Integration of the Time-Domain Green Function

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1. Introduction

The analysis of wave loads and marine structure motions by time-domain free surface Green's function methods has attracted many investigations during past two decades. The method is easily extended to evaluate body surface hydrodynamic nonlinearlities and arbitrary motions of marine vehicles especially for forward speed problems. Nevertheless, many surface ships are non-wall sided, like ships with flared bow or with counter sterns. For some designed ships, the difficulty that time-stepping calculations give divergent results will be encountered. The urge to realize the origin advantage of transient Green function method motivate the present research. A stable numerical scheme is proposed.

2. TIME-DOMAIN FORMULATION

Consider an arbitrary three-dimensional body floating on the free surface of an ideal fluid. Let o-xyz be a right-handed coordinate system with the x-y plane coincident with the calm-water level, and z-axis is positive upwards. For simplicity, only the radiation problem will be considered in this paper. Thus, starting from an initial state of rest at t=0, the body is forced to moving and/or oscillate with a prescribed velocity. Assuming incompressible, irrotational flow and linearized free surface boundary condition, and the body boundary condition satisfied on the instantaneous wetted surface. This is the so called body nonlinear formulation.

Then, as shown in Lin(1990) or Dai(1998), applying Green's theorem and the transient free-surface Green function, the velocity potential $\Phi(p\ ,\ t)$ is solved by the source distribution integral equation

$$\begin{aligned}\Phi(p,t) = \frac{1}{4\pi}\Bigg\{ & \iint_{B(t)} \sigma(q,t)\frac{\partial}{\partial n_q}\left(\frac{1}{r_{pq}} - \frac{1}{r_{p\bar{q}}}\right) ds_q + \int_0^t d\tau \Bigg\{ \iint_{B(\tau)} \sigma(q,\tau)\tilde{G}(p,t,q,\tau) ds_q \\ & - \frac{1}{g}\int_{\Gamma(t)} \sigma(q,\tau)\tilde{G}(p,t,q,\tau) V_N V_n dl_q \Bigg\}\Bigg\} \qquad p \in V(t)\end{aligned} \tag{1}$$

With V_n denotes the normal velocity to the body surface $B(t)$ and g represents acceleration of gravity. $\Gamma(\tau)$ the instantaneous waterline and V_N the normal velocity of $\Gamma(\tau)$.

where

$$\tilde{G}(p,t,q,\tau) = 2\int_0^\infty \sqrt{gk}\, e^{k(z+s)} J_0(kR) \sin\left[\sqrt{gk}(t-\tau)\right] dk\ ,\ z \le 0\ ,\ \zeta \le 0 \tag{2}$$

is the memory part of the time-domain Green function, with p=(x,y,z), q= (ξ,η,ζ) the field and source points,

$$R^2 = (x-\xi)^2 + (y-\eta)^2$$

Using the body surface condition to equation (1), the integral equation to determine the source strength $\sigma(q,t)$ is then given by

$$V_n(p,t)=\frac{1}{2}\sigma(p,t)+\frac{1}{4\pi}\left\{\iint_{B(t)}\sigma(q,t)\frac{\partial}{\partial n_q}\left(\frac{1}{r_{pq}}-\frac{1}{r_{p\bar{q}}}\right)ds_q\right.$$
$$\left.+\int_0^t d\tau\left\{\iint_{B(\tau)}\sigma(q,\tau)\frac{\partial\widetilde{G}(p,t,q,\tau)}{\partial n_p}ds_q-\frac{1}{g}\int_{\Gamma(\tau)}\sigma(q,\tau)\frac{\partial\widetilde{G}(p,t,q,\tau)}{\partial n_p}V_N V_n dt_q\right\}\right\} \quad (3)$$
$$p\in B(t)$$

Once the source strength is found, velocity potential may be evaluated by (1), and the velocity is obtained using vector of (3). The hydrodynamic pressure is obtained by Bernoulli's equation.

3. General numerical method of the integral equation

Up to now, constant panel method is most used for solving equation (3). The body surface $B(t)$ is divided into $N(t)$ quadrilateral elements over which the source strength is assumed constant. The body waterline $\Gamma(\tau)$ is divided into $N_W(t)$ straight line segments on which the source strength is assumed to be the same as that of the adjacent body panel. The equation is satisfied at control points of the panels. For the convolution integral with respect to time variable, a constant time step, Δt ,is used and a trapezoidal rule is adopted by all the published research work. The discretized form of (3) is given by

$$\sum_{j=1}^{N^M}\sigma_j^M\overline{A}_{ij}=\overline{B}_i \quad (4)$$

where

$$\overline{A}_{ij}^M=\begin{cases}\vec{n}_i\cdot\nabla_i\iint_{\Delta S_j}\left(\frac{1}{r_{ij}}-\frac{1}{r_{-ij}}\right)^M ds_j & i\neq j\\ 2\pi-\vec{n}_i\cdot\nabla_i\iint_{\Delta S_j}\left(\frac{1}{r_{-ij}}\right)^M ds_j & i=j\end{cases} \quad (5)$$

$$\overline{B}_i=\Phi_{ni}^M-\Delta t\sum_{m=1}^{M-1}\left(\sum_{j=1}^{N^m}\sigma_j^m\vec{n}_i\cdot G_{fij}^{M-m}-\frac{1}{g}\sum_{k=1}^{N_W^m}\sigma_k^m G_{fik}^{M-m}\cdot V_{Nk}^m V_{nk}^m\right) \quad (6)$$

With $$G_{fij}^{M-m}=\mathrm{n}_i\iint_{\Delta s_j}\nabla_i\widetilde{G}\left(p_i\ ,\ q_j\ ,\ (M-m)\Delta t\right)ds_j \quad (7)$$

$$G_{fik}^{M-m}=\mathrm{n}_i\cdot\int_{\Delta L_k}\nabla_i\widetilde{G}\left(p_i\ ,\ q_k\ ,\ (M-m)\Delta t\right)dl_k \quad (8)$$

In the above, M and m are the indices for t and τ respectectively; i and j are respectively the panel indices for control points p and field points q . It should be noted that the convolution summation ends at $m=M-1$ due to a trapezoidal rule adopted and a property of $\widetilde{G}$. The numerical integration of $\widetilde{G}$ and its derivatives over the quadrilateral panels is performed using two-dimensional Gauss-Legendre quadrature . A one-dimensional quadrature is used for the waterline elements.

4. Modified time-stepping method for the convolution integral equation

These numerical method described above for the space integration and time integration of memory part of Green function is most adopted by the similar research work. We also use it for many years. Nevertheless, unstable result for ships with large flare are always encountered. This is a main block for hydrodynamic analysis of the practical ship using time-domain Green function based method.

Recently, a modified numerical method for calculating the convolution integral with respect to time variable is presented and find to improve the stability of calculation over the above trapezoidal rule based method. Let's examine the derivation of the integral over body surface in (3)

$$I=\int_0^t d\tau \iint_{B(\tau)} ds_q \sigma(q,\tau)\tilde{G}_{np}.=\sum_{m=1}^{M}\int_{(m-1)\Delta t}^{m\Delta t} d\tau \iint_{B(\tau)} ds_q \sigma(q,\tau)\tilde{G}_{np}$$
$$=\sum_{m=1}^{M}\left[\iint_{B(m\Delta t)} ds_q \sigma(q,m\Delta t)\int_{(m-1)\Delta t}^{m\Delta t}\tilde{G}_{np}(\,t-\tau\,)\,d\tau\right] \tag{9}$$

Where a middle value theorem of integration has been used. After integral of $\tilde{G}$, we get

$$I=\iint_{B(t)}\left[2\frac{\partial}{\partial n_p}\left(\frac{1}{r_{p\bar{q}}}\right)-\frac{\partial\bar{G}}{\partial n_p}(\Delta t)\right]\sigma(q,t)ds_q+$$
$$\sum_{m=1}^{M-1}\left[\iint_{B(m\Delta t)} ds_q \sigma(q,m\Delta t)\left(\frac{\partial\bar{G}}{\partial n_p}((M-m)\Delta t)-\frac{\partial\bar{G}}{\partial n_p}((M-m+1)\Delta t)\right)\right] \tag{10}$$

Where $\overline{G}(p,t,q,\tau)=2\int_0^{\infty}e^{k(z+s)}J_0(kR)\cos\left[\sqrt{gk}(t-\tau)\right]dk$ is a new function. The line integration in (3) can be calculated similarly.

Comparing (10) with (4-8), we can find two different characteristic: The first is different form of the convolution integration algorithms; the second is the resulted left hand side matrix. The new one may be regarded as an implicit algorithm due to the memory part involved in the present calculation.

5. An improved space integration

Similar to $\tilde{G}$, the numerical integration of $\overline{G}$ and its derivatives over the quadrilateral panels may be performed using two-dimensional Gauss-Legendre quadrature. But when the control and field point is near the waterline. $\overline{G}$ and its derivatives are examined to show highly oscillating property. An improved numerical integration of $\overline{G}$ and its derivatives over the quadrilateral panels is derived.

$$I_x=\iint_{\Delta s_j}\frac{\partial}{\partial x}\overline{G}\left(p_i\,,\,q_j\,(M-m)\Delta t\right)ds_j$$
$$=\sum_{l=1}^{4}\left\{\left[\frac{n_1^2-1}{|n_3|}\beta_{1,2}-\frac{n_1 n_2}{|n_3|}\alpha_{1,2}\right]\int_{L_{1,2}^j}\overline{G}dl-\left[\frac{n_1 n_3}{|n_3|}(\xi_1-x)\beta_{1,2}-(\eta_1-y)\alpha_{1,2}\right]\int_{L_{1,2}^j}\frac{H}{R}dl\right\} \tag{11}$$

$$I_y = \iint_{\Delta s_j} \frac{\partial}{\partial y} \overline{G}\left(p_i \ , \ q_j \ (M-m)\Delta t\right) ds_j$$
$$= \sum_{l=1}^{4} \left\{ \left[\frac{1-n_2^2}{|n_3|} \alpha_{1,2} + \frac{n_1 n_2}{|n_3|} \beta_{1,2} \right] \int_{L_{1,2}^j} \overline{G} dl - \left[\frac{n_2 n_3}{|n_3|} (\xi_1 - x) \beta_{1,2} - (\eta_1 - y) \alpha_{1,2} \right] \int_{L_{1,2}^j} \frac{H}{R} dl \right\} \quad (12)$$

$$I_z = \iint_{\Delta s_j} \frac{\partial}{\partial z} \overline{G}\left(p_i \ , \ q_j \ (M-m)\Delta t\right) ds_j$$
$$= \sum_{l=1}^{4} \left\{ \left[\frac{n_2 n_3}{|n_3|} \alpha_{1,2} - \frac{n_1 n_2}{|n_3|} \beta_{1,2} \right] \int_{L_{1,2}^j} \overline{G} dl + \left[\frac{n_3^2}{|n_3|} (\xi_1 - x) \beta_{1,2} - (\eta_1 - y) \alpha_{1,2} \right] \int_{L_{1,2}^j} \frac{H}{R} dl \right\} \quad (13)$$

Where $L_{1,2}^l$ means the four side of the quadrilateral panels , and n_1, n_2, n_3 stands for the normal components of that panel,

$\alpha_{1,2} = \dfrac{\xi_2 - \xi_1}{L}; \beta_{1,2} = \dfrac{\eta_2 - \eta_1}{L}; L^2 = (\xi_2 - \xi)^2 + (\eta_2 - \eta_1)$ 1,2 means the staring and end point corresponding one of the side following right hand laws. For vertical panels, similar formulas can be derived. It can be seen that the space integration of Green function are now transferred to the line integration. An high accuracy integration results can be derived.

6. Results and discussion

Using the modified integration scheme of the transient free surface Green function presented in this paper, numerical calculation for half submerged sphere radiation problem validated well with the results of Hulem(1982). Particularly interested to us is the stable time-stepping result for large flared ship. For linear zero forward speed hydrodynamic calculation of S175 and SL7 container ship, the result comparing well with the result by frequency domain Green function based method. For large amplitude nonlinear calculation and forward speed problem, the results is stable comparing with the divergent results of the general numerical model. Validation with experiment are being performed now, and detailed results will be reported on the workshop.

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