2-D Body-Exact Computations in the Time Domain

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Abstract

A numerical method is presented for the time-domain simulation of large amplitude motions of a 2-D surface piercing body with arbitrary shape in deep water. Based on potential theory, panels are distributed on the body surface and desingularied sources are distributed above the calm water surface. The body boundary condition is satisfied on the exact submerged surface. The free surface boundary conditions are linearized and satisfied on the calm water level. The solution is stepped forward in time by integrating the free surface kinematic and dynamic conditions. The numerical solutions for the radiation problem are compared with experimental results and other numerical results, and found to agree well. The results for the impact problem are compared with a similarity solution. The results for the diffraction problem are also presented.

1 Introduction

The accurate prediction of the wave-induced motions is very important in ship and offshore design. Severe motions and extreme loads can lead to operability problems and in extreme cases structural failure and capsize. Traditionally, the problem is formulated using potential flow theory and linearized by assuming the motions are small. Linear system theory and random process theory are then used to predict the extreme responses and loads. There are many variations to this linear approach. Strip theory developed by Salvesen, et al.[1] is probably the most popular for long slender ships.

The other extreme is to solve the Navier-Stokes equations using some type of CFD technique. At the present time, the required computational costs are too large for practical use. Fully nonlinear potential flow calculations greatly reduce the necessary computation time, but they have problems with numerical stabilities and wave breaking.

A compromise approach is to use a blended method for nonlinear seakeeping calculations. Blended methods are computationally very fast and use a blend of linear and nonlinear approaches. Typically, nonlinear equation of motions, hydrostatic and Froude-Krylov forces are used with linear predictions of the radiation and diffraction forces. Strip theory is often used to compute the radiation and diffraction forces. To improve the validity of the blended method, we have developed a body-exact technique. In this approach, the two-dimensional boundary value problem is solved using an exact body boundary condition and linearized free surface boundary conditions satisfied on the calm water level. The solution is time stepped using the known motion of the body and the linear free surface conditions. As will be shown, the advantages of the method are that it introduces the nonlinearities associated with the changing wetted surface of the body, while retaining the computational efficiency of the linearized free surface conditions.

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2 Mathematical Formulation

We consider a general two-dimensional body floating on a free surface and undergoing arbitrary three-degree-of-freedom motion. An earth-fixed Cartesian coordinate system is chosen with the **y** axis coincident with the quiescent free surface, and **z** is positive upward. The fluid is assumed to be homogeneous, incompressible, inviscid and its motion is irrotational. Surface tension is neglected and water depth is infinite. The fluid motions can be described by a velocity potential $\Phi(y, z, t)$. In the fluid domain, Φ satisfies Laplace equation

$$\nabla^2\Phi = 0 \tag{1}$$

On the mean free surface, the linearized free surface boundary condition is imposed

$$\eta_t - \Phi_z = 0 \qquad \text{on} \quad z = 0 \tag{2}$$

$$\Phi_t + g\eta = 0 \qquad \text{on} \quad z = 0 \tag{3}$$

where $z = \eta(y, t)$ is the free surface amplitude, g is the acceleration due to gravity. On the *instantaneous* body boundary, no normal flux is permitted

$$\frac{\partial\Phi}{\partial n} = V_n \quad \text{on} \quad S_b \tag{4}$$

where the unit normal vector into the body **n** is positive out of the fluid. V_n is the instantaneous velocity in the normal direction including rotational effects. In the far field, a radiation boundary condition is imposed that there are no incoming waves.

The initial conditions at $t = 0$ are

$$\Phi = \Phi_t = 0 \quad \text{in the fluid domain} \tag{5}$$

At each time step a mixed boundary value problem must be solved; the potential is given on the free surface and the normal derivative of the potential is known on the body surface. In terms of the desingularized sources above the free surface and sources distributed on the body surface, the potential at any point in the fluid domain can be given

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n} \sigma(\xi_i) \ln|\mathbf{x} - \xi_i| + \int_{S_w} \sigma(\xi) G(\mathbf{x}; \xi) dl \tag{6}$$

where S_w represent the instantaneous wetted body surface. $|\mathbf{x} - \xi_i|$ represents the distance between any point in the fluid domain and the desingularized source point. $G(\mathbf{x}; \xi)$ is a Rankine source Green function

$$G(\mathbf{x}; \xi) = \ln r \tag{7}$$

$$r = |\mathbf{x} - \xi| \tag{8}$$

where r is the distance between a source point and a collocation point; ξ is the source point on the body boundary. Applying the boundary conditions, the descritized integral equations can be solved to determine the unknown source strength.

Once the source strength is found, Φ can be evaluated by (6), and the velocity on the body $\nabla\Phi$ can been obtained. The total pressure is given by Bernoulli's equation

$$p = -\rho(\frac{\partial\Phi}{\partial t} + \frac{1}{2}|\nabla\Phi|^2 + gz) \tag{9}$$

The forces acting on the body can be obtained by integrating (9) over the instantaneous submerged body surface, which can be written as

$$\mathbf{F} = \int_{S_w} p\mathbf{n}\, dl \tag{10}$$

3 Numerical Method

The integral equation (6) is discretized in the usual manner. Desingularized sources are distributed above the calm water level, and constant strength flat panels are distributed on the body surface. A 2nd-order Adams-Bashforth scheme is used to time step the solution. At each time step, the body surface is repanelized.

4 Results

4.1 Linear Radiation Problem

Added mass and damping calculations for heave, sway and roll of a circular cylinder and

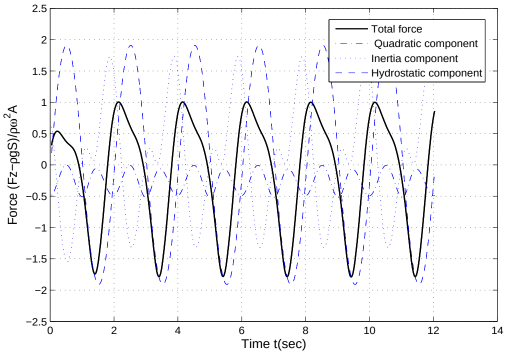

Figure 1: Vertical force acting in the circular with $a = 0.5R$, $\omega^2 g/R = 1.0$

box have been compared with Vugts'[2] experimental and other numerical results. The differences were less than 5% for all cases tested.

4.2 Body-Exact Problem

As an example, forced large amplitude motion of a circular cylinder of radius R is studied here. The cylinder is initially submerged such that the center is at the calm water line. The forced heave motion is $z(t) = -a \sin \omega t$, a is the body motion amplitude. We set $\frac{\omega R}{g} = 1.0$. The panel number is $N = 40$ on the body surface and $T/dt = 100$. As addressed earlier, at each time step, the submerged portion of the cylinder is repanelized, and the influence matrix is reevaluated.

Fig.1 shows the different components of the vertical force on the circular cylinder for the case of $a/R = 0.5$. Steady state is rapidly reached. For this case, the hydrostatic force is the largest part of the total force. The inertia term $\partial\Phi/\partial t$ shows higher harmonic components. The quadratic component $(-|\nabla\Phi|^2/2)$ is primarily a second-order harmonic.

4.3 Water Entry and Exit Problem

The impact problem of 30, 45, and 60 degree wedges was investigated. The pressures over the wedges were compared with the similarity solution presented by Zhao et al.[3]. The initial conditions were set such that wedges had negligible initial draft and a constant downward velocity of 10 meters-per-second. The pressure measurements were taken once the solution reached a relative steady state. At the same time, by extrapolating the free surface, the wave elevation at the intersection between the free surface and the body surface can be determined. Once the wetted surface was known, the pressure distribution on the body can be stretched up to that point. Fig.2 and Fig.3 show the pressure distribution and impact force acting on the 45-degree wedge. The values are compared with the similarity solution. The comparison shows that the impact pressures and forces calculated by the present method after stretching are much better that the unstretched results. While the agreement is not perfect, the results do show that this computationally fast, simplified model gives reasonable results. Fig.4 shows the force acting on a 45-degree wedge undergoing large amplitude sinusoidal motion. The body enters the water at $t = 0$sec, and reaches the bottom of the down stroke at $t = 0.5$sec. At $t = 1$sec the body exits the water and remains out of the water until it reenters at $t = 2$sec. The cycle then repeats itself. As shown, for small times, the impact force is consistent with the water entry problem. The water exit problem shows a significantly different force from the force in the entry problem. This contrasts with linear theory that says both the water entry and exit force are same.

4.4 Diffraction Problem

Fig.5 shows the forces acting on the oscillating cylinder at the same time it is impacted by incident waves of amplitude A, and wave length λ. F_y and F_z are the forces in the y and z directions respectively. The motion amplitude of the circular cylinder is $a = 0.5R$. The nondimensional wave number of the incident wave is $KR = 0.6$, and the amplitude of of the incident wave is $A = \lambda/20$. The phase angle between the oscillation of the cylinder and the incident wave is 0^o. The incident wave frequency and body oscillating frequency are same. As shown in Fig.5, the incident waves

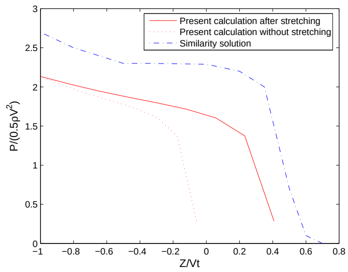

Figure 2: Pressure distribution over a 45-degree wedge

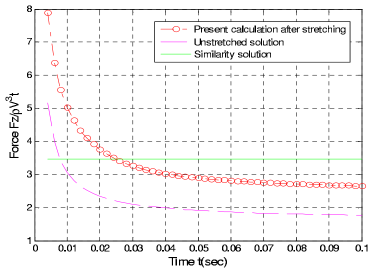

Figure 3: Impact force acting on a 45-degree wedge

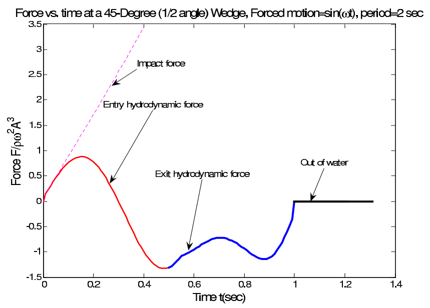

Figure 4: Force acting on a 45-degree wedge with large amplitude motion

cause a force in the y-direction(F_y), and a significant mean shift in the vertical force(F_z).

5 Conclusions

Two-dimensional, large-amplitude body motions are studied in this paper with a linearized free surface and an exact body boundary condition. Numerical results are obtained for small-amplitude motion, large-amplitude motion, water entry and exit, and wave diffraction. These results are compared with experiments and other numerical solutions. Good agreement is achieved for the large-amplitude oscillation problem of circular cylinder. For the

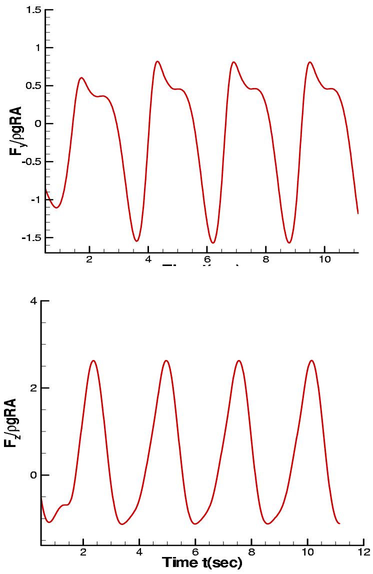

Figure 5: Forces acting on oscillating circular cylinder impacted by incident wave with $a = 0.5R$, $KR = 0.6$

wedge impact problem, the calculated values are smaller than the similarity solution. This method also works for the wave exciting problem. Investigation of the exciting forces are continuing and will be shown at the workshop.

References

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[3] Zhao, R, Faltinsen, O., Aarsnes, J., 1997, "Water entry of Arbitrary Two - Dimensional Sections with and without Flow seperation," Proceedings of 21^{st} Symposium on Naval Hydrodynamics, Santa Barbara, CA, 408-423.

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Discusser - Y. Kim:

I remember that W.M Lin (1995) showed the similar works for large amplitude motion. Can you describe the advantage of your method, compared with his?

Reply:

Lin and Yue (1990, 1995) used a time domain free surface Green function to solve the body-exact problem. In our method, we use the desingularized sources distributed over the free surface. The advantages of the desingularized sources are that we do not have to compute a complex free surface Green function and also do not have to evaluate the resulting convolution integrals at each time step. The disadvantage is that we have to worry about the far field radiation condition which is met by the free surface Green function. In addition, we have a large influence matrix to invert. Both the techniques have difficulties in dealing with the free surface-body intersection points that must be properly accounted for.