When is the bow wave of a ship in steady motion unsteady ?

Francis Noblesse, Dane Hendrix, Gabor Karafiath
NSWCCD, 9500 MacArthur Blvd, West Bethesda, MD 20817, U.S.A. (francis.noblesse@navy.mil)

Introduction

A ship that advances along a straight path, with constant speed U, through a large body of calm water is usually expected to generate a steady bow wave; but this is not necessarily always true. Indeed, steady motion of a body through a fluid at rest does not necessarily result in a steady flow; the von Karman vortex sheet that can be observed, under some conditions, behind a bluff body advancing steadily through a fluid at rest is a well-known example of unsteady flow generated by steady motion of a body.

A simple criterion that determines when the bow wave generated by a ship, advancing steadily in calm water, cannot be steady is given here. This criterion, based on the Bernoulli equation and a simple analytical expression for the height of a ship bow wave, shows that a ship with a sufficiently fine waterline (with waterline entrance angle smaller than approximately $25°$) may be expected to generate a steady bow wave at any speed. However, a ship having a fuller waterline can only generate a steady flow if the ship speed is higher than a critical speed, for which a simple analytical expression is given below.

A simple analytical expression for the height of a ship bow wave

Noblesse et al. (2006) shows that the height Z_b of the bow wave generated by a ship that advances at constant speed U in calm water is approximately determined in terms of the ship speed U, the gravitational acceleration g, the ship draft T, and the waterline entrance angle $2\,\alpha_E$ by the analytical expression

$$\frac{Z_b\,g}{U^2} \approx \frac{C^Z}{1+F_T}\,\frac{\tan\alpha_E}{\cos\alpha_E} \tag{1}$$

with

$$F_T = U/\sqrt{gT} \tag{2}$$

and $C^Z \approx 2.2$.

This simple analytical expression — based on fundamental theoretical considerations (dimensional analysis, thin-ship limit, and deep-draft and shallow-draft limits) and experimental measurements, which are used to determine the constant C^Z in (1) — is in excellent agreement with experimental measurements for six wedge-shaped ship bows, and is also in good agreement with measurements for the Wigley hull and the Series 60 model, and similar ship-bow forms, especially if a simple procedure is used to define an effective draft T and an effective waterline entrance angle $2\,\alpha_E$. This agreement between experimental measurements and theoretical predictions can be observed in Fig.1, where the normalized bow-wave height $(Z_b\,g/U^2)\cos\alpha_E\,/\tan\alpha_E$ is depicted as a function of the draft Froude number F_T defined by (2). Experimental measurements for nine ship hulls are shown in Fig.1, where the solid line corresponds to the approximation $2.2/(1+F_T)$.

The Bernoulli constraint

For a steady free-surface flow that is observed from a system of coordinates (X,Y,Z) attached to a ship advancing along a straight path at constant speed U in calm water, the velocity of the total flow (uniform stream opposing the forward speed of the ship + flow due to the ship) is $(V_x - U, V_y, V_z)$. Here, (V_x, V_y, V_z) is the flow due to the ship. Furthermore, the Z axis is vertical and points upward with the mean free surface taken as the plane $Z=0$, and the X axis lies along the ship path and points toward the bow. The Bernoulli relation

$$P/\rho + g\,Z + [\,(V_x - U)^2 + V_y^2 + V_z^2\,]/2 = P_{atm}/\rho + U^2/2$$

applied at the free surface, where $P = P_{atm}$, shows that an upper bound for the free-surface elevation Z is

$$Z\,g/U^2 \leq 1/2 \tag{3}$$

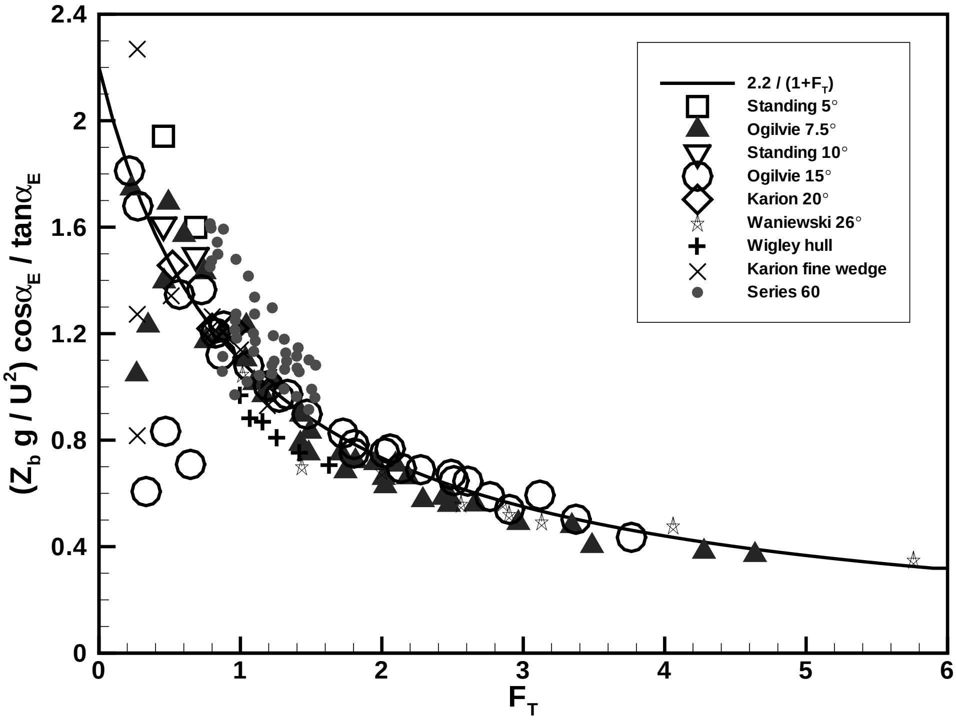

Figure 1: Bow-wave height $(Z_b\, g/U^2)\cos\alpha_E\, /\tan\alpha_E$ for nine ship hulls. The solid line corresponds to the approximation $2.2/(1+F_T)$

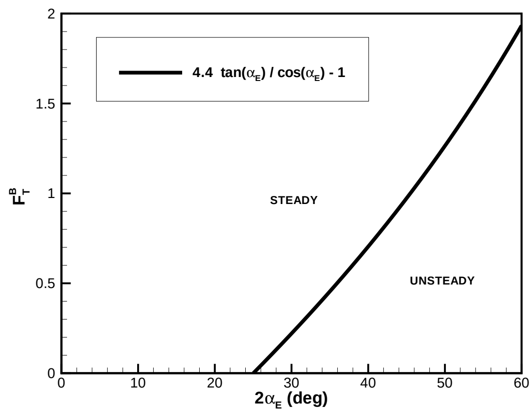

Figure 2: Bernoulli-bound Froude number F_T^B given by (4)

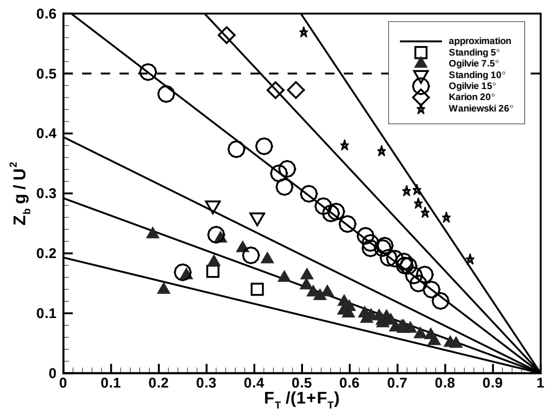

Figure 3: Bow-wave height $Z_b\, g/U^2$ for six wedge-shaped hulls. The solid lines correspond to the approximation (1) and the horizontal dashed line represents the Bernoulli upper bound (3)

This Bernoulli constraint is satisfied by expression (1) for the bow-wave height Z_b if $F_T^B(\alpha_E) \leq F_T$ where the function $F_T^B(\alpha_E)$ is defined as

$$F_T^B(\alpha_E) = \left\{ \begin{array}{c} 0 \\ 2\,C^Z \tan\alpha_E \,/\cos\alpha_E - 1 \end{array} \right\} \quad \text{if} \quad \left\{ \begin{array}{l} \alpha_E \leq \alpha_E^B \\ \alpha_E^B \leq \alpha_E \end{array} \right\} \tag{4a}$$

with

$$\alpha_E^B = \sin^{-1}(\sqrt{(C^Z)^2 + 1} - C^Z) \approx 12.51^\circ \quad \text{for} \quad C^Z \approx 2.2 \tag{4b}$$

Thus, the Bernoulli constraint is satisfied for every value of F_T if $\alpha_E \leq \alpha_E^B$, but is only satisfied for a sufficiently high value of the Froude number F_T if $\alpha_E^B < \alpha_E$. For $F_T < F_T^B(\alpha_E)$, the Bernoulli constraint does not permit a steady-flow solution, and unsteady flow must then be expected. The Bernoulli-constraint Froude number $F_T^B(\alpha_E)$ defined by (4a) is depicted in Fig.2 for $0 \leq 2\,\alpha_E \leq 60^\circ$.

The bow-wave height $Z\,g/U^2$ is depicted in Fig. 3 for the experimental measurements reported by Ogilvie, Standing, Waniewski, and Karion for six wedge-shaped hulls; see *Noblesse et al. (2006)*. The solid lines in Fig. 3 correspond to the approximation (1) and the horizontal dashed line represents the Bernoulli upper bound (3). Fig. 3 shows that the Ogilvie, Standing, Waniewski, and Karion measurements of bow-wave height for six wedge-shaped hulls are well approximated by (1). In particular, Ogilvie's measurements for $\alpha_E = 15^\circ$ are in excellent agreement with (1), except for three data points that appear to be outliers. Two data points in Fig. 3 lie above the Bernoulli horizontal line $Z\,g/U^2 = 1/2$. These data points may be associated with unsteady effects or may stem from measurement errors.

Concluding remarks

Expression (1) is shown in *Noblesse (2006)* to provide a fairly accurate estimate of the height of a ship bow wave for ship bows that are approximately wedge-shaped (like the Wigley hull and the Series 60 model), especially if an effective-bow transformation is used to define an effective draft T and waterline entrance angle $2\,\alpha_E$. However, expression (1), and the related "steady bow-wave criterion" (4), can only be expected to provide a rough estimate for a hull form equipped with a large bulb, notably a bulb that extends ahead of the ship bow. Thus, expression (1) needs to be refined for application to bulbous ship bows.

Common observations show that the steady bow wave generated by a ship — if the "steady bow-wave" criterion (4) is satisfied — typically consists in an overturning thin sheet of water. A simple theory of overturning ship bow waves is summarized in a companion paper also submitted to IWWWFB06; see *Delhommeau et al. (2006)*.

References

Delhommeau, G., Guilbaud, M., Noblesse F. (2006) "A simple theory of overturning ship bow waves", submitted to Workshop on Water Waves and Floating Bodies, Loughborough, UK

Noblesse F., Hendrix D., Faul L., Slutsky J. (2006) "Simple analytical expressions for the height, location, and steepness of a ship bow wave", *J. Ship Research*, in press

Noblesse, F., Hendrix, D., Karafiath, G.
‘When is the bow wave of a ship in steady motion unsteady’

Discusser - M. Kashiwagi

In Fig. 3 in the abstract, values above 0.5 imply that the flow is unsteady. In these cases, the wave amplitude might be fluctuating due to unsteadiness. How much does it fluctuate? How is the repeatability in the wave measurement in these unsteady cases?

Reply:

While I did not personally observe all these measurements, I have seen similar conditions tested. It is true that the wave profile appears unsteady but the amplitude of the unsteadiness is small compared to the wave amplitude.