Water wave scattering by two partially immersed barriers - an alternative method of solution

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1. Introduction

The classical problems of water wave scattering by two thin vertical barriers which are either partially immersed upto the same depth or completely submerged from the same depth and extending infinitely downwards in deep water admit of explicit but complicated solutions. The problem of two partially immersed barriers was first solved explicitly by Levine and Rodemich(1985) and later approximately by Evans and Morries(1972) (hereinafter referred to as LR and EM respectively). LR obtained the explicit expressions for the reflection and transmission coefficients in terms of six definite integrals, which involve complicated functions of elliptic integrals by employing Schwartz-Christoffel transformation of complex variable theory in a somewhat complicated manner. The complementary problem of two submerged barriers was investigated by Jarvis(1971) using a similar procedure. An alternative and simple method based on solution of coupled Abel integral equations is proposed here to solve the two-barrier problem of LR. This method is based on Fourier analysis for the expansion of the velocity potential function, known as Havelock's expansion. Ultimately the reflection and transmission coefficients are obtained explicitly in terms of one modified Bessel function and two definite integrals involving Bessel function which can be computed numerically fairly easily. These integrals are definitely simpler than the integrals obtained by LR. When the two barriers are closely spaced, the expressions for R and T reduces to those for a single barrier which was obtained by Ursell(1947). Also when the two barriers are widely separated, then the expressions for R and T coincide with the results obtained by LR and EM. Choosing the same set of values of different parameters as taken by EM, $|R|$ is depicted graphically against the wave number in a number of figures and these curves almost coincide with the corresponding curves given in EM.

2. Mathematical formulation and solution

Assuming linearised theory and irrotational motion, the problem of our interest is to solve for $\phi(x, y)$ satisfying

$$\nabla^2\phi = 0, \quad -\infty < x < \infty, \quad y \geq 0, \tag{1.1}$$

$$k\phi + \phi_y = 0 \quad y = 0, \tag{1.2}$$

$$\phi_x = 0, \quad x = \pm a, \ 0 < y < h, \tag{1.3}$$

$$r^{1/2}\nabla\phi = O(1) \quad \text{as } r = \{(x \mp a)^2 + y^2\}^{1/2} \to 0, \tag{1.4}$$

$$\nabla\phi \to 0 \ \text{ as } \ y \to \infty, \tag{1.5}$$

$$\phi(x, y) \sim \begin{cases} T\phi_0(x, y) & \text{as } x \to \infty, \\ \phi_0(x, y) + R\phi_0(-x, y) & \text{as } x \to -\infty, \end{cases} \tag{1.6}$$

where $\mathrm{Re}\{\phi_0(x, y)e^{-i\sigma t}\}$ denotes the velocity potential described in the fluid region ($e^{-i\sigma t}$ being dropped throughout), T and R denote the unknown transmission and reflection coefficients, $e^{-ky+ikx}$ is the potential of the wave train incident from the direction of $x = -\infty$, $x = \pm a$, $(0 < y < h)$ denote the positions of the two barriers partially immersed upto a length h below the mean free surface $y = 0$, y-axis being taken vertically downwards into the fluid region.

We now solve the BVP described by (1.1) to (1.6) by reducing it to a pair of coupled Abel integral equations. Using Havelock's expansion of water wave potential (cf. Ursell(1947)), $\phi(x, y)$ can be expressed as $\phi(x, y)$

$$= \begin{cases} e^{-ky}(e^{ikx} + Re^{-ikx}) + \dfrac{2}{\pi}\displaystyle\int_0^\infty A(\xi)L(\xi, y)e^{\xi x}d\xi, \\ x < -a, \\ e^{-ky}(\alpha e^{ikx} + \beta e^{-ikx}) + \dfrac{2}{\pi}\displaystyle\int_0^\infty \{B(\xi)e^{\xi x} \\ +C(\xi)e^{-\xi x}\}L(\xi, y)d\xi, \ -a < x < a, \\ Te^{-ky+ikx} + \dfrac{2}{\pi}\displaystyle\int_0^\infty D(\xi)L(\xi, y)e^{-\xi x}d\xi, \ x > a, \end{cases} \tag{2.1}$$

where $L(\xi, y) = \xi\cos\xi y - k\sin\xi y$, α,β are unknown constants, $A(\xi), B(\xi), C(\xi)$ and $D(\xi)$ are unknown functions such that the integrals in (2.1) and in the mathematical analysis below in which they appear, are convergent.

Using the condition of continuity of ϕ_x across the lines $x = \pm a$, $y > 0$, we obtain two relations involving the unknown functions $A(\xi), B(\xi), C(\xi), D(\xi)$ under integral sign and the four unknown constants α, β, R, T. Use of Havelock's inversion theorem (cf. Ursell(1947)) to each of these two relations gives rise to four equations, given by

$$e^{-ika} - Re^{ika} = \alpha e^{-ika} - \beta e^{ika}, Te^{ika} = \alpha e^{ika} - \beta e^{-ika}, \tag{2.3}$$

$$A(\xi) = B(\xi) - C(\xi)e^{2\xi a}, D(\xi) = C(\xi) - B(\xi)e^{2\xi a}. \tag{2.4}$$

Again, continuity of ϕ across the gap $x = \mp a$, $y > h$ (using (2.4)) produces two integral equations in $B(\xi)$ and $C(\xi)$ for $y > h$ and using the condition (1.3), we obtain another two integral equations in $B(\xi)$ and $C(\xi)$ for $0 < y < h$. These can be farther reduced by simple integration to

$$\int_0^\infty C(\xi)e^{\xi a}\sin\xi y d\xi = \frac{\pi}{4k}(\beta - R)e^{-ky+ika}, \quad y>h, \tag{2.5}$$

$$\int_0^\infty B(\xi)e^{\xi a}\sin\xi y d\xi = \frac{\pi}{4k}\beta e^{-ky-ika}, \quad y>h, \tag{2.6}$$

$$\int_0^\infty \xi\left\{B(\xi)e^{-\xi a} - C(\xi)e^{\xi a}\right\}\sin\xi y d\xi = -\frac{i\pi}{2}\left(\alpha e^{-ika} - \beta e^{ika}\right)\sinh ky, \quad 0<y<h, \tag{2.7}$$

$$\int_0^\infty \xi\left\{B(\xi)e^{\xi a} - C(\xi)e^{-\xi a}\right\}\sin\xi y d\xi = -\frac{i\pi}{2}\left(\alpha e^{ika} - \beta e^{-ika}\right)e^{-ky}, \quad 0<y<h. \tag{2.8}$$

Denoting left sides of (2.8) and (2.7) by the unknown functions $g_1(y)$ and $g_2(y)$ respectively for $y>h$ and using the sine inversion formula, $B(\xi)$ and $C(\xi)$ are obtained in terms of integrals involving $g_1(y)$ and $g_2(y)$. Substituting these in (2.5) and (2.6), we obtain two coupled integral equations for $g_1(t)$ and $g_2(t)$ as given by for

$$\frac{1}{\pi}\int_h^\infty g_1(t)\mathcal{K}_1(t,y)dt - \frac{1}{\pi}\int_h^\infty g_2(t)\mathcal{K}_2(t,y)dt = f_1(y), \quad y>h, \tag{2.9}$$

$$\frac{1}{\pi}\int_h^\infty g_1(t)\mathcal{K}_2(t,y)dt - \frac{1}{\pi}\int_h^\infty g_2(t)\mathcal{K}_1(t,y)dt = f_2(y), \quad y>h, \tag{2.10}$$

where

$$(\mathcal{K}_1,\mathcal{K}_2)(t,y) = \int_0^\infty \frac{\sin\xi t\sin\xi y}{\xi\sinh 2\xi a}\left(e^{2\xi a},1\right)d\xi,$$

$$\begin{aligned}(f_1(y),f_2(y)) =& \frac{\pi}{4k}e^{-ky}\left(\beta e^{-ika},(\beta-R)e^{ika}\right)\\ &+\frac{i}{2}\left(\alpha e^{ika}-\beta e^{-ika}\right)\int_0^h \sinh kt\left(\mathcal{K}_1,\mathcal{K}_2\right)(t,y)dt\\ &-\frac{i}{2}\left(\alpha e^{-ika}-\beta e^{ika}\right)\int_0^h \sinh kt\left(\mathcal{K}_2,\mathcal{K}_1\right)(t,y)dt.\end{aligned} \tag{2.11}$$

Addition and substraction of (2.9) and (2.10) produces the decoupled equations

$$\frac{1}{\pi}\int_h^\infty g(t)\mathcal{K}(t,y)dt = f(y), \quad y>h, \tag{2.12}$$

$$\frac{1}{\pi}\int_h^\infty G(t)\mathcal{L}(t,y)dt = F(y), \quad y>h, \tag{2.13}$$

where

$$(g,G)(t) = g_1(t)\mp g_2(t), (f(y),F(y)) = f_1(y)\pm f_2(y)$$

$$(\mathcal{K},\mathcal{L})(t,y) = \mathcal{K}_1(t,y)\pm\mathcal{K}_2(t,y). \tag{2.14}$$

The kernels $\mathcal{K}(t,y)$ and $\mathcal{L}(t,y)$ are given by

$$\mathcal{K}(t,y) = \frac{1}{2}\ln\left|\frac{y+t}{y-t}\right| + \frac{1}{2}\ln\left|\frac{\coth\frac{\pi y}{2a}+\coth\frac{\pi t}{2a}}{\coth\frac{\pi y}{2a}-\coth\frac{\pi t}{2a}}\right|,$$

$$\mathcal{L}(t,y) = \frac{1}{2}\ln\left|\frac{y+t}{y-t}\right| + \frac{1}{2}\ln\left|\frac{\mathrm{cosech}\frac{\pi y}{2a}+\mathrm{cosech}\frac{\pi t}{2a}}{\mathrm{cosech}\frac{\pi y}{2a}-\mathrm{cosech}\frac{\pi t}{2a}}\right|. \tag{2.15}$$

Now substituting $t=1/t_1$ and $y=1/y_1$ in (2.12) and (2.13) we get for $0<y_1<h_1(=1/h)$,

$$\frac{1}{\pi}\int_0^{h_1} g_1(t_1)\mathcal{K}_1(t_1,y_1)dt_1 = f_1(y_1), \tag{2.16}$$

$$\frac{1}{\pi}\int_0^{h_1} G_1(t_1)\mathcal{L}_1(t_1,y_1)dt_1 = F_1(y_1), \tag{2.17}$$

where $g_1(t_1)=g(1/t_1)/t_1^2, \mathcal{K}_1(t_1,y_1)=\mathcal{K}(1/t_1,1/y_1), f_1(y_1)=f(1/y_1)$ etc. Using the result that

$$\frac{1}{2}\ln\left|\frac{\psi(y)+\psi(t)}{\psi(y)-\psi(t)}\right| = \int_0^{\min(\psi(y),\psi(t))}\frac{\psi'(\eta)\psi(\eta)}{\left[\{\psi^2(y)-\psi^2(\eta)\}\{\psi^2(t)-\psi^2(\eta)\}\right]^{1/2}}d\eta,$$

where $\psi(y)$ is an increasing function, the integral equation (2.16) reduces to the coupled Abel type integral equation

$$\int_0^{y_1}A_1(y_1,\eta)p(\eta)d\eta + \int_0^{y_1}B_1(y_1,\eta)q(\eta)d\eta = f_1(y_1), \quad 0<y_1<h_1, \tag{2.18}$$

in $p(\eta),q(\eta)$ where

$$p(\eta) = \int_\eta^{h_1}\frac{g_1(t_1)}{(t_1^2-\eta^2)^{1/2}}dt_1,$$

$$\eta^2 q(\eta) = \int_\eta^{h_1}\frac{g_1(t_1)}{\left(\coth^2\frac{\pi}{2at_1}-\coth^2\frac{\pi}{2a\eta}\right)^{1/2}}dt_1,$$

so that $p(h_1)=0, \quad q(h_1)=0, \quad f_1(0)=0$. Elimination of $g_1(t_1)$ from the expressions of $p(\eta)$ and $q(\eta)$ produces

$$\int_{y_1}^{h_1}A_2(y_1,\eta)p(\eta)d\eta - \int_{y_1}^{h_1}B_2(y_1,\eta)q(\eta)d\eta = 0, \quad 0<y_1<h_1. \tag{2.19}$$

In (2.18) and (2.19)

$$A_1(y_1,\eta)=\eta(y_1^2\text{-}\eta^2)^{-\frac{1}{2}}, A_2(y_1,\eta)=\eta(\eta^2\text{-}y_1^2)^{-1/2},$$

$$B_1(y_1,\eta) = \frac{\pi\coth\frac{\pi}{2a\eta}}{2a\sinh^2\frac{\pi}{2a\eta}}\left(\coth^2\frac{\pi}{2at_1}-\coth^2\frac{\pi}{2a\eta}\right)^{-\frac{1}{2}},$$

$$B_2(y_1,\eta) = \frac{\pi\coth\frac{\pi}{2a\eta}}{2a\sinh^2\frac{\pi}{2a\eta}}\left(\coth^2\frac{\pi}{2a\eta}-\coth^2\frac{\pi}{2at_1}\right)^{-\frac{1}{2}}.$$

Similarly, the integral equation (2.17) is equivalent to the coupled Abel integral equations

$$\int_0^{y_1}C_1(y_1,\eta)P(\eta)d\eta + \int_0^{y_1}D_1(y_1,\eta)Q(\eta)d\eta = F_1(y_1), \quad 0<y_1<h_1, \tag{2.20}$$

$$\int_{y_1}^{h_1}C_2(y_1,\eta)P(\eta)d\eta - \int_{y_1}^{h_1}D_2(y_1,\eta)Q(\eta)d\eta = 0, \quad 0<y_1<h_1, \tag{2.21}$$

where $C_1(y_1,\eta)=\eta(y_1^2\text{-}\eta^2)^{-\frac{1}{2}}, C_2(y_1,\eta)=\eta(\eta^2\text{-}y_1^2)^{-1/2}$,

$$D_1(y_1,\eta)=\frac{-\pi\coth\frac{\pi}{2a\eta}}{2a\sinh^2\frac{\pi}{2a\eta}}\left(\mathrm{cosech}^2\frac{\pi}{2at_1}-\mathrm{cosech}^2\frac{\pi}{2a\eta}\right)^{-\frac{1}{2}},$$

$$D_2(y_1,\eta)=\frac{-\pi\coth\frac{\pi}{2a\eta}}{2a\sinh^2\frac{\pi}{2a\eta}}\left(\mathrm{cosech}^2\frac{\pi}{2a\eta}-\mathrm{cosech}^2\frac{\pi}{2at_1}\right)^{-\frac{1}{2}},$$

$$P(\eta) = \int_\eta^{h_1}\frac{G_1(t_1)}{(t_1^2-\eta^2)^{1/2}}dt_1,$$

$$\eta^2 Q(\eta) = \int_\eta^{h_1}\frac{G_1(t_1)}{\left(\mathrm{cosech}^2\frac{\pi}{2at_1}-\mathrm{csech}^2\frac{\pi}{2a\eta}\right)^{1/2}}dt_1.$$

3. Solution of coupled Abel integral equation

The coupled Abel integral equations(2.18) and (2.19) are rewritten as

$$\int_0^{x_1} A_1(x_1,\eta)p(\eta)d\eta + \int_0^{x_1} B_1(x_1,\eta)q(\eta)d\eta = f_1(x_1), \quad 0 < x_1 < h_1, \tag{3.1}$$

$$\int_{x_1}^{h_1} A_2(x_1,\eta)p(\eta)d\eta - \int_{x_1}^{h_1} B_2(x_1,\eta)q(\eta)d\eta = 0, \quad 0 < x_1 < h_1, \tag{3.2}$$

We introduce a new complex unit $j(=(-1)^{1/2})$ different from the complex unit i used earlier, to define the complex variable $z = x_1 + jy_1$, and let

$$\Psi_1(z) = \int_0^{h_1} A_2(z,\eta)p(\eta)d\eta,$$

$$\Psi_2(z) = \frac{\pi}{2a}\int_0^{h_1} B_2(z,\eta)q(\eta)d\eta.$$

Then $\Psi_1(z)$, $\Psi_2(z)$ are analytic in the complex z-plane cut along the real axis from $-h_1$ to h_1. Let $\Psi_j^{\pm}(x) = \lim_{y\to\pm 0}\Psi_j(z)$ $(j = 1,2)$. We find for $0 < x_1 < h_1$,

$$\Psi_1^{\pm}(x) = \pm j\int_0^{x_1} A_1(x_1,\eta)p(\eta)d\eta + \int_{x_1}^{h_1} A_2(x_1,\eta)p(\eta)d\eta,$$

$$\Psi_2^{\pm}(x) = \pm j\int_0^{x_1} B_1(x_1,\eta)q(\eta)d\eta + \int_{x_1}^{h_1} B_2(x_1,\eta)q(\eta)d\eta,$$

and for $-h_1 < x_1 < 0$

$$\Psi_1^{\pm}(x) = \mp j\int_0^{-x_1} A_1(x_1,\eta)p(\eta)d\eta + \int_{-x_1}^{h_1} A_2(x_1,\eta)p(\eta)d\eta,$$

$$\Psi_2^{\pm}(x) = \mp j\int_0^{-x_1} B_1(x_1,\eta)q(\eta)d\eta + \int_{-x_1}^{h_1} B_2(x_1,\eta)q(\eta)d\eta.$$

Thus for $-h_1 < x_1 < 0$,

$$\Psi_l^{\pm}(x_1) = \overline{\Psi_l^{\mp}(-x_1)}, \quad l = 1,2 \tag{3.3}$$

where the bar denotes the complex conjugate.Thus for $0 < x_1 < h_1$,

$$\int_0^{x_1} A_1(x_1,\eta)p(\eta)d\eta = \frac{\Psi_1^{+}(x_1) - \Psi_1^{-}(x_1)}{2j},$$

$$\int_{x_1}^{h_1} A_2(x_1,\eta)p(\eta)d\eta = \frac{\Psi_1^{+}(x_1) + \Psi_1^{-}(x_1)}{2}, \tag{3.4}$$

and

$$\int_0^{x_1} B_1(x_1,\eta)q(\eta)d\eta = \frac{\Psi_2^{+}(x_1) - \Psi_2^{-}(x_1)}{2j},$$

$$\int_{x_1}^{h_1} B_2(x_1,\eta)q(\eta)d\eta = \frac{\Psi_2^{+}(x_1) + \Psi_2^{-}(x_1)}{2}. \tag{3.5}$$

Using these relations in (3.1) and (3.2), we obtain two relations involving $\Psi_1^{\pm}(x_1)$ and $\Psi_2^{\pm}(x_1)$ for $0 < x_1 < h_1$. These relations can be extended to $-h_1 < x_1 < 0$ by using (3.3) and the fact that $f_1(x_1)$ is real with respect to j. Thus for $-h_1 < x_1 < h_1$

$$(\Psi_1^{+}(x_1) + \Psi_2^{+}(x_1)) - (\Psi_1^{-}(x_1) + \Psi_2^{-}(x_1)) = 2jl(x_1), \tag{3.6}$$

$$(\Psi_1^{+}(x_1) - \Psi_2^{+}(x_1)) + (\Psi_1^{-}(x_1) - \Psi_2^{-}(x_1)) = 0. \tag{3.7}$$

where $l(x_1) = f_1(x_1)$ for $0 < x_1 < h_1$ and $-f_1(-x_1)$ for $-h_1 < x_1 < 0$. Thus (3.6)and (3.7) define two independent Riemann Hilbert problems for the functions $\Psi_1(z) + \Psi_2(z)$ and $\Psi_1(z) - \Psi_2(z)$ analytic in the complex z-plane cut along the real line from $-h_1$ to h_1. Noting that $\Psi_1(z) = O\left(z^{-1}\right)$, $\Psi_2(z) = O\left(z^{-1}\right)$, as $z \to \infty$ we obtain the solution of (3.6) and (3.7) as

$$\Psi_1(z) + \Psi_2(z) = \frac{1}{\pi}\int_{-h_1}^{h_1} \frac{l(\eta)}{\eta - z}d\eta, \tag{3.8}$$

$$\Psi_1(z) - \Psi_2(z) = \frac{C_0}{\left(h_1^2 - z^2\right)^{1/2}}. \tag{3.9}$$

where C_0 is an arbitrary constant.Thus for $0 < x_1 < h_1$,

$$2\Psi_1^{\pm}(x_1) = \pm jf(x_1) + \frac{1}{\pi}\int_0^{h_1}\frac{f(\eta)}{\eta - x_1}d\eta + \frac{1}{\pi}\int_0^{h_1}\frac{f(\eta)}{\eta + x_1}d\eta \pm \frac{C_0}{\left(h_1^2 - x_1^2\right)^{1/2}},$$

$$2\Psi_2^{\pm}(x_1) = \pm jf(x_1) + \frac{1}{\pi}\int_0^{h_1}\frac{f(\eta)}{\eta - x_1}d\eta + \frac{1}{\pi}\int_0^{h_1}\frac{f(\eta)}{\eta + x_1}d\eta \mp \frac{C_0}{\left(h^2 - x^2\right)^{1/2}}.$$

Using these expressions in (3.6) we obtain for $0 < x_1 < h_1$,

$$\int_0^{x_1}\frac{\eta p(\eta)}{\left(x_1^2 - \eta^2\right)^{1/2}}d\eta = \frac{1}{2}f_1(x_1) - j\frac{C_0}{\left(h_1^2 - x_1^2\right)^{1/2}}. \tag{3.10}$$

Since $f_1(0) = 0$, for the consistency we must have $C_0 \equiv 0$. Thus $\Psi_1(z) = \Psi_2(z) = \frac{1}{2\pi}\int_{-h_1}^{h_1}\frac{l(\eta)}{\eta - z}d\eta$. Now (3.10) becomes

$$\int_0^{x_1}\frac{\eta p(\eta)}{\left(x_1^2 - \eta^2\right)^{1/2}}d\eta = \frac{1}{2}f_1(x_1), \ 0 < x_1 < h_1. \tag{3.11}$$

By Abel inversion, (3.11) produces

$$p(\eta) = \frac{1}{2\pi}\int_0^{\eta}\frac{f_1'(x_1)}{\left(\eta^2 - x_1^2\right)^{1/2}}dx_1, \quad 0 < \eta < h_1. \tag{3.12}$$

Since $p(h_1) = 0$ and substituting $x_1 = 1/x$, we get

$$\int_h^{\infty}\frac{xf'(x)}{\left(x^2 - h^2\right)^{1/2}}dx = 0. \tag{3.13}$$

It may be noted that the expression for $q(\eta)$ can be obtained from (2.19) by Abel inversion.

A similar procedure is adopted to solve the coupled Abel integral equations (2.20) and (2.21). This will ultimately produce

$$\int_h^{\infty}\frac{xF'(x)}{\left(x^2 - h^2\right)^{1/2}}dx = 0. \tag{3.14}$$

The equations (3.13), (3.14) together with (2.3) will produce the four unknown constants α, β, R, T explicitly.

4. Reflection and Transmission coefficients

Substituting the appropriate expressions for $f(x)$ and $F(x)$ in (3.13) and (3.14), carrying out the necessary integrations, and substituting for α, β in terms of R, T from (2.3), we ultimately obtain two equations involving R and T. These give R and T as

$$(R,T) = \frac{1}{2}\left[\frac{i\pi K_1(kh) + 4ie^{-ika}U\sin ka}{i\pi K_1(kh) + 4ie^{ika}U\sin ka} \pm \frac{4ie^{-ika}V\cos ka - \pi K_1(kh)}{4ie^{ika}V\cos ka + \pi K_1(kh)}\right] \tag{4.1}$$

where $(U, V) =$

$$\frac{\pi}{2}\int_0^h\int_0^\infty \frac{\sinh kt \sin\xi t\cos\xi y}{\sinh 2\xi a}\left(e^{2\xi a}\mp 1\right)J_1(\xi h)d\xi dt$$

and $K_1(x)$ is the modified Bessel function, $J_1(x)$ is the Bessel function. It is verified that R and T satisfies the energy identity $|R|^2+|T|^2=1$. Also α, β can be found from equation (2.3) using these expressions for R and T. Using (2.4) we obtain $A(\xi)$, $B(\xi)$, $C(\xi)$ and $D(\xi)$, and then we can be obtain $\phi(x,y)$ explicitly.

5. Discussion

5.1 *Approximation of R, T for small separation length:*

As $a\to 0$, $U\sin ka\to U_0$, $\quad V\to -\frac{\pi^2}{4}I_1(kh)$ where

$$U_0 = -\frac{\pi k}{2}\int_0^h \sinh kt\left\{\int_0^\infty \frac{\sin\xi t J_1(\xi h)}{\xi}d\xi\right\}dt.$$

Using these results in (4.1), we find as $a\to 0$,

$$R\to R_0,\quad T\to T_0,$$

where

$$R_0=\frac{\pi I_1(kh)}{\pi I_1(kh)+iK_1(kh)}, T_0=\frac{iK_1(kh)}{\pi I_1(kh)+iK_1(kh)}.$$

These results coincide with results of Ursell(1947).

5.2 *Approximation of R, T for large separation length:*

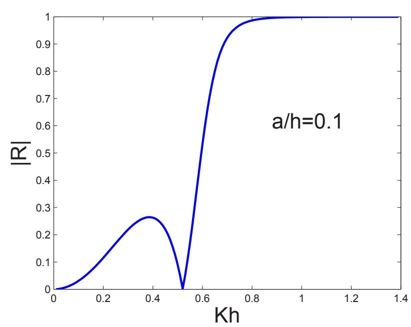
We introduce two dimensionless parameters $\lambda = a/h$ and $\mu = kh$. For large separation length of the two barriers *i.e.*, as $\mu\to\infty$, we get, $U, V\approx -\frac{\pi^2}{2}I_1(\mu)$. Using these,as $\mu\to\infty$, we find $R\approx e^{-2i\lambda\mu}$ $+\frac{2\pi\sin 2\lambda\mu K_1(\mu)I_1(\mu)-K_1^2(\mu)e^{-2i\lambda\mu}}{\{K_1(\mu)-2\pi\sin\lambda\mu e^{i\lambda\mu}I_1(\mu)\}\{K_1(\mu)-2\pi i\cos\lambda\mu e^{i\lambda\mu}I_1(\mu)\}}$ and

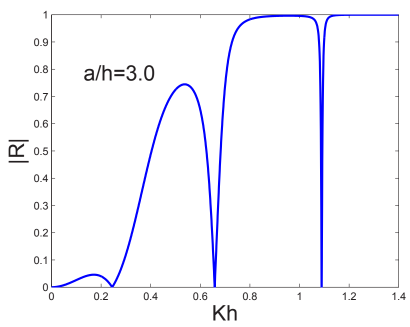
$$T\approx\frac{K_1^2(\mu)}{\{K_1(\mu)-2\pi\sin\lambda\mu e^{i\lambda\mu}I_1(\mu)\}\{K_1(\mu)-2\pi i\cos\lambda\mu e^{i\lambda\mu}I_1(\mu)\}}.$$

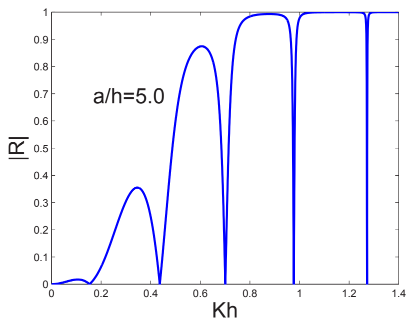
These results agree with LR and EM for large separation between the two barriers.

5.3 *Numerical Results:*

$|R|$ obtained in (4.1) is depicted graphically against kh for same values of a/h, used by EM. It observed that all the figures almost coincide with the corresponding curves given in EM.







6. Conclusion

An alternative but simple method has been employed here to obtain the explicit solution of the problem of water wave scattering by two partially immersed barriers. The method ultimately reduces to solving two pairs of coupled Abel type integral equations. The reflection coefficients, $|R|$ is depicted graphically for different values of a/h. The curves for $|R|$ almost coincide with the curves given by EM.

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