

An efficient hydroelastic model for wave induced coupled torsional and horizontal ship vibrations

Malenica Š¹, Senjanović I.² & Tomašević S.²

(1) BUREAU VERITAS - DR, Paris, France (*sime.malenica@bureauveritas.com*)

(2) Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia

Introduction

The wave induced hydroelastic response of a ship becomes an important issue for large ships, because of the structural natural frequencies which can fall into the range of the frequencies contained in the typical sea spectra. In those cases it is likely that the ship will experience an important hydroelastic response, usually called springing. For those ships with closed cross-section (tankers, bulk, general cargo,) the lowest natural frequencies are usually associated with the vertical bending, while for the ships with open cross-section, such as container ships, the lowest elastic natural frequency might be the coupled mode of torsion and horizontal bending. Also, for this type of ships coupling between horizontal and torsional vibration is very pronounced due to the fact that center of torsion (shear center) is below the keel, Figure 1. Since only the first few elastic modes (with lowest natural frequencies) may be excited, the simple non-uniform beam model of the ships is usually considered to be sufficient for springing analysis. However, the beam model is sufficiently simple only in cases of uncoupled responses so that for the container ships the full 3D FEM models are usually preferred, Figure 2. Since the 3D FEM modeling is very complex and time consuming, there is a need for simplified structural models which can be used in the pre-design process. Here we propose an improved beam model for coupled torsional and horizontal vibrations, and we use it together with 3D hydrodynamic panel code HYDROSTAR.

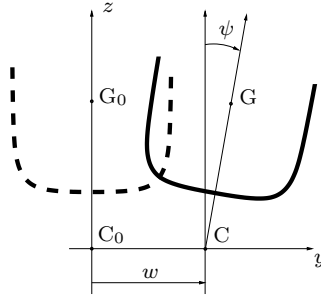


Figure 1: Initial and deformed ship section.

Mathematical model

Structural model

The basic definitions are shown in Figure 1. The w denotes the horizontal deflection, ψ the twist angle, G_0 and G the initial and final position of the center of gravity and C_0 and C the initial and final position of the center of torsional rotation (shear center). The coupled system of differential equations for prismatic beam becomes [4]:

$$EI \frac{\partial^4 w}{\partial x^4} - \left(\frac{EI}{GF} m + J \right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + m \left(\frac{\partial^2 w}{\partial t^2} + \frac{J}{GF} \frac{\partial^4 w}{\partial t^4} + z_{CG} \frac{\partial^2 \psi}{\partial x^2} \right) = q \quad (1)$$

$$EI_w \frac{\partial^4 \psi}{\partial x^4} - GI_t \frac{\partial^2 \psi}{\partial x^2} + J_t \frac{\partial^2 \psi}{\partial t^2} - J_w \frac{\partial^4 \psi}{\partial x^2 \partial t^2} + m z_{CG} \left(\frac{\partial^2 w}{\partial t^2} - \frac{EI}{GF} \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) = \mu \quad (2)$$

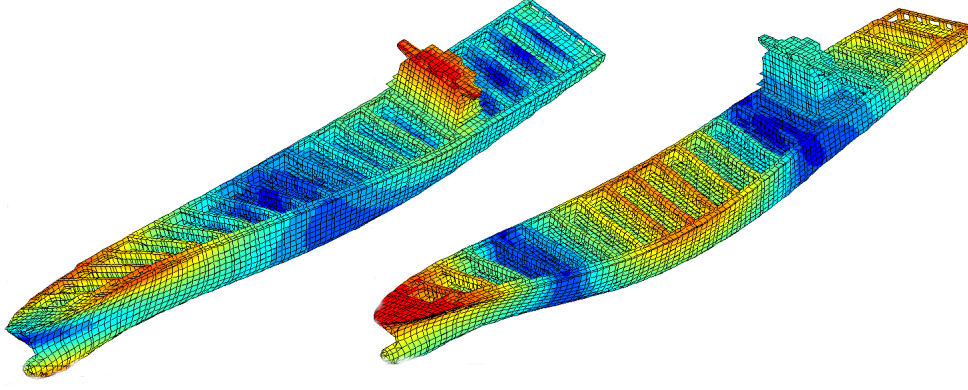


Figure 2: Typical deformation modes of container ship.

where E and G are Young's and shear modulus of elasticity, I and F are bending modulus and shear area, m is the distributed ship mass per unit length, J is mass moment of inertia about z axis, I_w is warping modulus, I_t is torsional modulus, J_t is polar mass moment of inertia, J_w is mass bimoment of inertia, q is the distributed lateral load and μ is the distributed torsional moment.

Harmonic excitation is assumed and the finite element method is used for solving the above coupled differential equations. The resulting matrix equation takes the following form:

$$([K] - \omega^2[M])\{U\} = \{P\} \quad (3)$$

where $[M]$ and $[K]$ are the mass and stiffness matrix respectively, ω is the excitation frequency, $\{U\}$ is the displacement vector containing the amplitudes of both horizontal deflections and torsional rotations and their first derivatives, and $\{P\}$ is the amplitude vector of the external hydrodynamic pressure loads. The corresponding eigenvalue problem is then formulated:

$$([K] - \omega^2[M])\{u\} = \{0\} \quad (4)$$

and the, so called, "dry" natural frequencies and modes are defined:

$$\omega_i, \{u\}_i, i = 1, N \quad (5)$$

$$[\Xi] = [\{u\}_1, \{u\}_2, \dots, \{u\}_i, \dots, \{u\}_N] \quad (6)$$

The forced vibration problem may be solved by the mode superposition method. In that case, the displacement vector is expressed as series of the natural modes:

$$\{U\} = [\Xi]\{\xi\} \quad (7)$$

where $\{\xi\}$ is the vector of modal amplitudes.

By multiplying (4) with $[\Xi]^T$ from the left, and substituting (7) the modal matrix equation for forced ship vibration becomes:

$$([\mathbf{k}] - \omega^2[\mathbf{m}])\{\xi\} = \{F^h\} \quad (8)$$

where $[\mathbf{k}]$ and $[\mathbf{m}]$ are the modal stiffness and mass matrix respectively and $\{F^h\}$ is the modal loads:

$$[\mathbf{m}] = [\Xi]^T[M][\Xi], \quad [\mathbf{k}] = [\Xi]^T[K][\Xi], \quad \{F^h\} = [\Xi]^T\{P\} \quad (9)$$

Note that, as result of orthogonality, the matrices $[\mathbf{k}]$ and $[\mathbf{m}]$ are diagonal.

Hydrodynamic model and coupling

The above mentioned modal superposition method is used for coupling of structural and hydrodynamic models. In that respect the displacement of any point belonging to the ship structure is found by the following expression:

$$\mathbf{H}(x, y, z) = \sum_{i=1}^N \xi_i \mathbf{h}^i(x, y, z) = \sum_{i=1}^N \xi_i [h_x^i(x, y, z)\mathbf{i} + h_y^i(x, y, z)\mathbf{j} + h_z^i(x, y, z)\mathbf{k}] \quad (10)$$

where the vector functions \mathbf{h}^i denotes the dry structural modes and ξ_i their amplitudes. The functions \mathbf{h}^i transfer the modal beam displacement vector $\{\mathbf{u}\}_i$ from the beam center of deformation (point C in Figure 1) to the ship cross-section and subsequently to whole ship structure.

We adopt the potential theory and define the velocity potential $\varphi(x)$ by the following boundary value problem (BVP):

$$\left. \begin{aligned} \Delta\varphi &= 0 && \text{in the fluid} \\ -\nu\varphi + \frac{\partial\varphi}{\partial z} &= 0 && z = 0 \\ \frac{\partial\varphi}{\partial n} &= -i\omega\mathbf{H}\mathbf{n} && \text{on } S_B \end{aligned} \right\} \quad (11)$$

where ν is the wavenumber $\nu = \omega^2/g$.

Thanks to the linearity, we decompose the total potential as follows:

$$\varphi = \varphi_I + \varphi_D - i\omega \sum_{j=1}^N \xi_j \varphi_{Rj} \quad (12)$$

where φ_I is the incident potential, φ_D is the diffraction potential and φ_{Rj} are the radiation potentials. From (10) and (11), we can deduce the body boundary conditions for each potential:

$$\frac{\partial\varphi_D}{\partial n} = -\frac{\partial\varphi_I}{\partial n} \quad , \quad \frac{\partial\varphi_{Rj}}{\partial n} = \mathbf{h}^j \mathbf{n} \quad (13)$$

Let us finally note, that the diffraction and radiation potentials should also satisfy the radiation condition. The classical 3D panel method, based on source formulation, is used to solve the above defined BVP's using the numerical code HYDROSTAR. Once the different potentials are found, the pressure is calculated from Bernoulli's equation:

$$p = i\omega\rho\varphi - \rho gz \quad (14)$$

and the corresponding forces are obtained after integration over the wetted body surface. The forces are then subdivided into the part independent of ship motions/deformations and the parts depending on the ship motions/deformations. We write (see [2] for details):

$$\{\mathbf{F}^h\} = \{\mathbf{F}^{DI}\} + (\omega^2[\mathbf{A}] + i\omega[\mathbf{B}] - [\mathbf{C}])\{\boldsymbol{\xi}\} \quad (15)$$

The coupled motion equation can now be written in the following form:

$$\{-\omega^2([\mathbf{m}] + [\mathbf{A}]) - i\omega[\mathbf{B}] + ([\mathbf{k}] + [\mathbf{C}])\}\{\boldsymbol{\xi}\} = \{\mathbf{F}^{DI}\} \quad (16)$$

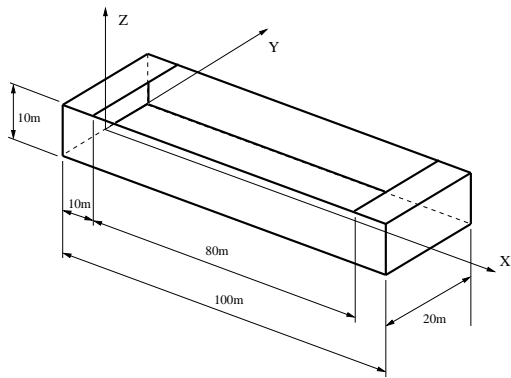
where:

- $[\mathbf{m}]$ - modal genuine mass
- $[\mathbf{k}]$ - modal structural stiffness
- $[\mathbf{A}]$ - hydrodynamic added mass
- $[\mathbf{B}]$ - hydrodynamic damping
- $[\mathbf{C}]$ - hydrostatic stiffness
- $\{\boldsymbol{\xi}\}$ - modal amplitudes
- $\{\mathbf{F}^{DI}\}$ - modal pressure excitation

The solution of the above equation gives the motion amplitudes ξ_i and the problem is formally solved.

Preliminary results and comments

Here below we show some preliminary results for simplified barge with open section through the midship part but closed at its ends. The geometry of the barge is presented in Figure 3 (the wall thickness is 1cm), together with its first three dry natural frequencies. Good agreement between the 1D and 3D



Mode No.	1	2	3
1D Beam model	0.42	1.43	1.61
3D FEM	0.41	1.41	1.73

Figure 3: Barge geometry and the first 3 dry natural frequencies [Hz].

results is found. The corresponding mode shapes, obtained with 1D beam model and calculated with respect to the center of gravity, are shown in Figure 4. All the modes show an important coupling between the torsion and horizontal bending. The first and second modes are dominated by the torsion, while the third one is dominated by the horizontal bending.

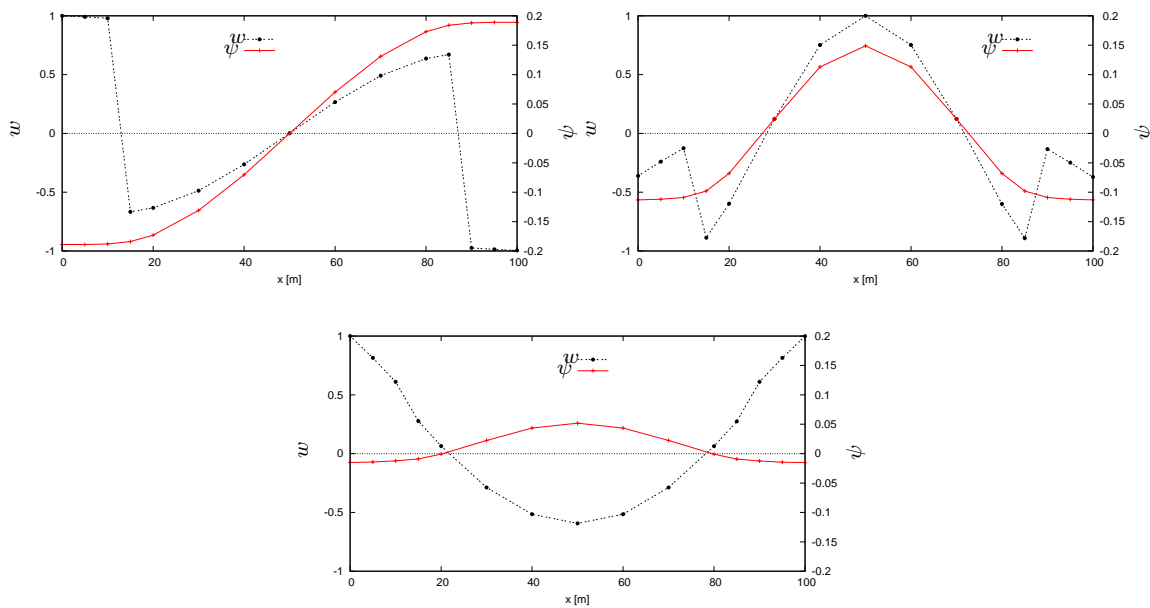


Figure 4: Natural modes.

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Discussor - H. B. Bingham:

As you are clearly interested in ships, but have only discussed zero speed problems here, I wonder how you intend to treat forward speed problem?

Reply:

The method that we presented here is formally independent on the presence or not of the forward speed. This means that it can be directly applied to the problem with forward speed, provided the hydrodynamic coefficients (added mass, damping, excitation & restoring) are properly calculated. Indeed, in the case of elastic body with forward speed, main problems will be related to the calculation of the hydrodynamic coefficients and, in that respect, the difficulties involved will be the same for rigid and elastic body. As we all know, the problem with forward speed is much more difficult and, we think, it is fair to say that it is not properly solved yet, even for linear rigid body seakeeping problem. Actually, one big part of the Bureau Veritas research activities concerns the rigid body seakeeping analysis for a floating body animated with forward speed, and these developments are led by Dr. Xiao-Bo Chen.

Anyway, in this paper, the main accent is put on hydroelasticity and on the coupling procedure. The zero speed case was chosen for detailed validation because of the existence of the experimental results. However, once the seakeeping model for floating rigid body with forward speed is ready, it will be straightforward to apply it for hydroelastic applications.

Discussor - R. Eatock Taylor:

While I understand the objective of developing a simple 1D model, and am aware of earlier work in this area by Bishop, Price et al., I am surprised that you have chosen the extreme simplification of a uniform beam. While your results from this model shows fair comparisons of the lowest three natural frequencies from those calculated using 3DFEM for a barge having an open cross section over 80 % of its length and a closed section over 10 % at each end, will such a very simple model be useful in practical situations? It should not be much more difficult to develop a 1D nonuniform beam model which would be appropriate for preliminary design study of container ships.

Reply:

Simplified 1D beam model is used only for the barge case, and that in order to check the functionality of the proposed approach. In the case of a real ship with variable cross-section (alternatively open and closed), a more complex 1D nonuniform beam FEM model is used. The beam equations (1) and (2) directly allow for this. Moreover, the stiffness parameters present in these equations are modified in order to take into account the contribution of the transverse bulkheads, and (we believe) this represent an improvement over the methods proposed in the past by Bishop, Price et al. On the other hand the hydrodynamic model, which is used here, is based on the full 3D method, while the method used by Bishop, Price et al. is based on strip theory approach (at least to our knowledge).

Discussor - Y.Kim :

Since the springing is higher-order phenomena, I assume that the second-harmonic wave excitation is important. Your frequency domain formulation shows only linear excitation. Do you have more explanation on such linear formulation?

Reply:

We do not think that the springing is necessarily higher order phenomena. We think that it is more exact to say that the springing is a just resonant phenomena. Would it be considered as a linear or higher order, will depend on the natural frequency of the elastic modes and on the ship operating conditions (ship speed, wave heading, frequency content of the sea state, ...). It is important to note that, the actual projects on ultra large container or LNG ships, lead to a relatively "soft" structures for which the combination of the operating conditions and low elastic natural frequencies, can lead to a linear types of excitations. Indeed, these ships are required to run faster and faster and, since the ship excitation is related to the encounter frequency and not to the fundamental wave frequency, these ships can be excited in a linear sense. In that respect, the present model could be directly applied for springing assessment of these ships, since the higher order parts of the excitation are likely to be of less importance. However, as you correctly pointed out, this would not always be the case, so that the source of the excitation might be of higher order (second, third, ...). The typical example concerns the springing of TLP platforms where the second order effects were identified as a main source of excitation. In that respect, important experience already exists in Bureau Veritas and will also be applied for the ship type hydroelastic applications. However, due to the extreme complexity of the seakeeping problem with forward speed (see the answer to H.Bingham question) even in the linear rigid body case, it should be expected that only approximated models will be considered first (Froude-Krylov approximation, incomplete second order model, local second order diffraction models, ...). In any case, and before going further, it is absolutely necessary first to validate the linear hydroelastic models, and that was the purpose of the present paper.