Three-dimensional capillary-gravity waves generated by moving disturbances

Jean-Marc Vanden-Broeck, E. Parau, M. Cooker
School of Mathematics, University of East Anglia, Norwich, NR4 7TJ, UK
e-mail: h010@uea.ac.uk

Three-dimensional capillary-gravity waves generated by a moving disturbance moving at a constant velocity c on or below a free-surface are considered. The fluid is assumed to be of infinite depth and steady solutions in a frame of reference moving with the disturbance are sought. A classical application is the calculation of the wave pattern generated by a moving ship. The problem is often modelled by potential flow and by neglecting surface tension. It is then necessary to impose the radiation condition which requires that there is no energy coming from infinity. This condition requires that the waves are behind the disturbance. It can easily be imposed numerically by forcing the free-surface to be flat at some distance in front of the disturbance (see for example [7], [8] and [1]).

For small disturbances (insects or probes), the effect of surface tension can be significant. The situation is then more complicated. There is a minimum value c_{min} of c such that there are no waves on the free-surface when $c < c_{min}$. The value of c_{min} is given by

$$c_{min} = (\frac{4gT}{\rho})^{1/4} \tag{1}$$

where T is the coefficient of surface tension (assumed to be constant), ρ is the fluid density and g is the acceleration of gravity. For an interface between water and air, $c_{min} \approx 0.23ms^{-1}$.

Parau, Vanden-Broeck and Cooker ([2] and [3]) calculated numerically nonlinear solutions for $c < c_{min}$. They showed that the free surface profiles are characterised by decaying oscillations in the direction of the motion of the disturbance and monotonic decay in the direction perpendicular to the direction of motion of the disturbance. As the size of the disturbance approaches zero, the solutions reduce to either a uniform stream or a three-dimensional solitary wave.

When $c > c_{min}$, two different wave systems can occur on the free-surface. Analytic solutions have been derived by assuming a small disturbance and seeking a solution as a small perturbation around a uniform stream (see for example [4], [5] and [6]). These linear results show that the radiation condition forces the waves of longer wavelength to accur behind the disturbance and those of shorter wavelength to occur at the front of the disturbance.

In this talk we supplement the linear theories for $c > c_{min}$ with nonlinear computations. Since waves occur both at the front and at the back of the disturbance, the radiation condition cannot easily be imposed (as it was the case when $T = 0$). Here we adapt to the nonlinear regime a technique introduced by Rayleigh to calculate analytically linear solutions. The idea is to include a dissipative term in the dynamic boundary condition. This term is characterised by an artificial viscosity $\mu > 0$ known as the Rayleigh viscosity. Rayleigh showed that the linear problem with $\mu \neq 0$ has a unique solution and that the correct solution satisfying the radiation condition is selected by taking the limit $\mu \to 0$.

We show that nonlinear solutions satisfying the radiation condition can be calculated numerically by using a boundary integral equation formulation in which a small Rayleigh viscosity $\mu > 0$ is introduced. The boundary integral equation formulation is based on ideas developed by [7], [8], [1], [2] and [3]. For simplicity we assumed that the disturbance is a distribution of pressure with bounded support (qualitatively similar results can be obtained for different disturbances, for example moving submerged objects). We note that related approaches were used before for two-dimensional free-surface flows ([9] and [10]).

Our solutions are not truly non-dissipative because $\mu \neq 0$. The effect of $\mu \neq 0$ on the solutions can be estimated by comparing solutions with $\mu \neq 0$ to known solutions with $\mu = 0$.

These known solutions include the solutions with $T = 0$ of Parau and Vanden-Broeck [1] and the solutions of Parau, Vanden-Broeck and Cooker ([2] and [3]) for $c < c_{min}$. In both cases we show that the effect of $\mu \neq 0$ is relatively small, provided μ is small.

We conclude the talk by presenting nonlinear time dependent results obtained by a boundary integral equation formulation. In particular we show how the steady gravity-capillary solutions described in the first part of the talk can be computed as the long time behaviour of a time dependent calculation.

Acknowledgments
This work was supported by EPSRC.

References

[1] E.I. Parau, J.-M. Vanden-Broeck, Eur. J. Mechanics B/Fluids, **21**, 643 (2002).

[2] E.I. Parau, J.-M. Vanden-Broeck, M. Cooker, J. Fluid Mech. **536**, 99 (2005).

[3] E.I. Parau, J.-M. Vanden-Broeck, M. Cooker, Phys. Fluids, **17**, 122101 (2005).

[4] Lord Rayleigh, Proc. London, Math. Soc., **15**, 69 (1883).

[5] E.Raphael, P.-G. de Gennes, Physical Review E, **53**, 3448 (1996).

[6] S.-M. Sun, J.B. Keller, Phys. Fluids, **13**, 2146 (2001).

[7] L. Landweber, M. Macagno, Iowa Institute of Hydraulic Research Report (IIHR), **123**, 1 (1969).

[8] L.K. Forbes, J. Comp. Physics, **82**, 330 (1980).

[9] B. Spivak, J-M. Vanden-Broeck, T. Miloh, Eur. J. Mech./B Fluids, **21**, 207 (2002).

[10] J.-M. Vanden-Broeck, Euro. Jnl. Appl. Mathss, **12**, 257 (2001).