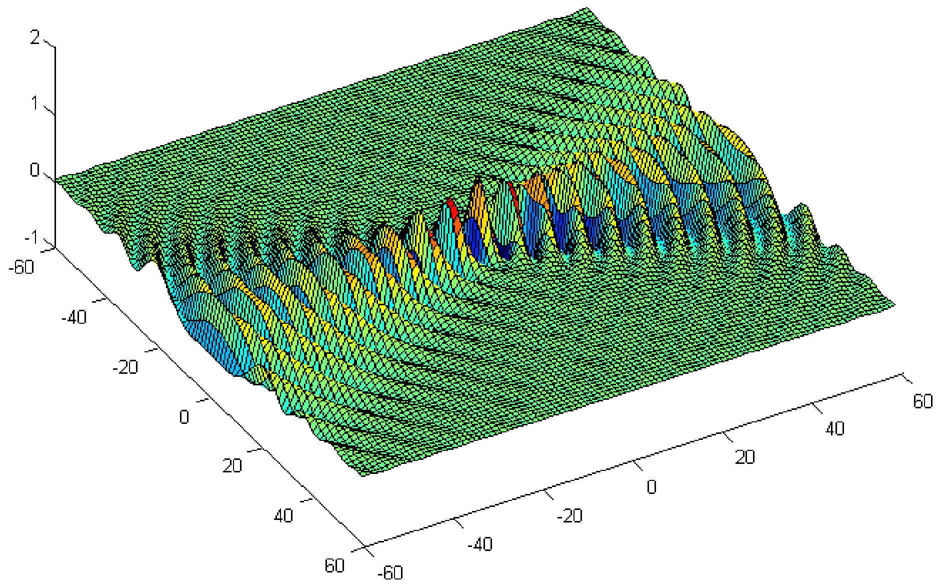

Figure 1 Non-planar wave with single frequency

By applying Stokes' perturbation method, the wave velocity potential in directionally spread seas to second order can be expressed by

$$\Phi(x,y,z,t) = \sum_{i=1}^{N} \phi_i^{(1)}(x,y,z)e^{-i\omega_i t} + \sum_{i=1}^{N}\sum_{j=1}^{N}[\phi_{ij}^{+}(x,y,z)e^{-i\omega_{ij}^{+}t} + \phi_{ij}^{-}(x,y,z)e^{-i\omega_{ij}^{-}t}] + \dots$$

where ω is wave frequency, N is the total components of frequency components. The velocity potential ϕ is only associated with the spatial variables, its first and second order value can be given by

$$\phi_i^{(1)}(x,y,z) = \sum_{m=1}^{M} \phi_{im}^{(1)}(x,y,z)e^{i(k_i(x\cos(\theta_m)+y\sin(\theta_m)))}$$

$$\phi_{ij}^{\pm}(x,y,z) = \sum_{m=1}^{M}\sum_{n=1}^{M} \phi_{ijmn}^{\pm}(x,y,z)e^{i(k_i(x\cos(\theta_m)+y\sin(\theta_m))\pm k_j(x\cos(\theta_n)+y\sin(\theta_n)))}$$

where k is wave number, θ is wave direction, and M is total components of wave direction.

Discussion of results

The non-linear interaction of spread seas with a simple representation of an FPSO is considered in this paper. The length of the body is 1.124m, the width is 0.324m, and the draft is 0.125m. For further details see [4]. A Gaussian directional spreading function

$$D(\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{-\frac{(\theta-\bar{\theta})^2}{2\sigma_\theta^2}}$$

with various values for σ_θ is used to model waves of various angular spreading. For uni-directional waves (σ_θ=0), our numerical results show excellent agreement with experiments conducted at Imperial College [3,4].

The QTFs for mean drift forces with directional spreading σ_θ=15^o and 30^o in main wave direction β_o=0.0^o (head-on), 15^o, 30^o, and 45^o are obtained and compared with the results from uni-directional waves. The results shown in this paper are normalised by

$$\text{Force:} \quad QTFs = Force/\pi\rho gbA_1A_2, \quad \text{Moment:} \quad QTFs = moment/\pi\rho gb^2A_1A_2$$

where A is the wave amplitude and b is the draft. Examples of the results are presented in Figure 2 to Figure 4. The influence of wave directionality on the mean drift surge force in the head-on case is shown in Figure 2. For this case where the mean-wave direction is head-on, the uni-directional forces are largest at all frequencies. As the spreading angle is increased the surge QTFs are increasingly reduced below the uni-directional values at high frequency.

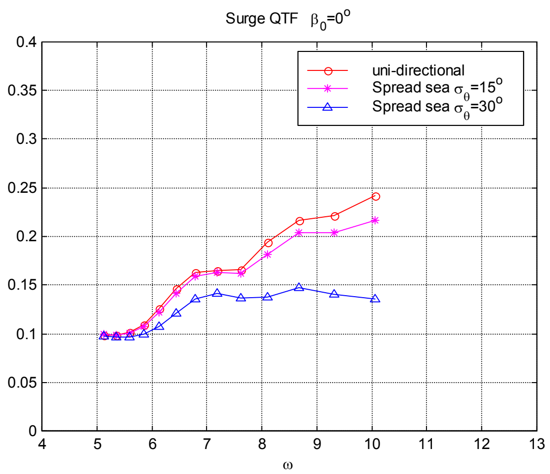

Figure 2 Surge QTFs for various values of Gaussian spreading function in head-on case.

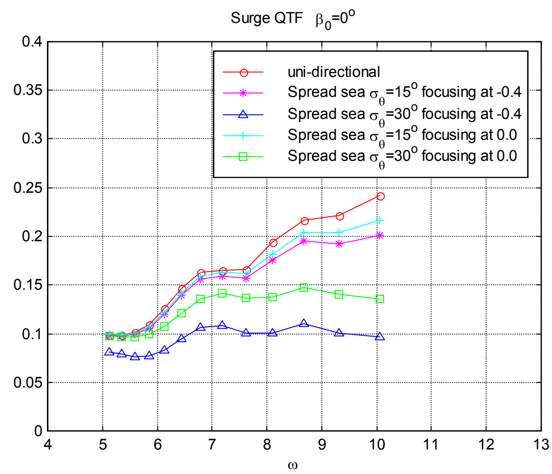

Figure 3 Surge QTFs for various values of Gaussian spreading function focused at different locations in head-on case

Figure 3 shows the effect of focusing position on the surge force. As discussed in [2], the local behaviour of a spread wave close to the focus point leads to the idea of a 'focus spot' over which the spread wave looks locally uni-directional. The relative size of this 'focus spot' as compared to the size of the body plays an important role in the diffraction of spread waves. As the FPSO used in the calculation is relatively large compared with the wavelength for the entire range of the wave frequencies considered, a shift in position of the focus point also affects the wave force, but a larger degree of wave spreading always leads to a reduction of the surge force.

Some of the results for non-head on cases are presented in Figure 4. It is interesting to see that uni-directional waves still produce the maximum mean drift surge forces, but the mean drift sway QTFs obtained from spread waves are bigger than those from uni-directional waves for some certain frequencies. It seems that, at certain wave frequencies in certain wave directions, spread waves may lead to a more severe design case for wave loading.

The above results have shown the significance of wave directional spreading on mean-drift forces. They also demonstrate that the influence of wave directionality on the FPSO depends on a number of factors, such as spreading function, main incoming wave direction, wave frequency, focusing position, and relative ratio of body size and spread wave characteristics etc. More details will be discussed in the presentation.

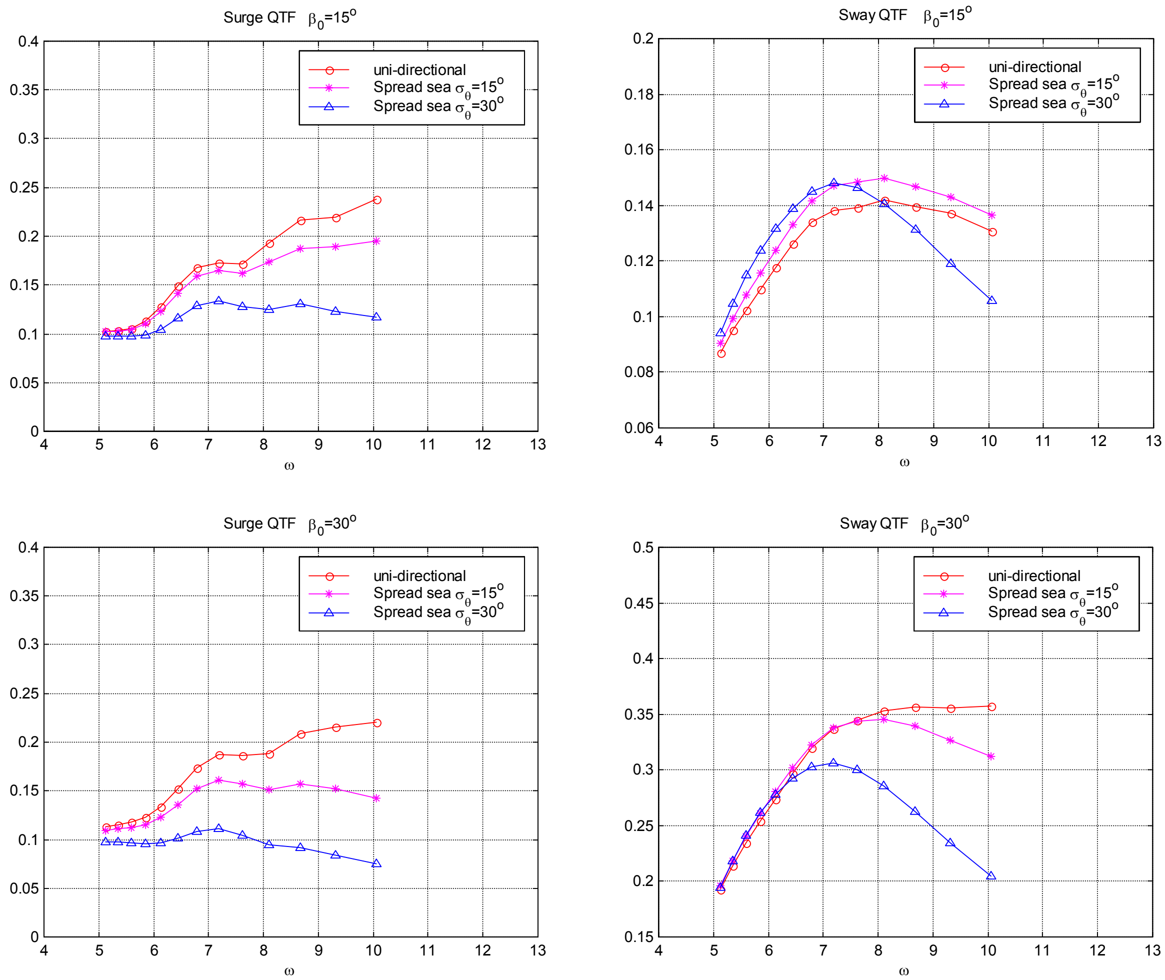

Figure 4 QTFs of mean drift forces for various bandwidths of Gaussian spreading function in main wave direction $\beta_o=15.0^o$ and $\beta_o=30.0^o$

Acknowledgements

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