Experiments on the velocity and acceleration fields in overturning waves in deep water

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The extreme waves on the ocean, how they form and look like, represent a current research focus in the field of marine hydrodynamics. Several sources are leading to the build-up of the large waves. The most important mechanisms are the wave fields interacting with a variable current such as the Aghulas current on the east coast of South Africa, superposition of the wave fields, and nonlinear focusing. A fourth effect is wind. How this factor acts on the formation of freak waves is less investigated due to the more complicated mathematical representation of the motion of the two fluid phases that are involved; the air and water. The theoretical research is pursued along the lines where these effects are modelled. What is not known is the organization that takes place leading to the very energetic conditions with one ore more extreme waves as the end product, such as the Draupner wave, however.

Observations of the very large waves at sea and precise experiments in laboratory represent important complements to the theoretical investigations. Our purpose here is to report the results of an exhaustive measurement campaign of very large waves in the laboratory. More specifically, we report the measured kinematics of the six largest wave events in a campaign where 122 different large waves in deep water were measured. The results for the six largest events presented here include a total of 36000 experimental velocity vectors measured by Particle Image Velocimetry (PIV). Focussing waves were generated in the 24.6 m long wave tank in the PIV and wave laboratory at the University of Oslo. The tank width is 0.5 m and the water depth 0.72 m. The velocities and the material acceleration fields of the waves is obtained by employing an extended PIV system. The system, particularly designed to measure accelerations, consists of two CCD cameras and CW argon ion laser together with a scanning beam system (Jensen and Pedersen 2004). Polyamid particles with diameter $50\mu m$ were used for seeding.

Velocity fields. For the velocity fields we shall use the scaling introduced by Grue et al. (2003). The appropriate velocity scale is obtained by first defining a wavenumber, k and a wave slope, ϵ of the individual large wave event. A reference velocity is then defined by $u_{ref} = \epsilon\sqrt{g/k}$, and non-dimensional velocity vectors by $(\hat{u}, \hat{v}) = (u, v)/u_{ref}$. The k and ϵ are obtained from the time record of the surface elevation where the trough-to-trough period of the event, T_{TT} and the maximal elevation, ζ_m are determined. We then define $\omega = 2\pi/T_{TT}$ and compute k and ϵ from

$$\frac{\omega^2}{gk} = 1 + \epsilon^2, \qquad k\zeta_m = \epsilon + \tfrac{1}{2}\epsilon^2 + \tfrac{1}{2}\epsilon^3. \tag{1}$$

The usefulness of this scaling is illustrated in figure 1 (upper) which shows the velocity profile below crest of eight different waves with ϵ in a rather broad range: $0.29 < \epsilon < 0.46$. The six largest waves are experimental and are indicated in the figure by the square symbols. The values of ϵ is in the range 0.40–0.46 for the six waves. The experimental waves are made in the wave tank by a standard superposition, or focussing technique. The theoretical curve that appears in the figure is the computed velocity profile below crest of a wave with $\epsilon = 0.29$ (estimated by using eq. 1). This (and other) large event(s) occur when an initially long wave packet splits onto several group solitons that also become interacting (Clamond and Grue 2002). The wave number eight is due to the LDA measurements published by Baldock et al. (1996) with estimated $\epsilon = 0.29$. Their velocity measurements fit very much with our measurements when put on non-dimensional form. The curve indicates the velocity profile below crest of any large deep water wave at sea.

Figure 1 (mid and lower) shows the (non-dimensional) velocity vector plots due to the two largest waves. For reference of the non-dimensional values we estimate the wave propagation speed by evaluating ω/k. In non-dimensional terms the wave speed is estimated by $\sqrt{1+\epsilon^2}/\epsilon$, see eq. (1), giving a non-dimensional wave propagation speed of 2.5 in figure 1 (mid) and 2.4 in figure 1 (lower). The horizontal velocities below crest are identified along the line corresponding to $\hat{v} = 0$. Maximal horizontal velocities are observed all along the top part of the wave for the overturning waves and has a maximal non-dimensional value of 2.3 in all the experiments. This is always smaller than the estimated wave propagation speed. The absolute value of the velocity vector $\sqrt{\hat{u}^2 + \hat{v}^2}$ has a maximal value of 2.4 in the front part – the tip – of the wave in figure 1 (lower). This corresponds to the magnitude of the wave propagation speed. The magnitude of the velocity vector is less than the wave speed in all other cases we have investigated, however.

Accelerations. We plot here the acceleration vectors that are scaled by the acceleration of gravity (g). The material (full) acceleration vector is obtained, including the local and the convective terms. Two acceleration plots are given: the vertical and horizontal components of the acceleration vector as function of the vertical coordinate (figure 2). The vast majority of the acceleration vectors are in the range determined by the estimated wave slope $\epsilon \sim 0.40 - 0.46$. The accelerations in the strongest overturning waves exhibit a maximal vertical acceleration of 1.5 g, occurring about half-way to the level of the crest. The results for the horizontal accelerations indicate a level up to about 1.1 g, and a few (three) vectors that exceed 2 g. The maximal horizontal accelerations occur in the front part of the wave at level about half-way to the crest. The maximal accelerations that can be identified in these experiments are at most 2 g and occur in the base in the front part of the wave. This should be compared to the maximal computed accelerations in such waves with absolute value of 3.25 g (Chen et al. 1999) and up to 6 g in both the vertical and horizontal directions (New et al. 1985).

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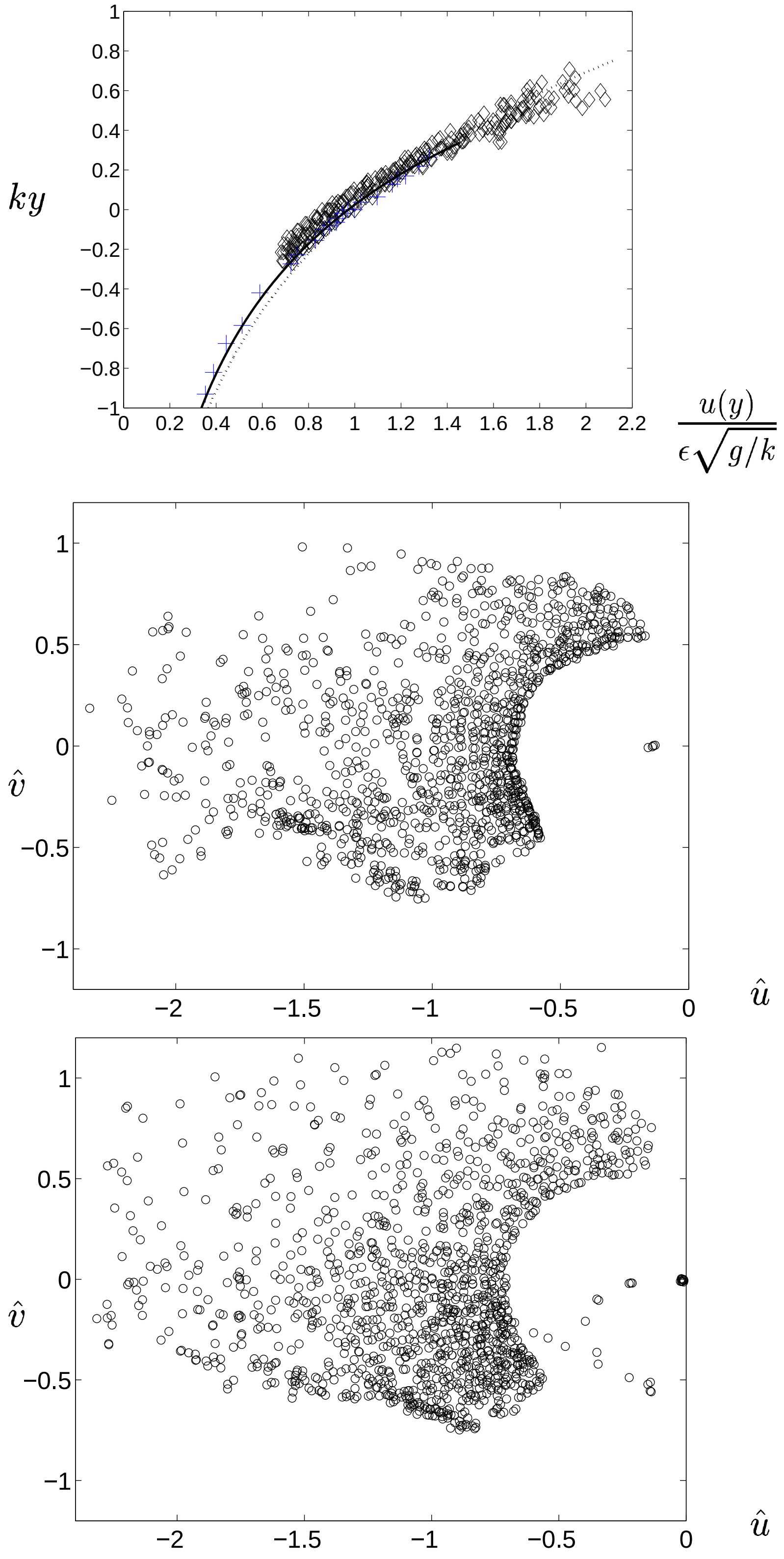

Figure 1: Upper: Non-dimensional velocity profile below crest, present experiments ($\diamond$ $\diamond$), LDA-measurements by Baldock et al. (1996) with $\epsilon = 0.29$ ($+ + +$), fully nonlinear computations by Clamond and Grue (2002) (solid line), $\exp(ky)$ (....). Mid: Velocity plane plot of vectors $(\hat{u}, \hat{v}) = (u, v)/[\epsilon\sqrt{g/k}]$ for overturning wave with $\epsilon = 0.44$. Lower: same with $\epsilon = 0.46$.

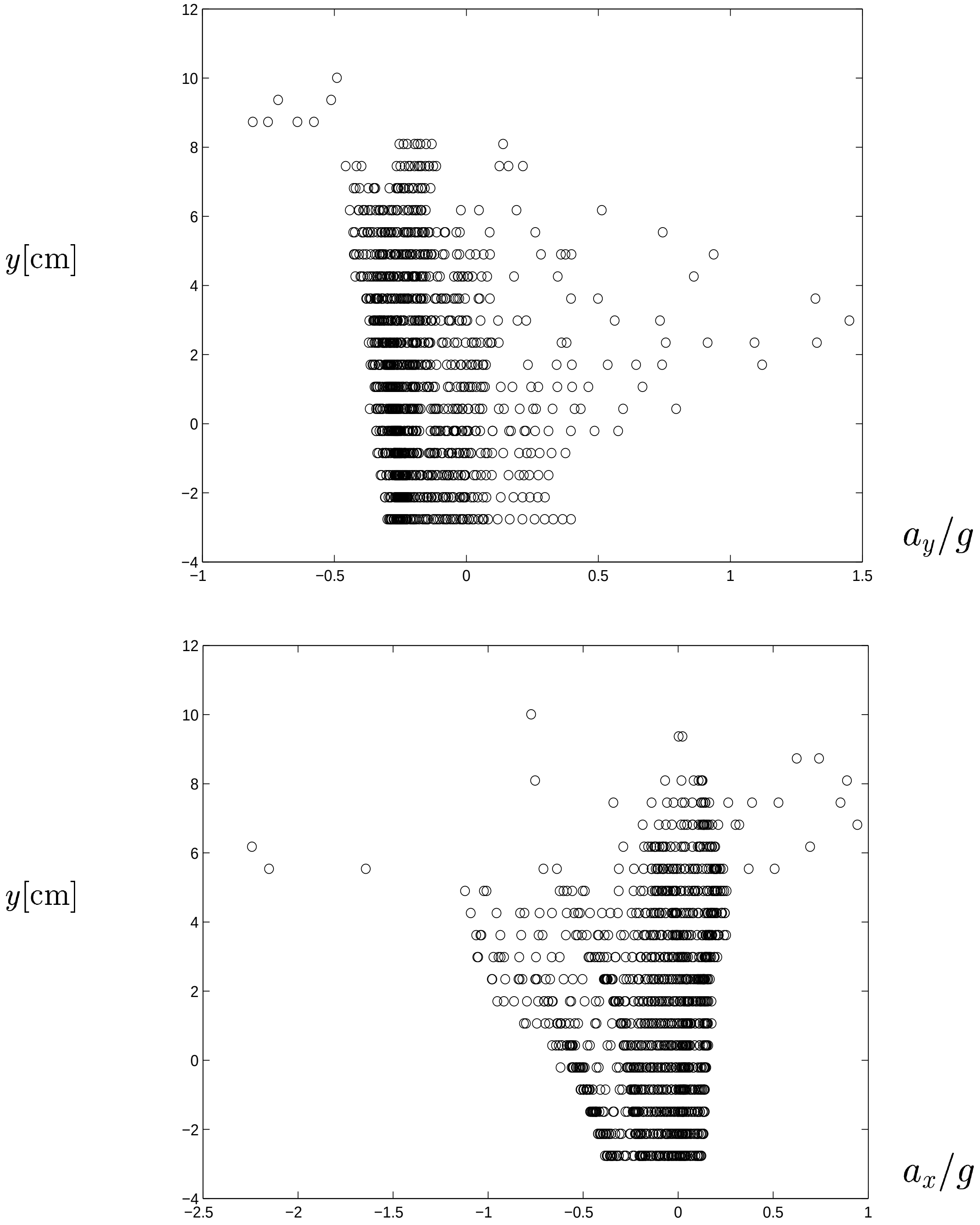

Figure 2: Vertical acceleration a_y/g (upper) and horizontal acceleration a_x/g (lower) vs. vertical position y in waves, three strongest cases, with g the acceleration of gravity.