

## A Test of Linearity in the Generation of Ship Waves

Lawrence J. Doctors

The University of New South Wales, Sydney, NSW 2052, Australia

Email: L.Doctors@UNSW.edu.au

### Summary

An elaborate set of towing-tank experiments on the wave generation of three geosim catamaran models was conducted in order to test the hypothesis of linearity in the wave theory. It was found that for the different beams of the ship models, the wave pattern was very accurately predicted by the theory, except in the region very close to the critical depth Froude number.

### 1 Introduction

The matter of wave generation of vessels is of great importance when considering the operation in rivers, because of the concern of damage to the shores. There is now a considerable body of literature on this subject that has been produced over the last 15 years. An early example was the work of Doctors, Renilson, Parker, and Hornsby (1991). A total of ten candidate vessels, both catamarans and trimarans, was studied for operation on the Parramatta River, leading into Sydney Harbor in Australia. It was demonstrated experimentally that increasing the spacing between the two demihulls of a catamaran would reduce the height of the generated waves. This outcome correlated well with theoretical predictions of the wave resistance, using the traditional thin-ship theory based on Michell (1898) and modified for the case of a restricted waterway (such as a canal) by Sretensky (1936).

Doctors (2003) carried out an experimental investigation on a model catamaran, in which longitudinal wave profiles were measured and compared with the predictions of inviscid theory. Generally, excellent correlation between the experiments and the predictions for the wave profiles was achieved. Similarly, comparisons of the root-mean-square wave elevation were equally promising. The influence of viscosity was also included, in an approximate manner, following the approach of Tuck,

**Table 1: 80%-Beam Series 64\* Demihull**

Item	Symbol	Value
Displacement mass	$\Delta$	3.745 kg
Waterline length	$L$	1.500 m
Waterline beam	$B_1$	0.1113 m
Draft	$T$	0.04623 m
WP-area coef.	$C_{WP}$	0.7866
Max. section coef.	$C_M$	0.7292
Block coefficient	$C_B$	0.4860
Prismatic coefficient	$C_P$	0.6665
Slenderness coef.	$L/\nabla^{1/3}$	9.654

Scullen, and Lazauskas (2000).

This work was later extended by Doctors and Zilman (2004), who also incorporated the effect of surface tension and elasticity of surfactants at the free surface. It was shown that the presence of these additional physical properties, while relatively small, had a measurable effects on small ship models towed at the lower end of the Froude-number range.

### 2 Linearity in Wave-Generation Theory

The traditional theories referred to above are linear in that the wave elevation at a specific location in the wave field is predicted to be proportional to the beam of the vessel —

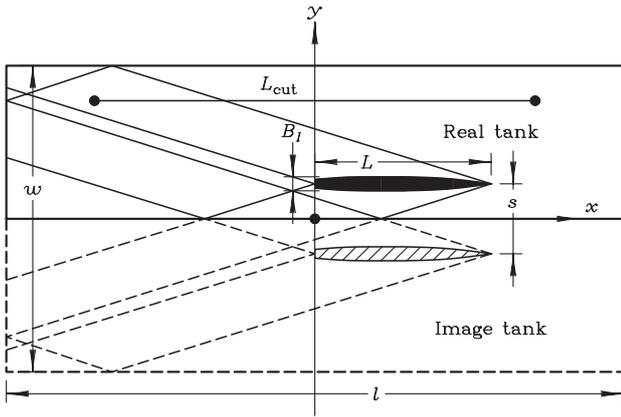


Figure 1: Definition of the Problem  
(a) Experimental Setup

on the assumption that the considered vessels are all geosims (affine transformations) of each other. With this in mind, it was thought that it would be an instructive exercise to directly test this hypothesis by measuring the wave pattern generated by three similar models, which differed only in their beams.

### 3 Experiments in Towing Tank

The two parts of Figure 1 provide a schematic of the towing tank setup as well as a pictorial view of the thinnest model, referred to as the 80%-beam model. This model possesses a transom stern and it is a suitable candidate for a high-speed vessel. The model was tested at three offsets from the side wall of the towing tank. Thus, the experiment simulated tests on a catamaran in a tank of twice the actual width. The geometric data pertaining to this vessel is presented in Table 1.

In addition, a 100%-beam version and a 120%-beam version of the vessel were tested. The test conditions and experimental matrix were as follows: effective demihull spacing:  $s = 0.300(0.150)0.600$  m, lateral offsets of wave probes:  $y = 1.000(0.5000)3.000$  m, effective tank width:  $w = 7.100$  m, water depth:  $d = 0.750$  m and  $1.500$  m, and Froude number:  $F = 0.2$  to  $1.0$ .

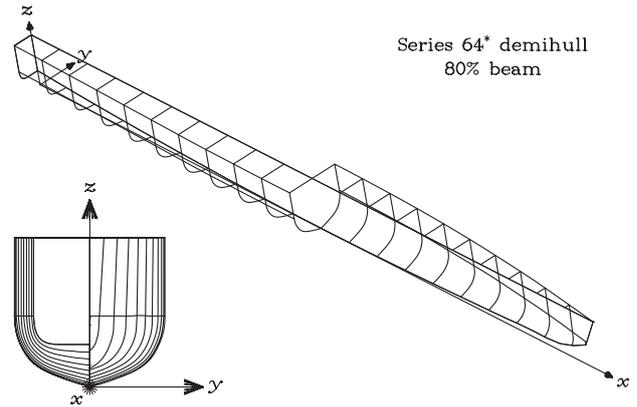


Figure 1: Definition of the Problem  
(b) Demihull with 80% Beam

### 4 Wave-Elevation Curves

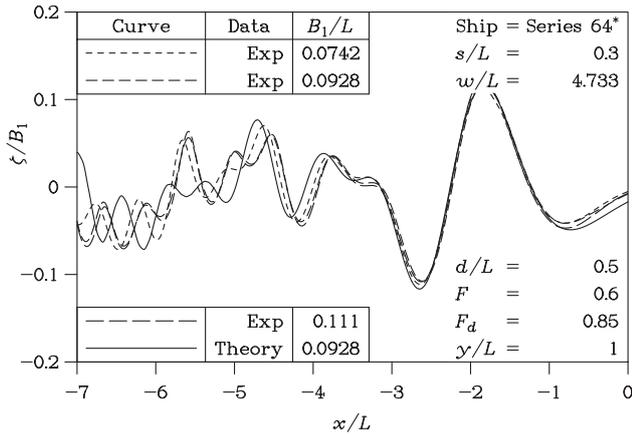
Figure 2(a) and Figure 2(b) are plots of the wave elevation on longitudinal cuts at two different lateral offsets, respectively. The data corresponds to the intermediate demihull spacing ( $s/L = 0.3$ ), the smaller depth ( $d/L = 0.5$ ) and a Froude number  $F$  of  $0.6$ .

The wave elevation  $\zeta$  is rendered dimensionless against the demihull beam  $B_1$ , so that according to linear theory, the three experimental curves corresponding to the three different demihull beams should collapse together. This is seen to be very nearly the case, particularly for the greater offset of  $y/L = 2$  in Figure 2(b), where the nonlinear effects are likely to be less. It is difficult to state whether the agreement with the theory from Doctors and Zilman (2004) is better when the demihull beam is smaller, as one would anticipate.

In the theoretical calculations, the following water properties were used: turbulent kinematic viscosity:  $\nu = 2 \times 10^{-5}$  m<sup>2</sup>/s, surface tension:  $\tau = 0.0735$  N/m, and surface elasticity:  $\epsilon = 0.0050$  N/m.

### 5 Root-Mean-Square Wave Elevation

As a measure of the overall wave-generation

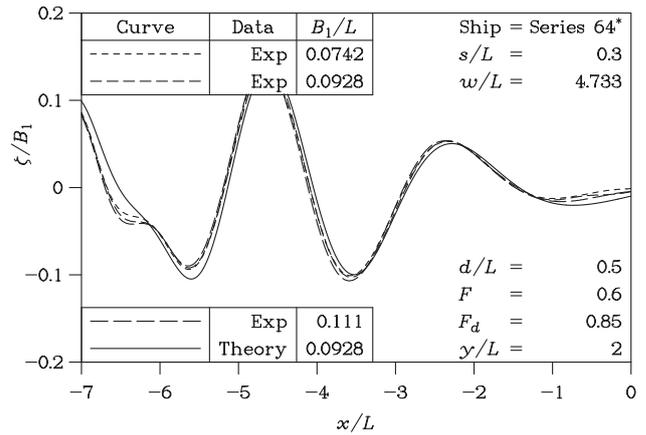


**Figure 2: Similarity of Wave Profiles**  
(a)  $d/L = 0.5$ ,  $F = 0.6$  and  $y/L = 1$

characteristics of the catamarans, we now turn to Figure 3. This is a pair of plots of the nondimensional root-mean-square wave elevation  $\zeta_{\text{RMS}}/L$  against the Froude number  $F$  for the smallest demihull spacing  $s/L = 0.2$ . The shallower case of  $d/L = 0.5$  in Figure 3(a) shows the excellent agreement between the theory and the experiment for all three model demibeams. The linear theory, of course, breaks down near the critical depth Froude number  $F_d = 1$ . In the same conditions, it is unlikely that the experimental data is any more reliable, because of the great difficulty in achieving a steady-state condition.

The deeper case of  $d/L = 1$  is shown in Figure 3(b). In this case, the critical depth Froude number occurs at the right-hand side of the graph.

Finally, we replot this data in a normalized form, namely  $\zeta_{\text{RMS}}/B_1$ , in the two parts of Figure 4. It is encouraging to observe the very high degree of collapse of the data onto a single curve — confirming the essential linearity. It may be observed also that, for the high-speed end of the range, there is better agreement between the theory and the data for the smallest demibeam, as one would expect. Also, the theoretical curves do not collapse perfectly (specifically at the low-speed end of the range), because the transom-stern



**Figure 2: Similarity of Wave Profiles**  
(b)  $d/L = 0.5$ ,  $F = 0.6$  and  $y/L = 2$

hollow model introduces a minor geometric nonlinear feature into the calculations.

## 6 Conclusions

The extensive experiments have confirmed that linear wave theory can be applied to the case of practical marine vessels, even those with a relatively high beam-to-length ratio.

An interesting future extension of this work could involve further increasing the beam to test the ultimate useful limit of linear theory.

## 7 Acknowledgments

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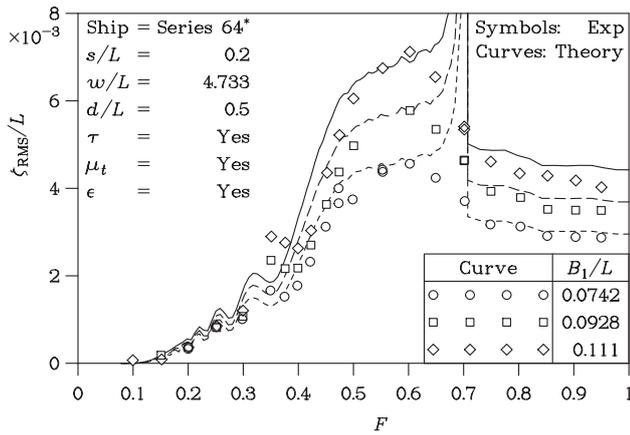


Figure 3: Dimensionless RMS Wave Elevation (a)  $d/L = 0.5$  and  $s/L = 0.2$

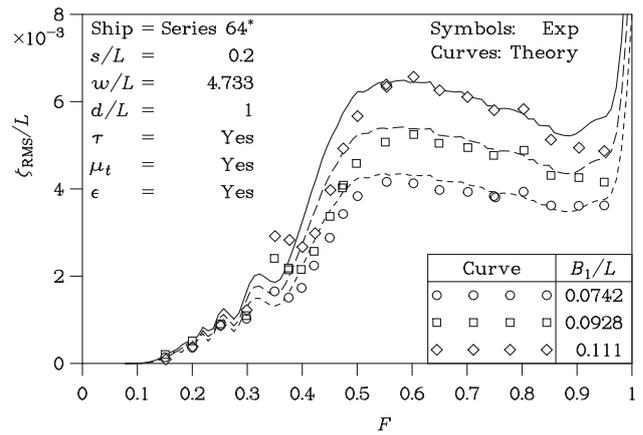


Figure 3: Dimensionless RMS Wave Elevation (b)  $d/L = 1.0$  and  $s/L = 0.2$

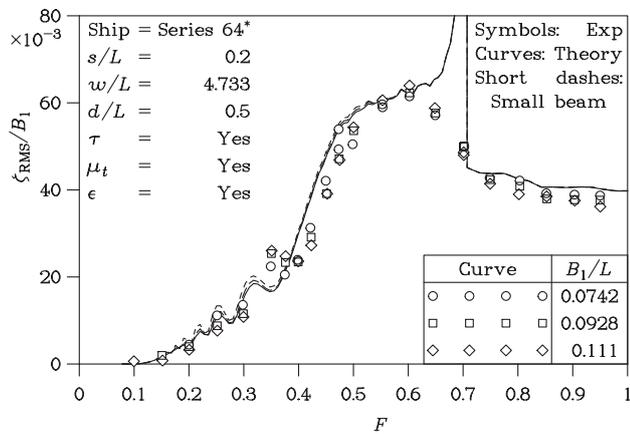


Figure 4: Normalized RMS Wave Elevation (a)  $d/L = 0.5$  and  $s/L = 0.2$

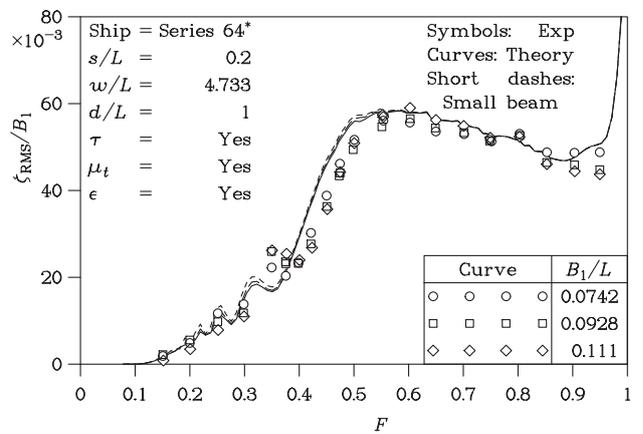


Figure 4: Normalized RMS Wave Elevation (b)  $d/L = 1.0$  and  $s/L = 0.2$

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