WAVE FORCES ACTING ON A SEMISUBERGED AQUACULTERE FISH CAGE

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1.Introduction

Recently, with the reduction of fishery products, more and more effort and fund are invested to the aquaculture in the bay or coastal area. Fishes are fed up in fish cages with various shapes. These fish cages are made of porous materials to allow fresh seawater to flow in and out. Generally, the sea condition at the fish farm is quite rough, it necessary to investigate the wave loads acting the cages.

In the present work, a porous circular-cylindrical cage semi-submerged in waves is studied. The problem is solved semi-analytically by means of the eigen-function expansion. The water domain is divided into two regions and different eigen-functions are applied. In the interior region, it is equivalent to the problem of two porous disks, which leads to a complicated transcendental equation for the eigen values and eigen functions in the interior region.

2. Formulation of the problem

A circular-cylindrical fish cage with radius a is fixed in a water of depth h. A polar coordinate system (r, θ, z) is adopted to describe the problem. The origin is put on the still wafer level. The axis of the fish cage is taken as the z-axis, pointing upward (see Fig.1). The upper plate of the cylindrical cage is located at z=- d_1 while the lower one is situated at z = - d_2 ($1 < d_1 < d_2 < h$). The height of the cage $d = d_2 - d_1$. It is assumed that the fluid is inviscid and incompressible, and the flow is irrotational. There exists a velocity potential, which can be expressed as $\Phi(\boldsymbol{x},t) = \mathrm{Re}\{\phi(\boldsymbol{x})e^{-i\omega t}\}$ when the fluid motion is harmonic. Under the assumption of linear wave theory, the velocity potential $\phi(\boldsymbol{x})$ satisfies the following governing equation as well as boundary conditions on the free surface, sea bed and the Sommerfeld far field:

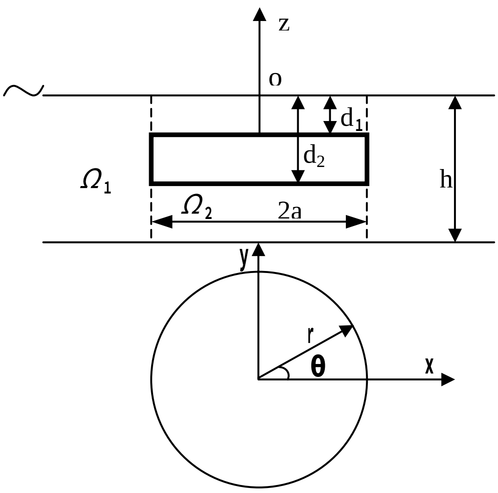

Fig.1 Definition of polar coordinate system and division of fluid domain

$$\nabla^2\phi = 0 \quad (-h < z < 0) \qquad \partial\phi/\partial z = 0 \quad (z = -h)$$
$$\partial\phi/\partial z - \nu\phi = 0 \quad (z = 0) \qquad \lim_{r\to\infty}\sqrt{r}\,(\partial\phi/\partial r - ik_0\phi) - 0 \tag{1}$$

Here, $\nu=\omega^2/g$ with gas the gravity acceleration. The frequency and wave number of incident waves are denoted by ω and k_0 respectively, which satisfies the dispersion relation $\nu = k_0 \tanh k_0 h$

The fish cage is made of porous materials to let the sea water flow through it. Following the fine-pore assumption made by Chwang(1983) and used in many other works, the normal velocity is continuous through the porous boundary and is proportional to the pressure difference between two sides of the boundary. Hence, the boundary condition on the body surface may be written as:

$$\begin{aligned} &\partial\phi(z=-d_j+0)/\partial z = \partial\phi(z=-d_j-0)/\partial z \\ &= i\sigma_j\left[\phi(z=-d_j-0)-\phi(z=-d_j+0)\right] \quad (j=1,\ 2) \end{aligned} \tag{2-a}$$

$$\begin{aligned} &\partial\phi(r=a+0)/\partial r = \partial\phi(r=a-0)/\partial r \\ &= i\sigma_3\left[\phi(r=a-0)-\phi(r=a+0)\right] \end{aligned} \tag{2b}$$

In the above body surface conditions, σ_j is the porous-effect parameter, which is real, positive and is defined as $\sigma_j = \rho\omega b_j/\mu$ with ρ as the water density and μ as the dynamic viscosity of fluid. The parameter b_j is the porosity coefficient with a dimension of length and j = 1, 2, 3 represents the upper and lower plates and sidewall of the cylindrical fish cage respectively.

3. Solution to the problem

To solve the above boundary value problem, the fluid domain is divided into two regions, i.e. an exterior region Ω_1($r > a$, $-h < z < 0$) and an interior one Ω_2 ($r < a$, $-h < z < 0$) (see Fig. 1). The velocity potential has two parts accordingly, i.e. $\phi^{(e)}(\boldsymbol{x})$ and $\phi^{(i)}(\boldsymbol{x})$ valid in exterior and interior region respectively.

The solution of the eigen-function expansion is readily obtained for the exterior region:

$$\begin{aligned} \phi^{(e)}(x) = \frac{g\zeta_0}{i\omega}\sum_{n=0}^{\infty}\varepsilon_n\Big[&\big(i^n J_n(k_0 r) + A_{n0}\, H_n(k_0 r)/H_n^{'}(k_0 a)\big)Z_0(z) \\ &+\sum_{m=1}^{\infty} A_{nm}\, K_n(k_m r)/K_n^{'}(k_m a)\, Z_m(z)\Big]\cos n\theta \qquad Z_m(z) = \begin{cases}\cosh k_0(z+h) & (m=0) \\ \cos k_m(z+h) & (m\ge 1)\end{cases} \end{aligned} \tag{3}$$

As well known, the first term in the first summation of the above expansion represents the incident wave potential. The parameter ε_n is equal to 1 as n = 0 and equal to 2 otherwise, and k_m (m = 1, 2, 3…) is the pure imaginary root of equation $\nu = k \tanh kh$.

In the interior region, by means of variable separation, the potential can be expressed as:

$$\phi^{(i)}(x) = \frac{g\zeta_0}{i\omega}\sum_{n=0}^{\infty}\varepsilon_n\sum_{l=1}^{\infty}[B_{nl}\, J_n(K_l r)/J_n^{'}(K_l a)]\, f(K_l z)\cos n\theta \tag{4}$$

The function $f(K_l z)$ and eigen value K_l satisfy the following differential equation and boundary conditions:

$$\begin{aligned} &f''-K^2 f = 0 \qquad (-h<z<-d_2, -d_2<z<-d_1, -d_1<z<0) \\ &f'-\nu f = 0 \qquad (z=0) \qquad\qquad f'=0 \qquad (z=-h) \\ &f'(z=-d_j+0) = f'(z=-d_j-0) \\ &= i\sigma_j[f(z=-d_j-0)-f(z=-d_j+0)] \quad (z=-d_j,\ j=1,\ 2) \end{aligned} \tag{5}$$

Taken into account of the governing equation, the free surface condition at z = 0, the sea bed condition at

z = -h and the continuity of the normal velocity at $z = -d_1$ and $z = -d_2$, the function $f(Kz)$ might be expressed as:

$$f(Kz)=\begin{cases} P(K\cosh Kz+\nu\sinh Kz) & (-d_1<z<0) \\ PD(Kd_1)\cosh K(z+d_2)/\sinh K(d_2-d_1) \\ -Q\sinh K(h-d_2)\cosh K(z+d_1)/\sinh K(d_2-d_1) & (-d_2<z<-d_1) \\ Q\cosh K(z+h) & (-h<z<-d_2) \end{cases} \quad (6)$$

$$D(Kd)\text{ is defined as}: D(Kd)=\nu\cosh Kd-K\sinh Kd$$

To determine the unknown constants P, Q and the eigen value K, the above expression for $f(Kz)$ is substituted into the remaining conditions at $z = -d_1$ and $z = -d_2$. The eigen value K is obtained by solving a complicated 'dispersion-dissipation relation' as follows:

$$\begin{aligned}&\sigma_1\sigma_2 D(Kh)+i\sigma_1 KD(Kd_2)\sinh K(h-d_2)+i\sigma_2 D(Kd_1)\sinh K(h-d_1)\\ &=K^2 D(Kd_1)\sinh K(h-d_2)\sinh Kd\end{aligned} \quad (7)$$

The porous effect parameter σ_j varies from 0, i.e. impermeable, to infinite which means completely permeable. Some special cases will be discussed. When both σ_1 and σ_2 tend to infinite, the dispersion relation of exterior region, i.e. $D(Kh) = 0$, is recovered since both plates in the interior region are completely permeable or 'disappeared'. When one of tends to infinite, it reduces to only one porous pate in the interior region. The results are the same as the previous works like in Chwang and Wu(1994). When σ_j vanishes, it gives a locally shallow water effect.

There are infinite number of discrete complex root $K_l = K_{lr} + iK_{li}$ (l = 1, 2, 3…) for the transcendental equation (7). It can be solved numerically, e.g. by means of the Newton-Raphson iteration method. One of the unknown constants P and Q can be arbitrarily chosen. In the present work, we put $P=K\sinh K(h-d_2)\sinh Kd-i\sigma_2\sinh K(h-d1)$, which yields $Q=-i\sigma_2 D(Kd_1)$.

It should be noted that the eigen-function $f(K_l z)$ is orthogonal though its form is complicated, i.e.

$$\int_{-h}^{0} f(K_l z)f(K_p z)dz=\begin{cases}0 & \text{for } p\neq l\\ N_l & \text{for } p=l\end{cases} \quad (8)$$

The expression for N_l is too complicated. It had better be omitted in the present compact abstract.

Once the eigen function $f(K_l z)$ and eigen value K_l are determined, the unknown coefficients A_{nm} and B_{nl} in (3) and (4) can be obtained by matching two parts of the velocity potential and normal velocity at the common interface $r = a$, $-h < z < 0$, i.e.

$$\partial\phi^{(i)}/\partial r=\partial\phi^{(e)}/\partial r$$
$$\phi^{(i)}=\begin{cases}\phi^{(e)} & (r=a,\ -d_1<z<0 \text{ and } -h<z<-d_2)\\ \phi^{(e)}+\dfrac{1}{i\sigma_3}\dfrac{\partial\phi^{(e)}}{\partial r} & (r=a,\ -d_2<z<-d_1)\end{cases} \quad (9)$$

By means of the orthogonal property of eigen-functions, the above matching conditions yield a system of linear algebraic equations for the unknown coefficients A_{nm} and B_{nl}, i.e.

$$A_{nm} = \frac{1}{C_m k_m} \sum_{l=1}^{\infty} S_{ml} K_l B_{nl} - i^n J_n^{'}(k_0 a)\delta_{m0} \qquad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$B_{nl} = \frac{J_n^{'}(K_l a)}{N_l J_n(K_l a)} \{ i^n [J_n(k_0 a) S_{0l} + J_n^{'}(k_0 a) T_{0l} k_0 / i\sigma_3] \qquad R_n(k_m a) = \begin{cases} H_n(k_0 a) & m = 0 \\ K_n(k_m a) & m > 0 \end{cases} \tag{10}$$

$$+ \sum_{m=0}^{\infty} [S_{ml} A_{nm} R_n(k_m a) / R_n^{'}(k_m a) + T_{ml} A_{nm} k_m / i\sigma_3] \}$$

In (10), the superscript prime denotes the derivative with respect to the argument. The coefficients C_m, S_{ml} and T_{ml} are all known and given by the integrals of eigen functions as follows:

$$C_m = \int_{-h}^{0} Z_m(z) Z_m(z) dz = \begin{cases} \dfrac{h(k_0^2 - \nu^2) + \nu}{k_0^2 - \nu^2} & m = 0 \\ \dfrac{h(k_m^2 + \nu^2) - \nu}{k_0^2 + \nu^2} & m > 0 \end{cases} \tag{11a}$$

$$\begin{aligned} S_{ml} &= \int_{-h}^{0} Z_m(z) f(K_l z) dz \\ T_{ml} &= \int_{-d_2}^{-d_1} Z_m(z) f(K_l z) dz \end{aligned} \tag{11b}$$

Once the potential is solved in both regions, the hydrodynamic forces acting on the fish cage may calculated by integrating the pressure difference on two sides of the cage surface. By using the porous boundary condition, the integrand can be replaced by the normal velocity at the corresponding surface.

$$\begin{aligned} \frac{F_x}{\rho g \zeta a^2} &= \frac{1}{i a \sigma_3} \int_0^{2\pi} \cos\theta d\theta \int_{-d_2}^{-d_1} \partial\phi^{(e)}(a,\theta,z) / \partial r \, dz \\ \frac{F_z}{\rho g \zeta a^2} &= \int_0^{2\pi} d\theta \int_0^a [\frac{\partial\phi^{(i)}(r,\theta,-d_2)/\partial z}{i a^2 \sigma_1} - \frac{\partial\phi^{(i)}(r,\theta,-d_1)/\partial z}{i a^2 \sigma_2}] r dr \\ \frac{M_y}{\rho g \zeta a^3} &= \frac{1}{i a^2 \sigma_3} \int_0^{2\pi} \cos\theta d\theta \int_{-d_2}^{-d_1} \partial\phi^{(e)}(a,\theta,z) / \partial r \, z dz \\ &\quad - \int_0^{2\pi} \cos\theta d\theta \int_0^a [\frac{\partial\phi^{(i)}(r,\theta,-d_2)/\partial z}{i a^3 \sigma_1} - \frac{\partial\phi^{(i)}(r,\theta,-d_1)/\partial z}{i a^3 \sigma_2}] r^2 dr \end{aligned} \tag{12}$$

Reference

1. Chwang A.T. (1983): A Porous-Wavemaker Theory. J. Fluid Mech., 132, pp395-406.
2. Chwang A. T. & Wu J. (1994): Wave Scattering by Submerged Porous Plate. J. Eng. Mech., 120(12), pp 2572-2587.