STEEP WAVE IMPACT ONTO ELASTIC WALL

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ABSTRACT

We consider two-dimensional unsteady problem of steep wave impact onto a vertical wall, part of which is elastic. The problem is solved within both acoustic approximation and incompressible liquid model. The problem is relevant to violent sloshing in LNG tank with low filling (see [1,2] for more details on sloshing-slamming analysis) and steep wave impact onto coastal structures. The present analysis can be used for interpretation of experimental data on behavior of a contaminant system under sloshing impact loads. In experimental set-up a part of LNG contaminant system is installed at a vertical wall, which is otherwise rigid, and is subject to steep wave impact. Depending on position of the elastic part with respect to the wave crest and its elastic characteristics, the response of the elastic body to the wave impact is rather different, which is demonstrated in the present analysis. The structural part of the analysis is limited here to a uniform and simply supported Euler beam, however, its extension to FE models of contaminant system response is straightforward.

1. FORMULATION OF THE PROBLEM

Two-dimensional problem of steep wave impact onto elastic wall is studied within the acoustic approximation. Before the impact ($t = -0$) the liquid occupies the half-strip, $x < 0$ and $0 < y < H$, where H is the liquid depth (Fig. 1). The line, $x < 0$ and $y = 0$, corresponds to the rigid bottom of the liquid and the line, $x < 0$ and $y = H$, to the liquid free surface, which is horizontal and flat. A part of the liquid boundary, $x = 0$ and $0 < y < H_w$, is in contact with the vertical wall before the impact, $H_w < H$. The boundary part, $x = 0$ and $H_w < y < H$, corresponds to the vertical face of the wave (hydraulic jump), which approaches the wall at constant speed U and hits the wall at $t = 0$. The liquid flow after the impact is assumed potential. Air-cushion effects in the impact place, gravity and surface tension are neglected. The fluid is assumed compressible with c_0 being the sound speed in fluid at rest and ρ_0 the fluid density. A part of the wall, $x = 0$, $H^{(-)} < y < H^{(+)}$, $H^{(-)} \geq 0$ and $H^{(-)} < H$, is elastic and modelled as Euler beam with corresponding end conditions, mass per unit length m_b, bending rigidity EJ and structural damping γ. The rest of the wall is rigid. For champed beam conditions $w = 0$ and $w_y = 0$ are hold at $y = H^{(\pm)}$, where $w(y, t)$ is the deflection of the elastic part of the vertical wall. For simply supported beam conditions $w = w_{yy} = 0$ should be satisfied at the end points. The damping coefficientγ is evaluated from dry–vibration experiments.

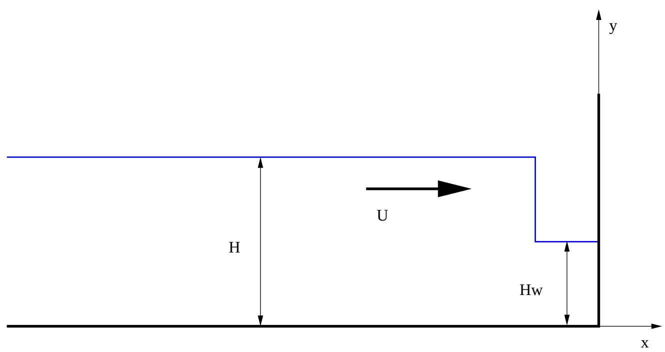

Figure 1. Hydraulic jump impact onto elastic wall.

In general, the fluid flow and the beam deflection should be determined at the same time. This is, the problem is considered as the linear problem of hydroelasticity. At the initial stage of the impact, which is of short duration, the fluid motion equations and the boundary conditions on the fluid free surface and in the contact region between the fluid and the elastic part of the wall are linearized. This implies, in particular, that the elastic deflections and bending stresses are linearly dependent on the impact velocity. We shall determine the elastic response of the wall and the hydrodynamic pressure distribution within the linear hydroelasticity theory. The problem is considered in non-dimensional variables

$$\tilde{x} = \frac{x}{H}, \quad \tilde{y} = \frac{y}{H}, \quad \tilde{t} = \frac{t}{T}, \quad \tilde{\varphi} = \frac{\varphi}{UH}, \quad \tilde{q} = \frac{q}{\rho_0 c_0 U}, \quad \tilde{w} = \frac{w}{UT}, \quad T = \frac{H}{c_0},$$

which are denoted with tilde. Here $\varphi(x, y, t)$ is the velocity potential of the flow induced by the impact, $q(x, y, t)$ is the hydrodynamic pressure, which is given by linearized Bernoulli equation, and T is the time scale of the problem. Below we omit the tilde and formulate the impact problem within the non–dimensional variables as

$$\varphi_{tt} = \varphi_{xx} + \varphi_{yy} \qquad (x < 0,\ 0 < y < 1,\ t > 0), \tag{1}$$

$$\varphi = 0 \qquad (x < 0,\ y = 1), \tag{2}$$

$$\varphi_y = 0 \qquad (x < 0,\ y = 0), \tag{3}$$

$$\varphi_x = -1 + w_t \qquad (x = 0,\ 1 - h_w < y < 1), \tag{4}$$

$$\varphi_x = w_t \qquad (x = 0,\ 0 < y < 1 - h_w), \tag{5}$$

$$q = -\varphi_t(0, y, t) \qquad (0 < y < 1), \tag{6}$$

$$\varphi = \varphi_t = 0 \qquad (t < 0), \tag{7}$$

$$\alpha w_{tt} + \beta(1 + \gamma_d \frac{\partial}{\partial t}) w^{IV} = q(y, t) \qquad (h^{(-)} < y < h^{(+)},\ t > 0), \tag{8}$$

$$w = w_t = 0 \qquad (t < 0), \tag{9}$$

$$w = w_{yy} = 0 \qquad (y = h^{(\pm)}), \tag{10}$$

where non-dimensional parameters are

$$h_w = \frac{H - H_w}{H}, \quad h^{(\pm)} = \frac{H^{(\pm)}}{H}, \quad \gamma_d = \frac{\gamma}{T}, \quad \alpha = \frac{m_b}{\rho_0 H}, \quad \beta = \frac{EJ}{\rho_0 c_0^2 H^3}.$$

The solution of the boundary-value problem (1) - (10) depends on three geometrical parameters h_w, $h^{(\pm)}$ and three elastic parameters α, β and γ_d. The hydroelastic problem (1)–(10) can be divided into two parts. In the hydrodynamic part (1)–(7) one can evaluate the loads acting along the wall once the wall deflection $w(y, t)$ is known. In the elastic part of the problem, which is represented by equations (8)-(10), the wall deflection and the stresses in the wall due to impact can be computed once one knows the load distribution $q(y, t)$.

2. NUMERICAL SOLUTION OF THE PROBLEM

The velocity potential $\varphi(x, y, t)$ and the hydrodynamic pressure distribution along the wall $q(0, y, t)$ can be presented as

$$\varphi(x, y, t) = \sum_{n=1}^{\infty} \varphi_n(x, t) \cos(\lambda_n y), \qquad q(0, y, t) = \sum_{n=1}^{\infty} q_n(t) \cos(\lambda_n y), \qquad \lambda_n = \frac{\pi}{2}(2n - 1).$$

Then equations (1) - (7) provide [3]

$$q_n(t) = -v_n J_0(\lambda_n t) - \frac{\partial^2}{\partial t^2} \int_0^t Y_n(\tau) J_0[\lambda_n(t - \tau)]\, d\tau, \qquad Y_n(t) = 2 \int_{h^{(-)}}^{h_a} w(y, t) \cos(\lambda_n y)\, dy,$$

$$v_n = \frac{2}{\lambda_n}(-1)^n [1 - \cos(\lambda_n h_w)], \qquad h_a = \min\{1, h^{(+)}\}, \tag{11}$$

where $J_0(z)$ is the Bessel function of zeroth order. The beam deflection $w(y, t)$ is sought in the form

$$w(y, t) = \sum_{m=1}^{\infty} a_m(t) \Psi_m(y). \tag{12}$$

The shape functions $\Psi_m(y)$ satisfy equations

$$\Psi_m^{IV} = \mu_m^4 \Psi_m \qquad (h^{(-)} < y < h^{(+)}), \tag{13}$$

the corresponding edge conditions, are orthogonal and normalized. Equation (8) with account for (12) and (13) leads to the system

$$\alpha \ddot{a}_k + \beta \mu_k^4 (1 + \gamma_d \frac{d}{dt}) a_k = \int_{h^{(-)}}^{h^{(+)}} q(0, y, t) \Psi_k(y)\, dy. \tag{14}$$

Note that $q(0, y, t) = 0$ where $h_a < y < h^{(+)}$. Combining equations (11) - (14), we arrive at the system of differential equations

$$\ddot{b}_k + \beta \mu_k^4 (a_k + \gamma_d \dot{a}_k) = p_k(t) \tag{15}$$

and the integral equations

$$\alpha a_k + \sum_{m=1}^{\infty} \int_0^t a_m(\tau) K_{km}(t - \tau)\, d\tau = b_k(t). \tag{16}$$

The system (15), (16) is solved numerically subject to initial conditions ($k \geq 1$)

$$b_k(0) = \dot{b}_k(0) = a_k(0) = 0. \tag{17}$$

Here $b_k(t)$ are auxiliary functions, $K_{km}(t) = K_{mk}(t)$,

$$K_{km}(t) = 2 \sum_{n=1}^{\infty} T_{nk} T_{nm} J_0(\lambda_n t), \qquad p_k(t) = -\sum_{n=1}^{\infty} v_n T_{nk} J_0(\lambda_n t), \qquad T_{nk} = \int_{h^{(-)}}^{h_a} \cos(\lambda_n y) \Psi_k(y).$$

It should be noted that the hydroelastic problem has at least two time scales. Up to now only the time scale T of the acoustic effects has been employed. In non-dimensional variables significant variations of the functions $K_{km}(t)$ and $p_k(t)$ occur at time intervals of the unity order. However, the principal coordinates $a_k(t)$ of the elastic wall deflection are governed, in general, by another time scale T_w which is of the order of the period of the lowest mode free vibration of the beam in contact with the fluid. If $T_w/T = O(1)$, elastic and acoustic effects are strongly coupled and equations (15) - (17) should be solved numerically. If $T_w \gg T$, then unknown functions in equations (15) and (16) are expected to be slow-varying functions. This makes it possible to simplify equations (15) and (16) by using the fact that the functions $K_{km}(t)$ and $p_k(t)$ decay as $t \to \infty$ and the approximations

$$\int_0^t a_m(\tau) K_{km}(t - \tau)\, d\tau \approx a_m(t) \mathcal{K}_{km}, \quad p_k(t) \approx \delta(t) \mathcal{P}_k, \quad \mathcal{K}_{km} = \int_0^{\infty} K_{km}(\tau)\, d\tau, \quad \mathcal{P}_k = \int_0^{\infty} p_k(\tau)\, d\tau.$$

Within this approximation equations (15) and (16) are reduced to the homogeneous ODE system

$$(\alpha \mathbb{I} + \mathbb{K}) \mathbf{a}_{tt} + \mathbb{D}[\mathbf{a} + \gamma_d \mathbf{a}_t] = 0, \qquad \mathbf{a}(0) = 0, \qquad \mathbf{a}_t(0) = (\alpha \mathbb{I} + \mathbb{K})^{-1} \mathbf{P}, \tag{18}$$

where $\mathbb{K} = \{\mathcal{K}_{km}\}$, $\mathbf{P} = \{\mathcal{P}_k\}$, $\mathbb{D} = \mathrm{diag}\{\beta \mu_k^4\}$ and $\mathbb{I}$ is the unit matrix. It can be checked that equations (18) become exact in the model of incompressible fluid, when the l.h.s. in equation (1) is replaced with zero.

Equations (15)-(16) and (18) are integrated in time after truncation by implicit scheme, which is based on the following approximations of the time derivatives

$$\frac{1}{2}[\dot{a}_k(t + \Delta t) + \dot{a}_k(t)] = \frac{1}{\Delta t}[a_k(t + \Delta t) - a_k(t)], \qquad \frac{1}{2}[\ddot{a}_k(t + \Delta t) + \ddot{a}_k(t)] = \frac{1}{\Delta t}[\dot{a}_k(t + \Delta t) - \dot{a}_k(t)].$$

The integrals in equation (16) are approximated by Simpson' rule as

$$\int_0^{t+\Delta t} a_m(\tau) K_{km}(t - \tau)\, d\tau = \frac{1}{3} \Delta t \cdot K_{km}(0) a_m(t + \Delta t) + S_{km}^{(j)},$$

where $S_{km}^{(j)}$ are dependent on the solution at previous time steps.

3. NUMERICAL RESULTS

Numerical calculations were performed for different geometrical and elastic parameters of the problem. Figure 2 shows the elastic deflection at the middle of the plate made of steel, of length 1m, thickness 2cm and $H = 2$m, $H_w = 1.5$m, $H^{(-)} = 1$m and $H^{(+)} = 2$m. Deflection is in meters and the time in seconds. It is seen that the deflections are very close to each other at the initial stage. Calculations were continued with the incompressible fluid model with, $\gamma = 0.0001sec$, and without, $\gamma = 0$, structural damping (see Figure 3). Note that even if the structural damping is very low, its influence on the deflections is visible. This influence is even more visible for the strains, evolution of which at the middle of the plate is shown in Figure 4.

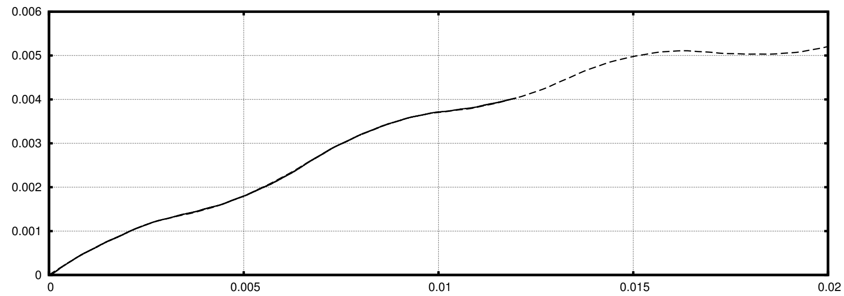

Figure 2. Deflection obtained within the compressible fluid model is shown with solid line and within the incompressible model by dashed line.

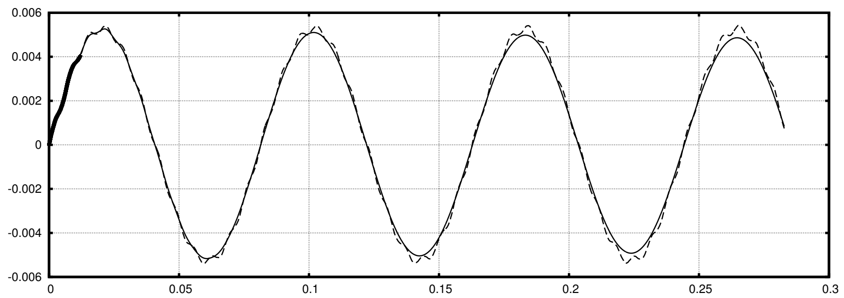

Figure 3. Deflection obtained within the incompressible fluid model with, $\gamma = 0.0001sec$ - solid line, and without, $\gamma = 0$ - dashed line, structural damping.

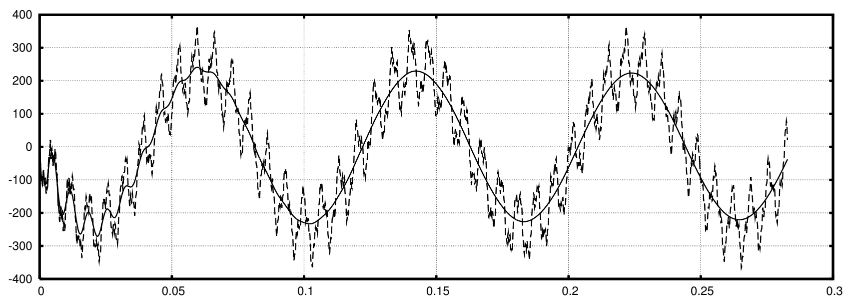

Figure 4. Strains (in μs) obtained within the incompressible fluid model with, $\gamma = 0.0001sec$ - solid line, and without, $\gamma = 0$ - dashed line, structural damping.

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