VERIFICATION OF THE METHOD OF FLAT CROSS-SECTIONS FOR THE CASE OF JET IMPACT ONTO ELASTIC PLATE

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The paper is concerned with 3D compressible liquid jet impact onto an elastic plate (Fig.1). The response of the plate is governed by the linear dynamic equation. The coupling between the fluid flow and the plate deflection is taken into account through the dynamic and kinematic conditions imposed on the wetted part of the plate. Solution of this problem was constructed by Korobkin, Khabakhpasheva & Wu [1,2] for a two dimensional problem, an axisymmetrical problem and a three dimensional problem of a rectangular jet impact onto a rectangular plate. Modal analysis was used for both fluid flow and structural response. However, the problem of impact by a jet of arbitrary cross section is still open.

In this study the hydroelastic problem of elongated jet impact onto rectangular elastic plate is solved approximately by using strips theory for hydrodynamic analysis. The plate response is evaluated in the same way as in [1,2]. This combined approach for solving 3D problems of hydroelasticity was proposed by Dr. S. Malenica (private communication at last WWWFB).

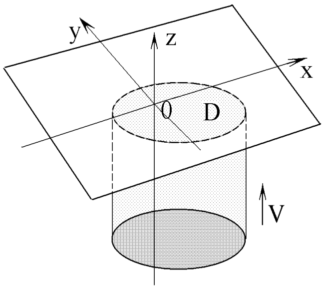

Figure 1

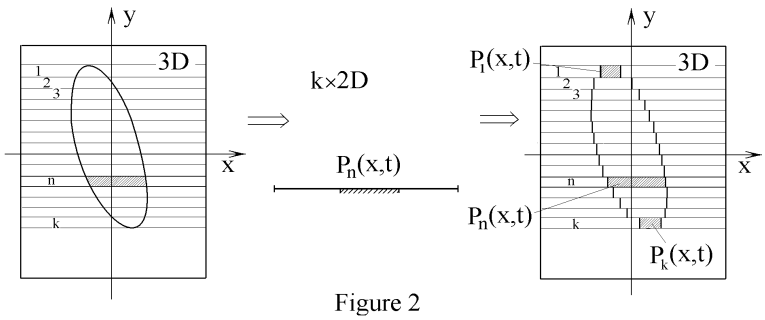

Figure 2

The present analysis is dedicated to verification of the method of 2D cross sections (strips method) and application of this method for the case of impact of the jet with arbitrary cross section onto rectangular elastic plate. To verify applicability of this method we can solve 3D problem of a rectangular jet impact onto a rectangular plate twice: first the problem solved as a full 3D problem (see [1,2]) and, second, by the method of 2D cross sections. Namely, we divide the part of the plate with the contact region into thin stripes and determine the hydrodynamic pressure at each stripe via solution of 2D problem, while vibration of the plate is determined as solution of 3D problem (Fig.2). For illustration of applicability of this approach elliptical and diamond shape of the jet cross section are considered.

Formulation of the problem

The coupled problem of jet-structure impact is considered. The structure is a single simply supported plate of uniform thickness. The jet with constant cross section and a flat head hits the plate from below in the normal direction. Gravity and surface tension effects are neglected and the liquid is assumed ideal. The jet speed V is assumed to be much smaller than the speed of the sound c_0. The disturbed flow due to the impact is then described within the linear acoustic approximation through the velocity potential theory. The initial stage of the impact is considered here only. Over this short period of time, the deformation of the jet surface is neglected and the boundary conditions are linearized.

The problem is considered in non-dimensional variables, where the characteristic dimension R of the jet cross section D is the length scale, jet speed V is the velocity scale, the "water hammer" pressure $\rho c_0 V$ is the pressure scale, ρ is the liquid density, the product VR is the scale of the velocity potential of the flow in the jet region $(x,y) \in D,\ z < 0$ and RV/c_0 is the plate deflection scale, $Oxyz$ is the Cartesian coordinate system with the plate being in the plane $z = 0$. The ratio R/c_0 is taken as the time scale.

The total velocity potential can be written as $z - \phi(x,y,z,t)$, where ϕ is the disturbed potential which satisfies the following equations and boundary conditions

$$\varphi_{tt} = \varphi_{xx} + \varphi_{yy} + \varphi_{zz} \qquad ((x,y) \in D,\ z < 0), \tag{1}$$

$$\varphi = 0\ \ ((x,y) \in \partial D,\ z < 0), \quad \varphi \to 0\ \ ((x,y) \in D,\ z \to -\infty), \tag{2}$$

$$\varphi_z = 1 - w_t(x,y,t) \qquad ((x,y) \in D,\ z = 0), \tag{3}$$

$$\varphi = \varphi_t = 0 \qquad (t = 0), \tag{4}$$

where $w(x,y,t)$ is the plate deflection, $p(x,y,z,t) = \varphi_t$ is the hydrodynamic pressure.

The plate deflection is governed by the following equation

$$\alpha w_{tt} + \beta \Delta^2 w = p(x,y,0,t) \qquad ((x,y) \in S,\ \ t > 0), \tag{5}$$

$$w = 0, \qquad \Delta w = 0 \qquad (x,y) \in \partial S, \tag{6}$$

$$w = w_t = 0 \qquad (t = 0), \tag{7}$$

where Δ is the Laplace operator on x and y variable,

$$\alpha = \frac{m}{\rho R}, \qquad \beta = \frac{D_p}{\rho c_0^2 R^3},$$

$m = \rho_p h$ is the plate mass per unit area, ρ_p is the density of the plate material and h is the plate thickness, $D_p = \frac{Eh^3}{12(1-\nu^2)}$ is the plate stiffness, E is the Young modulus, ν is the Poisson ratio and S is the surface area. It can seen that the problem under consideration is a coupled one of hydroelasticity. The liquid flow and the plate deflection have to be determined simultaneously.

3D and 2D cross sections methods of solution

By using normal mode method and Laplace transform, the problem can be reduced to a system of ordinary differential and integral equations with respect to time for principal coordinates of velocity potential and the plate deflection. The system can be truncated and solved numerically by the fourth order Runge-Kutta method, the integral terms can be computed by trapezoidal rule [1,2].

Modal analysis was used in [1,2] for both fluid flow and structural deflection. For the case of simple geometry (two-dimensional jet impact onto a beam, three-dimensional rectangular jet impact onto rectangular plate and axisymmetrical jet impact onto circular plate) the elements of linear system for principal coordinates were found by analytical formulae and are presented in [2]. But modal approach is not useful for the case of arbitrary geometry because it is very difficult to define eigenmodes both for the plate and for the jet cross-section. To solve the problem of the impact by jet with arbitrary elongated cross-section the following idea looks applicable: we can divide the part of the plate with the contact region into a number of thin strips and for a strip with number n we can determine hydrodynamic pressure distribution $P_n(x,t)$ via solution of a 2D problem. After that we find the total pressure as the sum of the $P_n(x,t)$, acting on corresponding strips, and determine the plate vibration under this pressure as a solution of full 3D problem. This is a modification of the method of 2D cross sections which is well known in the ship hydrodynamics.

Verification of 2D cross sections method

To verify accuracy of the 2D cross sections method we solve 3D problem for rectangular plate and rectangular jet cross section twice: as a full 3D problem (see [1,2]), and with the fragmentation into the number of thin strips. For each strip we can use analytical formulae for the hydrodynamics modes and, consequently, for the coefficients of the system for 2D hydrodynamic problem (see [2]). It is interesting to note, that for the case, when impact geometry (place of the jet center and jet width) in every strip is the same as for the other strips, we can find analytical formulae for the total pressure, which is independent of the number of the strips, but different from formulae for full 3D problem.

Pressure investigation for the jet - plate impact (see [2]) shows, that at the initial time instant pressure has discontinuity at the edge of a contact region, but as a result of liquid compressibility and propagation of rarefaction waves from the jet surface into the contact region (see [6] for details), at each subsequent time instant the pressure tends linearly to zero in the vicinity of the edge of the contact region. In nondimensional variables, rarefaction waves propagated with unite velocity.

But if we use the 2D cross sections method directly, without modification, we do not considere the rarefaction wave moving from the jet free surface on y-direction (strips are normal to y-direction). To take into account these waves we introduce decreasing coefficients k_i for the hydrodynamic pressure on the outer strips which are linearly proportional to time and change from zero to unite according to velocity of the rarefaction waves propagation (see sketch on Fig.3). We use these coefficients during the first unite of the nondimensional time, until the value of the pressure becomes small (see [2]).

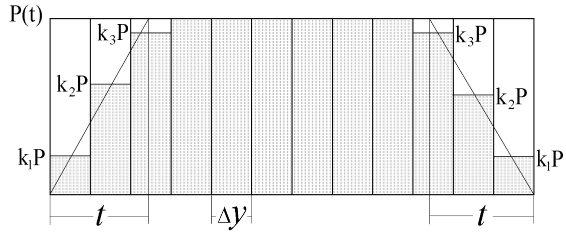

Figure 3

All results presented in this paper are obtained for a steel plate with $E = 21 \cdot 10^{10}\,\mathrm{N/m^2}$, $\rho_p = 7875\,\mathrm{kg/m^3}$, $\nu = 0.3$. The water jet parameters are $c_0 = 1500\,\mathrm{m/s}$, $\rho = 1000\,\mathrm{kg/m^3}$. The dimensions of the plate are $1\,\mathrm{m} \times 1\,\mathrm{m}$, the plate thickness is 2 cm. The impact velocity is $V = 10\,\mathrm{m/s}$. The dimensionless time step is $\Delta t = 0.04$, which corresponds to $5.3 \cdot 10^{-6}$ s. Calculations for the 3D model were performed with 100 modes for the plate and 100 modes for the jet. In 2D cross section approach the number of the 2D hydrodynamics modes was 10, the number of the 3D elastic modes was 100 and the number of strips was 29.

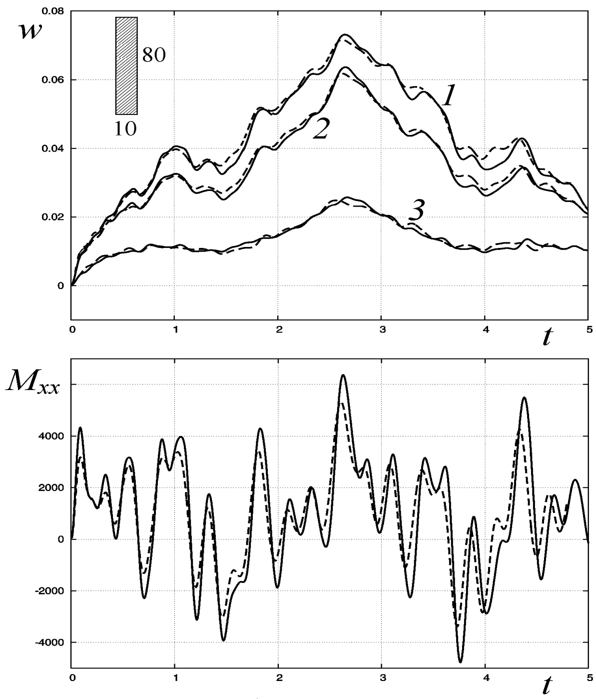

Figure 4

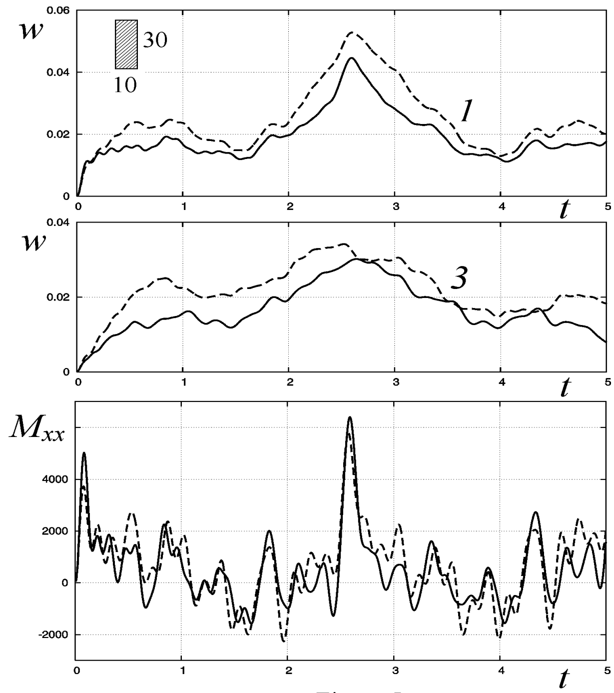

Figure 5

Figs.4-5 present the comparison of the time-histories of the plate deflections and bending moments for the 3D (solid lines) and 2D cross sections approach (dashed lines). The unit of time is the microsecond. The centre of the jet coincides with the centre of the plate. Fig.4 corresponds to the rectangular jet cross section with $a_j \times b_j = 10\,\mathrm{cm} \times 80\,\mathrm{cm}$, and Fig.5 is for the case $a_j \times b_j = 10\,\mathrm{cm} \times 30\,\mathrm{cm}$. Curves 1 on the plots for deflection and plots for the bending moments are for the centre of the plate ($x = y = 0.5\,\mathrm{m}$). Curves 2 and 3 on the plots are for the points on the plate with $x = 0.5\,\mathrm{m}$ and $y = 0.5 \pm b_j/2$ or $y = 0.5 \pm b_j/4$. One can see, that for more elongated jet cross sections (Fig. 4) results of both approaches for deflections are in very good agreement, but 2D cross section method underestimates the values of the bending moments. For the second case (Fig. 5) both deflection and bending moments are in qualitative agreement only. We can conclude, that elongation of the jet cross section is not sufficient for application of the strips method.

Numerical simulations show, that 2D cross section method for the jet plate impact is useful for the jet cross section with elongation $b_j/a_j > 4$ and arbitrary location of the jet center. We can predict well the evolution of the plate deflections, but we need to take into account, that for the bending moments we underestimate the maximum values about on 15-20%.

Results for elliptical and diamond shape of the jet cross section

Figs.7-8 present the comparison of the plate deflections and bending moments for the different shape of the jet cross section with the same area. Shapes and sizes of the jet cross section, considered here, are presented at the Fig.6.

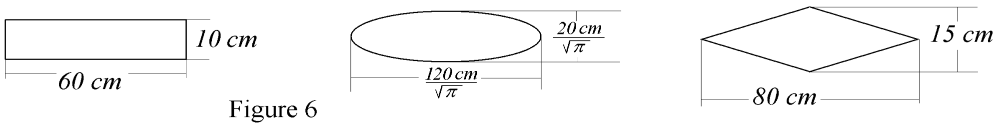

Figure 6

The centre of the jet coincides with the centre of the plate for the cases on Fig.7 and located at point (30 cm, 40 cm) for the Fig.8. Solid, dashed and dotted lines correspond to the rectangular (3D approach), elliptical and diamond shapes of the jet cross sections respectively. One can see, that the curves at each plot are quit close to each other. This fact indicates that, the vibration of the plate under impact of the jet of arbitrary cross section are very close to vibration of the plate under impact of rectangular jet with equal area, elongation and location.

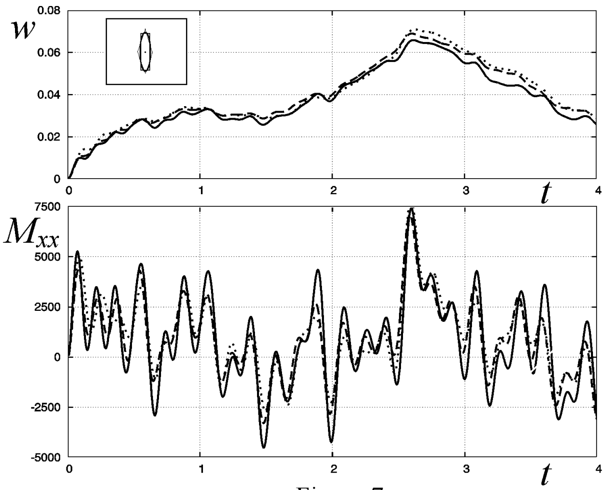

Figure 7

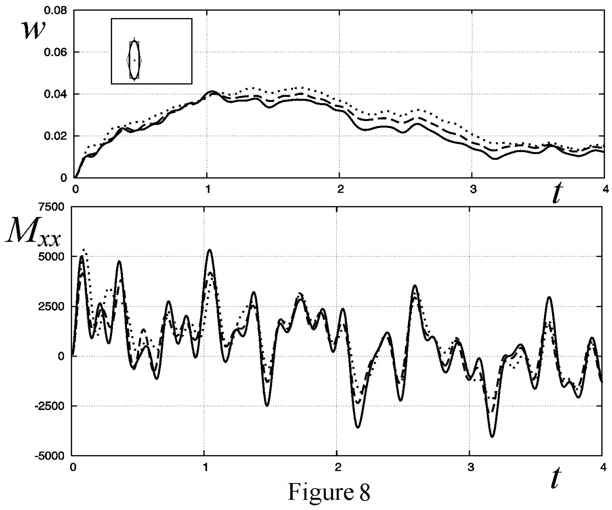

Figure 8

Conclusions

The following conclusions are made in the context of present study:

• 2D cross section method for the jet plate impact is useful for the jet cross section with elongation $b_j/a_j > 4$ and arbitrary location of the jet center. It is possible to predict the evolution of the plate deflections, but the maximum values of the bending moments underestimated by 15-20%.

• To predict vibration of the plate under impact of jet with arbitrary cross section we can use the results for rectangular shape of the jet with equal area, elongation and location.

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