

Hydrodynamic forces on a planing hull in forced heave or pitch motions in calm water

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Analysis of a planing hull is normally done by neglecting gravity. However, gravity matters. The gravity effects on a prismatic hull in steady motions have been studied by Sun and Faltinsen (2007) by using a 2D+t theory and a two-dimensional Boundary Element Method. In this paper, we will generalize this approach to study the loadings on a planing hull in unsteady motions at a moderate planing speed.

The forward speed of the planing hull is U . Forced oscillations in heave or pitch are given to the vessel. We assume no incident waves. An analysis similar as in Maruo and Song (1994) is followed. The space-fixed coordinates xyz and the body-fixed coordinates XYZ are defined in Fig. 1. The xy plane is in the calm water surface and the z -axis is pointing upwards. The planing hull is moving in the negative x -direction. The origin of XYZ is fixed at the centre of gravity (COG) of the hull. The X -axis is pointing to the stern and the Y -axis is towards the starboard. The instantaneous trim angle θ is defined positive when the bow is up. The position of the COG in the space-fixed coordinates is $(x_g, 0, z_g)$. The body-fixed coordinates can be related to the space-fixed coordinates by $X = (x-x_g)\cos\theta - (z-z_g)\sin\theta$; $Y = y$; $Z = (x-x_g)\sin\theta + (z-z_g)\cos\theta$.

A velocity potential $\varphi(x,y,z,t)$ is introduced to describe the water flow around the vessel. The velocity potential satisfies three-dimensional Laplace equation. Fully nonlinear free surface conditions and exact body boundary conditions are satisfied in three dimensions. Then we introduce the slenderness ratio as $\varepsilon = d/L$, where d is the draft and L is the length of the hull. By using slender body assumption, we have $\partial/\partial x \sim O(\varepsilon)$, $\partial/\partial y \sim O(1)$, $\partial/\partial z \sim O(1)$. Further, we have $\partial/\partial X \sim O(\varepsilon)$, $\partial/\partial Y \sim O(1)$, $\partial/\partial Z \sim O(1)$ and we assume that the trim angle is small. Neglecting the terms with order $O(\varepsilon^2)$ in the governing equation and the boundary conditions, one can obtain the 2D Laplace equation and boundary conditions in a cross- plane given by

$$\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad (1)$$

$$\frac{\partial \varphi}{\partial N} = -N_x U - N_z (U\theta - \dot{\eta}_3 + X\dot{\eta}_5) \quad \text{on the hull surface} \quad (2)$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}(\varphi_y^2 + \varphi_z^2) + g\zeta = 0 \quad \text{at } z = \zeta(x, y, t) \quad (3)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \zeta}{\partial y} \frac{\partial \varphi}{\partial y} - \frac{\partial \varphi}{\partial z} = 0 \quad \text{at } z = \zeta(x, y, t) \quad (4)$$

where g is the acceleration of gravity and the normal vector $\mathbf{N}=(N_x, N_z)$ is the 2D normal vector in the cross-plane. For a prismatic hull, we have $N_x=0$. The unsteady motions are described by the heave $\eta_3 = -\eta_{3a}\sin(\omega t)$ or the pitch $\eta_5 = -\eta_{5a}\sin(\omega t)$ for $t \geq 0$. The heave is positive when it is upward and the pitch is positive when the bow is up. The dot above η_3 , η_5 means time derivative. In addition, we have far-field conditions.

The Boundary Element Method described in Sun and Faltinsen (2007) is used to solve the 2D problem described by Eqs. (1)-(4). In the BEM, the thin jet along the body surface is cut off. When the water flow separates from the knuckles of the V-shaped section, a flow separation model similar as in Zhao et al. (1996) is applied. Thin sprays will rise up along both sides of the section after separation and turn over to hit the water surface underneath. To avoid wave breaking, the overturning thin spray is cut away before it hits the underlying water. The numerical schemes to cut the thin jet and to cut the thin spray are described in detail in Sun and Faltinsen (2007). However, numerical challenges are in particular found for the present unsteady problems.

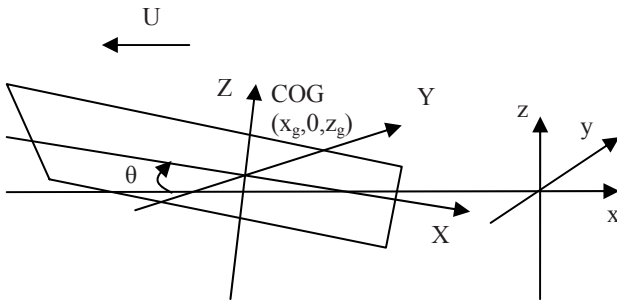


Fig. 1 Coordinate systems and definitions.

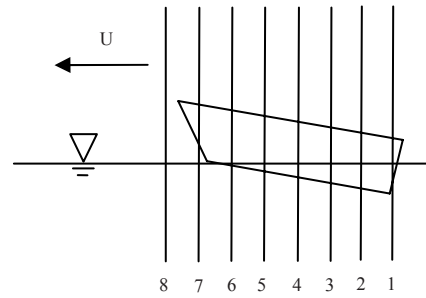


Fig. 2 Numerical scheme in the 2D+t theory.

Initially the hull is planing steadily on the free surface, so the initial conditions for the water flow can be obtained from the steady calculations in Sun and Faltinsen(2007). Then a forced unsteady oscillation in heave or pitch is given to the hull. The time histories of the total vertical force and the pitch moment around the COG of the hull can be calculated. The vertical force is positive in positive z-direction and the pitch moment is positive when it makes the bow go up and the stern go down.

The numerical calculations can be performed as follows. At first, a number of equally spaced vertical Earth-fixed planes intersecting the hull are introduced. The number of the planes is $NX = 6$ in Fig. 2. At each cross-plane, the free surface elevation and the velocity potential on the free surface are known from the steady calculations in advance. With these free surface conditions and the body boundary condition given by Eq. (2), the boundary value problem can be solved in the 2D cross-plane by the BEM. Then the pressure on the hull can be calculated by $p - p_a = -\rho \partial \phi / \partial t - [(\partial \phi / \partial y)^2 + (\partial \phi / \partial z)^2] - \rho g z$ where ρ is the water density and p_a is the atmospheric pressure. Integrating the pressure in this cross-section properly will result in a sectional vertical force. When the sectional forces in all the NX planes are known, we have an approximation of the sectional force distribution along the hull. From this force distribution, we can do integrations to obtain the total vertical force and the pitch moment. In the next time step, the free surface elevation and the velocity potential on the free surface at each plane are updated by using the free surface conditions in Eqs. (3) and (4) by the fourth-order Runge-Kutta method. We can continue the calculation until the hull advances forward for a distance equal to the interval of the cross-planes. Then we discard the plane No.1 and introduce a new plane No.7, so there are always $NX=6$ planes used in the calculations. The calculation starts in this new plane by using the initial conditions given from Wagner's approximation. As the hull moves further, we will continuously discard the plane after the hull and introduce a new plane in front of the hull. In this way, the calculations can be continued.

In front of the foremost plane, say No. 6 in Figure 2, there is a small part of hull that has been neglected in the procedure described above. Instead, the forces on this part will be calculated by a simplified method by neglecting the gravity effects. Actually, around this forward part of the hull the water moves so fast that the acceleration of the water flow is much larger than the acceleration of gravity. This simplified method is similar as what is used in Lin et al.(1995). The sectional vertical force is approximated by $f(x,t) = d(AV)/dt = VdA/dt + AdV/dt$, where $A(x,t)$ is the 2D heave added mass at infinite frequency and $V(x,t)$ is the vertical water entry speed of the section. From the 2D body boundary condition in (5), $V(x,t)$ can be written as $U\theta - \dot{\eta}_3 + X\dot{\eta}_5$. The added mass is approximated by $A(x,t) = C_m \rho \pi (\psi b)^2$ with $b(x,t)$ as the local beam measured from the calm water surface and C_m and ψ are correction factors. By using the similarity solution in Zhao and Faltinsen (1993), these two factors for deadrise angle $\beta=20^\circ$ are given by $C_m=0.787$ and $\psi=1.5$.

The numerical calculations are performed for the unsteady tests in Troesch (1992) with the following parameters. The beam of the prismatic hull is $B=0.318m$. Deadrise angle is $\beta=20^\circ$. The trim angle in steady conditions is $\tau=4.0^\circ$ and the mean wetted length-beam ratio is $\lambda_w = 3.0$. The

location of the COG is described by $l_{cg}/B = 1.47$ and $v_{cg}/B = 0.65$, where l_{cg} is the distance measured from the stern along the keel to the COG and v_{cg} is the normal distance from the COG to the keel line. The beam Froude number is $F_{nB} = U/(gB)^{1/2} = 2.5$ for the calculated cases. The amplitudes of the heave motion and the pitch motion are respectively $\eta_{3a} = 0.036B$ and $\eta_{5a} = 0.43^\circ$. Five frequencies for $\omega(B/g)^{1/2} = 0.85, 1.13, 1.4, 1.7, 1.95$ are used in the calculations.

The number of the cross-planes intersecting the hull is chosen as $NX=11$. The period of adding a new plane can be obtained by dividing the interval of the space-fixed planes by the speed of the hull, which is about 0.02s. So there are periodic small disturbances of this period in the time histories of the force and moment. However, the amplitudes of the noise are much smaller than the amplitudes of the overall oscillations and the period of the noise is much shorter than the investigated heave or pitch periods. Further, increasing the number of NX will result in more smooth time varying curves. However, the overall oscillations are not obviously affected.

From a time history of the vertical force or the pitch moment, we can do a Fourier analysis to find out the amplitudes of different harmonics. A time history $f(t)$ can be expanded as

$$f(t) = b_0 + \sum_{n=1}^{\infty} (a_n \sin(n\omega t) + b_n \cos(n\omega t)) \quad (5)$$

The amplitudes for the first harmonic a_1 and b_1 are used to estimate the added mass coefficient and the damping coefficient by $A_{ij} = -[(a_1)_{ij} - C_{ij}\eta_{ja}]/(\omega^2\eta_{ja})$ and $B_{ij} = (b_1)_{ij}/(\omega\eta_{ja})$ where the subscripts ij in $(a_1)_{ij}$ and $(b_1)_{ij}$ means that the amplitudes a_1 and b_1 are taken from the vertical force ($i=3$) or the pitch moment ($i=5$) when the forced heave ($j=3$) or pitch ($j=5$) is given to the hull. Because the definition of the positive direction of the pitch motion and pitch moment in the experiments is different from the present definition, the signs of the experimental A_{35}, A_{53}, B_{35} and B_{53} are changed in the comparisons between the experiments and the present calculations.

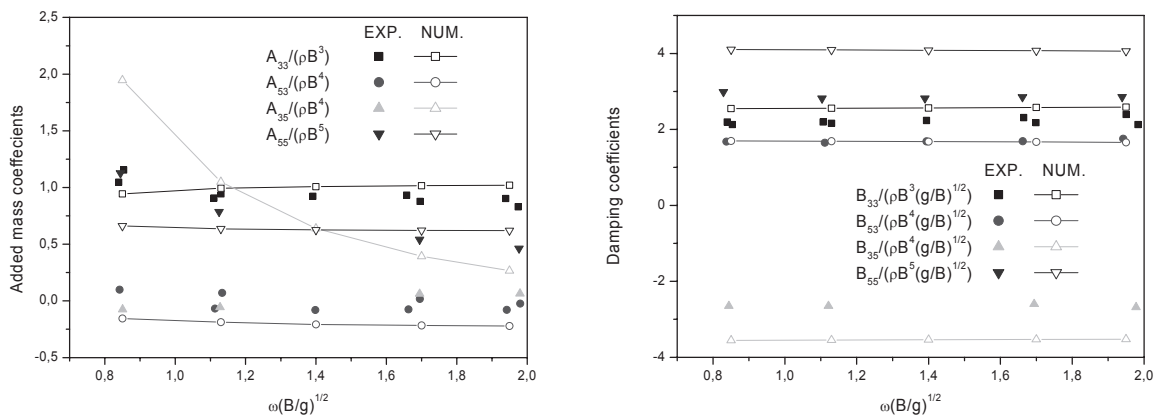


Fig. 3 Added mass and damping coefficients changing with frequency.

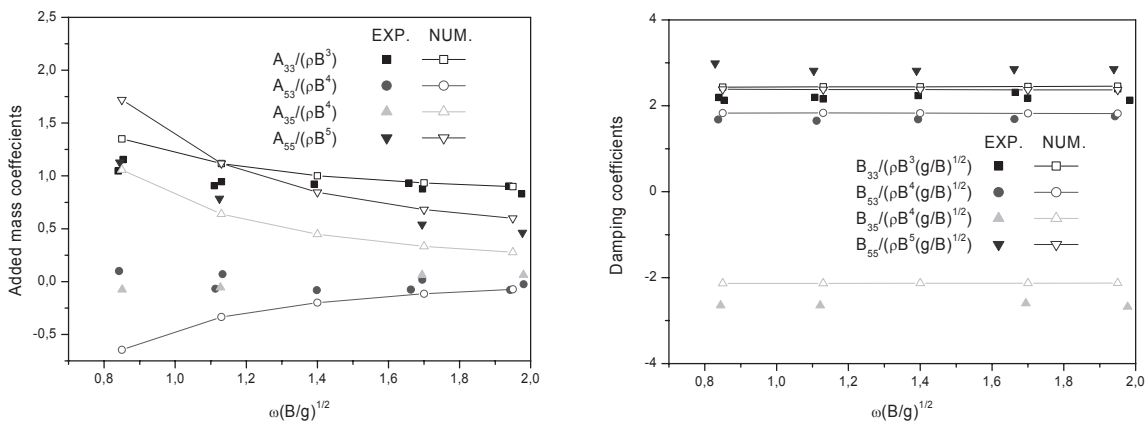


Fig.4 Added mass and damping coefficients after a three-dimensional correction.

The restoring force coefficients C_{ij} are nonlinear, Froude number and time-dependent. However, for simplicity Troesch (1992) used a linear modeling, in which C_{ij} were determined at $\eta_j=0$ from the experimental data in steady cases when a constant heave or pitch was given to the hull. To compare with the experiments, we follow the same way to decide C_{ij} . The calculated added mass and damping coefficients for the five different frequencies are compared with the experimental results in Fig.3. EXP. means experiments; NUM. means the present calculations.

All the damping coefficients and added mass coefficients except A_{35} seem to be independent of the frequency. There is large discrepancy between the experiments and calculations for A_{35} , however, the agreement looks better for higher frequencies. From the study of the unified theory and the traditional strip theory (Newman and Sclavounos, 1981), it can be shown that three-dimensional effects matter more for lower frequencies in the strip theory. This can also be true for the present method, because we have neglected some three-dimensionalities. It was shown in Sun and Faltinsen (2007) that the 3D effect at the transom stern will apparently affects the force and the moment on the hull. After a certain correction of this 3D effect, better results were obtained. The same correction for the 3D effects at the transom is applied here. The results after the 3D correction are shown in Fig. 4. Then the agreement in damping coefficients is improved and they are still almost frequency independent. The agreement for A_{35} looks better. The other three coefficients become slightly frequency dependent. Experiments also show slightly frequency dependent added masses. However, there is better agreement at higher frequencies and larger discrepancy at lower frequencies. We have to notice another effect which can influence the comparison. It is the influence of the estimated restoring force coefficients C_{ij} . Those coefficients used in Troesch (1992) are not directly given in his paper. Errors in C_{ij} will cause larger discrepancy in A_{ij} for lower frequencies because the added masses are calculated from $A_{ij} = -[(a_1)_{ij} - C_{ij}\eta_{ja}]/(\omega^2\eta_{ja})$. The error is proportional to $1/\omega^2$. Further, 3D effects near the chine wetted position, where the chine line starts to get wetted, cause overestimated force near this position, as shown in Sun and Faltinsen (2007). This effect will also cause errors in the results of the added mass and damping coefficients. Again, the effect is supposed to be more prominent for lower frequency cases.

Our future plan is to study the effect of nonlinearities which are particularly important for planing vessels in waves.

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