Active Sound Control Using a Floating Flexible Plate

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Introduction and Objectives:

In this study we deal with the scattering of a plane sound wave impinging on a floating circular and flexible plate. Particular attention will be given to the use of the flexibility of the plate as a mean of active noise control. Such problem was considered by the second author last year considering the use of external pressure to minimise the far field intensity of the scattered sound [1]. Eigenfunction expansion based on spheroidical functions was used and an approximation for a centred pressure force was derived for the low frequency limit. Here a computational approach is developed aiming to calculate the scattered sound wave and to optimize the external pressure acting on the plate so it will mimic a surface of zero impedance, i.e. a free surface. The result will be hiding or at least reducing significantly the acoustic signature of the plate to a detection device.

The interaction between sound and floating structure is of importance not just for issues of detection but also for structural integrity, e.g. metal fatigue. Of course the considered problem is of reduced complexity (e.g. plane incoming sound wave, circular plate), however it is hoped that this study will lead to further work on more generalized floating bodies and conditions.

Mathematical and Numerical Formulation:

Linear acoustics and plate dynamics is assumed. The plate is taken as circular and floating over the water. A pure tone incoming plane wave propagating perpendicular to the plate is assumed, thus the problem becomes axisymmetric. Two calculation methods are used. The first method is a spectral decomposition method based on the Fourier-Bessel expansion and the second method is a simulation technique, based on time marching the sound wave equation. The latter is achieved using finite-difference-central-schemes of second and fourth orders in time and space respectively. However, this method will only be used to validate the results of the spectral decomposition method and thus the emphasis in this section is on the spectral method.

The governing sound field equation in the space-frequency domain is the Helmholtz equation;

$$\left(\nabla^2 + \bar{k}^2\right)p = 0\,. \tag{1}$$

p is the sound pressure, $\bar{k} = \omega/c_0$, ω is the incoming wave frequency and c_0 is the speed of sound. The plate is positioned at z=0 and thus the boundary condition at that plane is taken as;

$$p = 0,\ \text{at}\ r > a, \tag{2a}$$

$$\partial p/\partial z = \rho\omega^2 w,\ \text{at}\ r < a \tag{2b}$$

where the co-ordinate z points down (towards increasing depth), a is the plate's radius, ρ is the fluid density and w is the plate's deflection. The water-air interface is assumed to be at rest and having zero impendence. The plate is assumed to have infinite impendence. Modifying the scheme to account for a finite impendence of the plate is straight forward. The plate deflection w is taken as governed by the following equation [2];

$$\left(D\nabla^4 - \rho_p h\omega^2\right)w = p + f\,. \tag{3}$$

D is the plate's rigidity, ρ_p is its density, h is its thickness and f is an external pressure allowing us to control the plate's deflection.

Following Eq. (1) the acoustic pressure p can be expressed as;

$$p = \sum_i A_i J_0(k_i r) e^{s_i z} + e^{j\bar{k}z} , \tag{4}$$

where $j \equiv \sqrt{-1}$ and

$$s_i = \begin{cases} -\sqrt{k_i^2 - \bar{k}} , k_i > \bar{k} \\ -j\sqrt{\bar{k} - k_i^2} , k_i < \bar{k} \end{cases} , \tag{5}$$

This method of solution is similar to the method used to investigate the super directive sound field emitted by a low speed jet (e.g. [3]). However, the Fourier series in Eq. (4) is taken as finite since a finite computational domain is assumed. Eq. (5) shows that high wave numbers are non-radiative (decays exponentially in the z direction) and low wave numbers are radiative. k_i is determined by the boundary condition used at the edge of the computational domain, i.e. at $r=R$. Computational experimentation found that the simple boundary condition of $J_0''(k_i R) = 0$ although is reflective in general, is satisfactory for our purposes as long as the computational domain is large in the radial direction. This is mainly because the incoming plane wave is taken as propagating in the z direction.

The coefficients A_i are determined by requiring the acoustic pressure p to comply with the boundary conditions at z=0, Eqs. (2). Dividing the edge of the computational domain at z=0 to N segments then A_i for i=1…N can be determined by complying the boundary conditions at the mid points of those segments. At the free surface this yields;

$$\sum_{i=0}^{N} A_i J_0(k_i r_j) + 1 = 0 \quad , r_j > a , \tag{6}$$

and at the plate's surface;

$$\sum_{i=0}^{N} A_i s_i J_0(k_i r_j) + j\bar{k} = \rho \omega^2 w \quad , r_j < a . \tag{7}$$

w the deflection is calculated using central finite difference schemes approximating Eq. (3) and leading to

$$w_j = L_{jm}^{-1} \left[\sum_{i=0}^{N} A_i J_0(k_i r_m) + f_m \right]. \tag{8}$$

L^{-1} is a finite difference matrix. Symmetry boundary conditions are used at the plate's centre (r=0) and simply hinged, clamped or free conditions are used at the plate's edge (r=a), see [1]. Eqs. (6) to (8) result in a full matrix equation for the coefficients A_i, which can be solved using LU decomposition [4]. This is basically a collocation method. However, numerical experimentation showed that the matrix may become ill conditioned, i.e. close to singular, when under certain loading causing difficulties to the matrix solver. Therefore a least square operation was added with respect to A_i and with a linear weight function of r. It led to a better conditioned matrix.

Our aim in this work is to mimic sound wave reflection from the plate as it were reflected from a surface of zero impendence, i.e. free surface. For this purpose the following target (cost) function was constructed;

$$J = (1 + A_0)^2 + \sum_{i=1}^{M} A_i^2 + \sum_{i=M+1}^{N} A_i^2 \big/ (i - M)^2 . \tag{9}$$

If J=0 the sound field is as of reflected only by the free surface. M is the highest wave mode which is still radiative. The contribution of the non-radiative modes is damped in J by the weight of the square of the wave mode number to emphasize the need to minimize the radiative modes. Numerical experimentation showed that neglecting all together the non-radiative modes in J led to similar results. Minimization of J was achieved by varying f and using the Powell's optimisation algorithm, which is widely available [4].

Results and Conclusions

The spectral method of solution was validated for the case of a rigid plate by comparing its results with results obtained from the simulation method. Good agreement was achieved. The finite difference solution of Eq. (3) was validated by comparing it to known analytical static load solutions [5], showing convergence of second order accuracy as required.

An aluminium plate of 0.04" (1 mm) thickness with a radius of 1m was assumed. The speed of sound was taken as of fresh water (1500 m/s) and the incoming acoustic pressure amplitude was taken as of 1 Pa (Sound Pressure Level of 120dB). The shown results are for pure tone of 5000 Hz (low to mid frequency sonar range) and a simply hinged plate. Other plate conditions such as clamped or free can be investigated similarly. The contour levels of the acoustic pressure level $|p|$ are shown in Figure 1 when no external pressure f is applied on the plate. As expected, there are two sound field patterns. One pattern (far field) is far from the plate which is of sound reflected from a free surface and the other (near field) is of reduced sound level above the plate, surrounding a beam of reflected sound around the plate's centre ($r \cong 0$). When applying the external pressure f and optimising every value at each node over the plate, the target function J was reduced by about 50dB, i.e. $10 \log_{10}(J_{f=0}/J_{\text{optimised } f})$ is about 50. This resulted in the contours of Figure 2 showing a sound field as it were reflected from a free surface. However, looking at the distribution of f along the plate as seen in Figure 4, there are some point to point oscillations. Their distribution suggests that assuming a uniform f distribution along the plate may also achieve a good reduction in the target function J. Optimising a uniform f but resolving only 512 radial modes (about 10 points per the incoming wave length), did not achieve any significant effect on the sound field. Increasing the resolution to 1024 modes produced a uniform f which converged to the average of the optimised variable f as can be seen in Figure 4. The value of the real part of the uniform f is of the same scale as of the incoming wave amplitude, while the imaginary part is much smaller. A reduction of about 10 dB was achieved in J, resulting in the dominance of a sound field as it were reflected from a free surface as seen in Figure 3, as it is the elastic plate were transparent.

These preliminary results are promising, particularly the success of using an optimised uniform pressure load over the plate which is more practical than a variable distributed load. It opens the path for further calculations for higher frequencies (above 20 KHz), investigating the effect of an oblique incoming wave and the ability of such device to reduce the acoustic signature of a nearby floating rigid body of a more complex geometry.

References:

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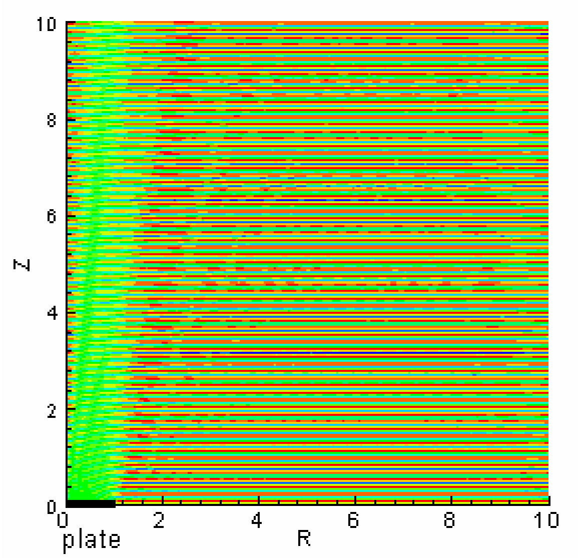

Figure 1: Contour levels of the acoustic pressure level varying from 0.1 to 1.9 and when no external pressure *f* is applied and for 5000 Hz. Number of calculated modes *N* is 512.

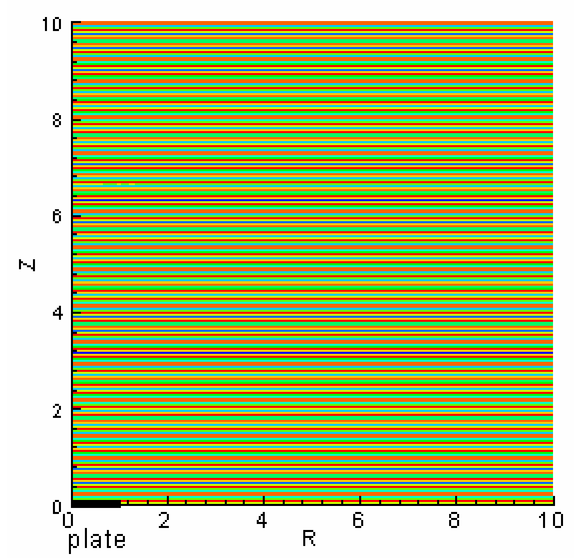

Figure 2: Contour levels of the acoustic pressure amplitude when optimised external pressure *f* is applied and allowed to vary along the plate. Rest of conditions are as in Figure 1.

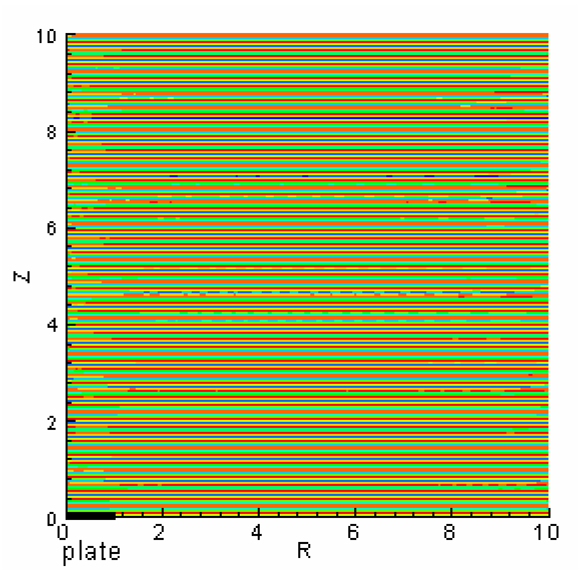

Figure 3: Contour levels of the acoustic pressure amplitude when optimised external pressure *f* is applied but taken as uniform along the plate. *N* is 1024.

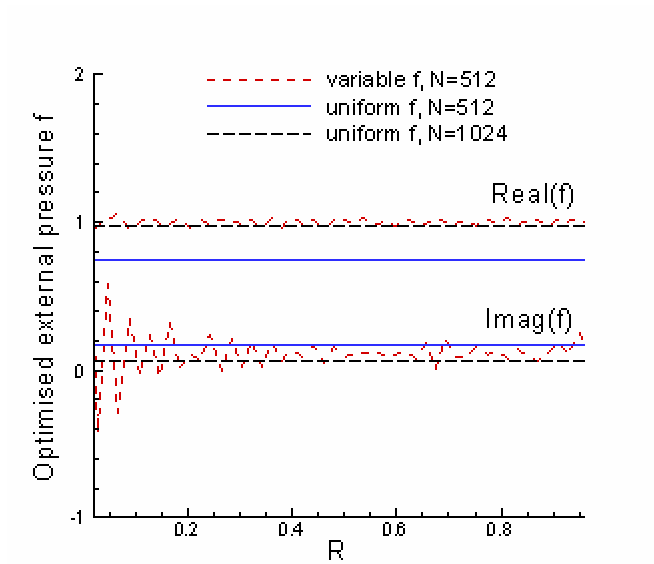

Figure 4: The distribution of the optimised external pressure *f* along the plate. *N* is the number of calculated modes.