

COUPLED COMPRESSIBLE AND INCOMPRESSIBLE APPROACH FOR JET IMPACT ONTO ELASTIC PLATE

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The paper is concerned with a compressible liquid jet impact onto an elastic plate. The response of the plate is governed by the linear dynamic equation. The coupling between the fluid flow and the plate deflection is taken into account through the dynamic and kinematic conditions imposed on the wetted part of the plate. The problem is solved through a coupled incompressible and compressible approach.

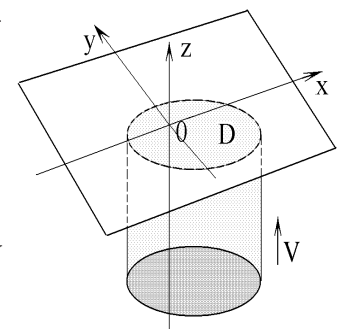
1. Introduction

The problem of jet impact onto an elastic panel is of importance in many fields of engineering such as naval architecture, offshore and coastal engineering. A well know example is sloshing in a LNG tank. The motion of the liquid can be so violent that strong jets can be formed. In extreme conditions the impact by the jet can damage the panels of the tank and cause leakage.

At the moment of jet colliding with the plate, its velocity will be reduced sharply. Because of this rapid temporal variation, the compressibility of the liquid is expected to be important, even if the Mach number is small. The compressible model is used by Korobkin, Khabakhpasheva & Wu [1,2] for a two dimensional problem, an axisymmetrical problem and a three dimensional problem. Modal analysis is used for both fluid flow and structural deflection. As a result, the partial differential equations are reduced to a set of linear ordinary differential equations for the principle coordinates. The importance of the compressibility is reflected in the 'memory' effect or a convolution term. The memory effect then takes major computational effort, which becomes increasingly significant when time progresses and a large number of modes are used. On the other hand, it is expected that after the initial contact period, the temporal variation the flow will become less rapid and the effect of compressibility will decrease. It is therefore proposed in this paper that the problem of jet impact onto an elastic plate should be tackled by the coupled compressible and incompressible approach. This means that the solution will start with the compressible model when the jet hits the plate. When the rate of temporal variation reduces, the incompressible theory will take over. As a result, the memory effect or the convolution term will be ignored. Comparison will be made to verify the effectiveness and accuracy of this approach.

2. Formulation of the problem

We consider a coupled problem of jet-structure impact. The structure is a single simply supported plate plate of uniform thickness. The jet with constant cross section and a flat head hits the plate from below in the normal direction. Gravity and surface tension effects are neglected and the liquid is assumed to ideal. The jet speed V is assumed to be much smaller than the speed of the sound c_0 , or the Mach number $M = V/c_0 \ll 1$. The disturbed flow due the impact is then described within the linear acoustic approximation through the velocity potential theory.



We consider the initial stage of the impact. Over this short period of time, the deformation of the jet surface is neglected and the boundary conditions are linearized. The problem is nondimensionalized through the typical dimension R of the jet cross section D as the length scale, the jet speed V as the velocity scale, R/c_0 as the time scale, $\rho c_0 V$ as the pressure scale (ρ is the density of the liquid), VR as the velocity potential scale and RM is the plate deflection scale.

A Cartesian system $Oxyz$ is defined with the plate in the plane $z = 0$. The total velocity potential can be written as $z - \phi(x, y, z, t)$, where ϕ is the disturbed potential which satisfies the following equations and boundary conditions

$$\varphi_{tt} = \varphi_{xx} + \varphi_{yy} + \varphi_{zz} \quad ((x, y) \in D, z < 0), \tag{1}$$

$$\varphi = 0 \quad ((x, y) \in \partial D, z < 0), \quad \varphi \rightarrow 0 \quad ((x, y) \in D, z \rightarrow -\infty), \tag{2}$$

$$\varphi_z = 1 - w_t(x, y, t) \quad ((x, y) \in D, z = 0), \tag{3}$$

$$\varphi = \varphi_t = 0 \quad (t = 0), \tag{4}$$

where $w(x, y, t)$ is the plate deflection, $p(x, y, z, t) = \varphi_t$ is the hydrodynamic pressure.

The plate deflection is governed by the following equation

$$\alpha w_{tt} + \beta \Delta^2 w = p(x, y, 0, t) \quad ((x, y) \in S, t > 0), \tag{5}$$

$$w = 0, \quad \Delta w = 0 \quad (x, y) \in \partial S, \tag{6}$$

$$w = w_t = 0 \quad (t = 0), \tag{7}$$

where Δ is the Laplace operator on x and y variable,

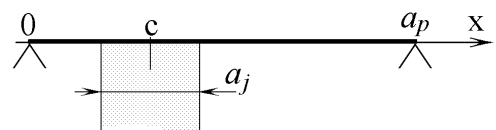
$$\alpha = \frac{m}{\rho R}, \quad \beta = \frac{D_p}{\rho c_0^2 R^3},$$

$m = \rho_p h$ is the plate mass per unit area, ρ_p is the density of the plate material and h is the plate thickness, $D_p = \frac{Eh^3}{12(1-\nu^2)}$ is the plate stiffness, E is the Young modulus, ν is the Poisson ratio and S is the surface area. It can be seen that the problem under consideration is a coupled one of hydroelasticity. The liquid flow and the plate deflection have to be determined simultaneously.

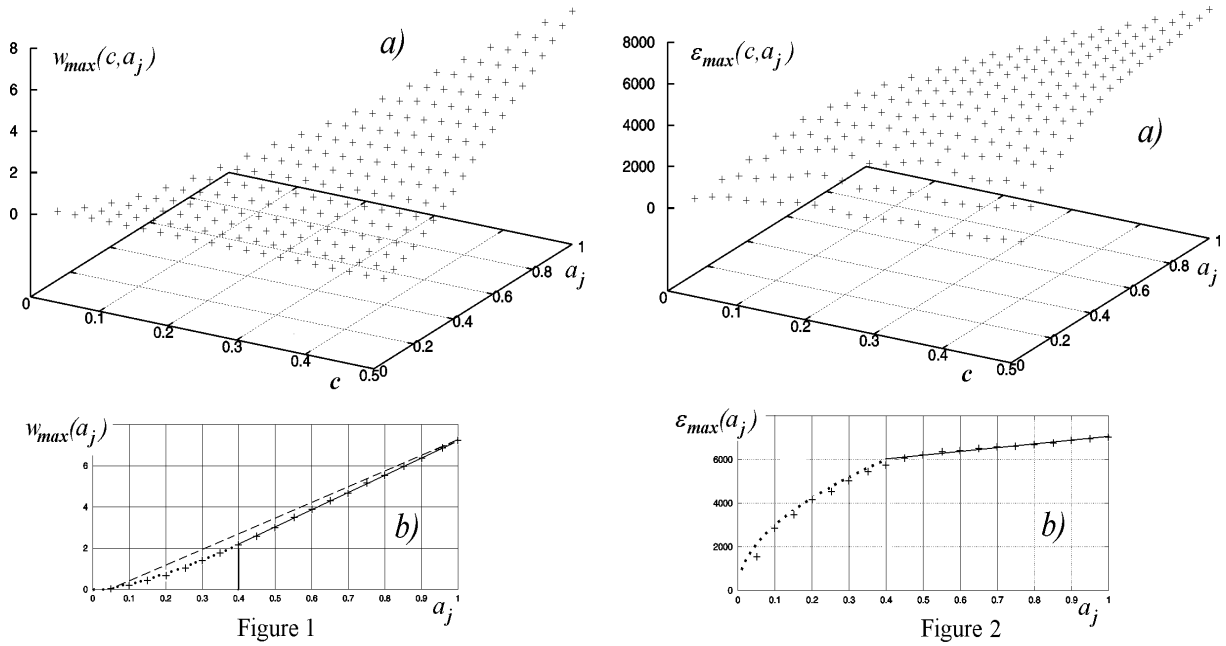
By using normal mode method and Laplace transform, the problem can be reduced to a system of ordinary differential and integral equations with respect to time for principal coordinates of velocity potential and the plate deflection. The 'memory' effects are taken into account in the framework of this statement. The system can be truncated and solved numerically by the fourth order Runge-Kutta method, the integral terms can be computed by trapezoidal rule (See [1,2]).

3. Results from compressible approach

We consider a two dimensional jet of width a_j hitting a two dimensional plate of length a_p and thickness h . The jet centre is at $x = c$ with the origin at the right edge of the plate. Calculation is made at $V = 10m/s$



(noticing the solution is a linear function of the speed), with the number of modes for plate $N_p = 10$ and for jet $N_j = 10$. Time step is $1.67 \cdot 10^{-4}s$. For the plate $E = 2.1 \cdot 10^{11} N/m^2$, $\rho_p = 7875kg/m^3$, $\nu = 0.3$. The jet parameters are $c_0 = 1500m/s$ and $\rho = 1000kg/m^3$.



Korobkin, Khabakhpasheva & Wu [1,2] have provided some detailed simulations for this case. Here we shall provide some further results to show the effect of position of the jet and the width of the jet. Figs. 1a and 2a provide the maximal deflection and strain for $0.05m \leq a_j \leq 1m$ and $a_j/2 \leq c \leq 0.5m$. It can be seen that the location of the jet has far less significant effect on these results than the width of the jet. The case with $c = 0.5m$ is provided in Figs. 1b and 2b (+). It can be seen that when $a_j > 0.4a_p$, the variations of the deflection and the strain with the width of the jet are mainly linear (solid line). When $x = a_j/a_p < 0.4$, the deflection can be approximated by $w_{max}(x) = w_{max}(0.4)(x/0.4)^{3/2}$ and the strain by $\epsilon_{max}(x) = 1.05\epsilon_{max}(0.4)\sqrt{x/0.4}$. These approximations are given in the dotted lines and they are in good agreement with the calculated results. The dashed line in Fig.1b is a straight line linking $w_{max}(0.05)$ and $w_{max}(1.0)$. It can be seen that the line actually gives fairly good approximation for the deflection.

4. Coupled compressible and incompressible approach

We consider a case with $a_p = 1m$, $h = 2cm$, $a_j = 20cm$ and $c = 0.25m$. The time step is taken as $1.3 * 10^{-4}s$ and $V = 25m/s$. Fig. 3a provides the temporal variation of the force against the nondimensional time t' . It can be seen that when $t' > 1$, the magnitude of the force becomes small, which is an indication that the compressibility effect is diminishing. To verify that, a coupled approach is used. When $t' < t^*$, the compressible model described in Eqs. (1) to (7) is used. When $t' > t^*$, the incompressible model is used. This means that the time derivative in Eq.(1) is deleted and the fluid flow is governed by the Laplace equation. Figs. 3b and 3c give the maximal deflection and strain based on this coupled approach with $t^* = 1$ and $t^* = 2$. It can be seen that the results from the coupled approach are quite close to that from the incompressible model (solid line). As expected, the result from $t^* = 2$ gives a better agreement than that from $t^* = 1$. One can further expect that the agreement can be improved when t^* increases.

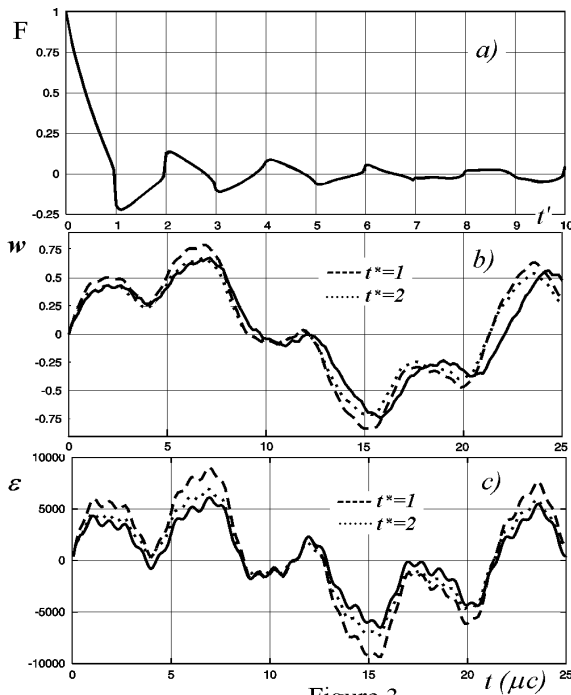


Figure 3

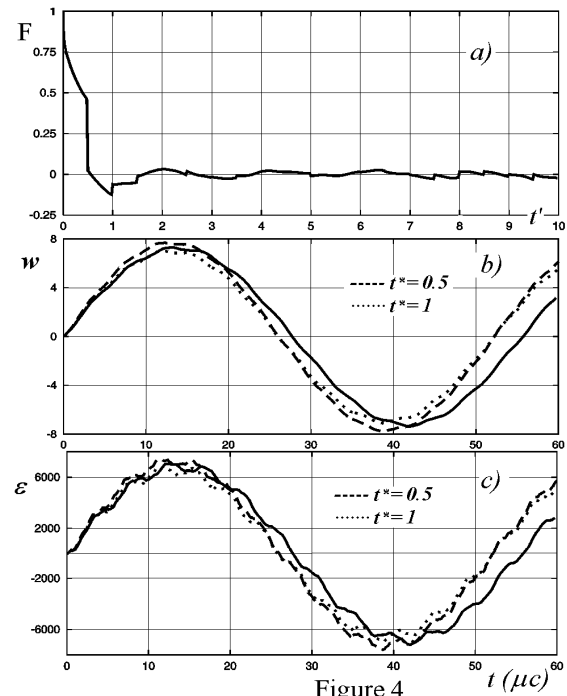


Figure 4

Another case is considered with $a_j = 1m$ and $h = 1.5cm$ while other parameters remain the same. The time step is taken as $6.7 \cdot 10^{-4}s$. Fig. 4a gives the force from the compressible flow. It shows that the force decreases faster than that in the previous case. Figs. 4b and 4c provide the results for maximal deflection and stain from the coupled approach with $t^* = 0.5$ and $t^* = 1.0$. Agreement with the compressible-flow method (solid) is better than that in the previous case. This is because the higher modes are less important for second case, when width of the jet is relatively large with respect to the width of the plate.

5. Conclusions

The following conclusions are made in the context of present results from a 2D rectangular jet impact onto a simply supported uniform plate.

- The location of the jet does not have strong effect on the maximal deflection and strain, while the width of the jet does. Within a wide range of the jet width, the dependence of the maximal deflection on the jet width is overwhelmingly linear.
- The impact problem can be modelled by the coupled compressible and incompressible model. The influence of compressibility must be taken into account only during a very short initial period of the time.

Acknowledgement

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