On Wave Amplification over Submerged Lenses.

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Introduction

The paper is dealing with two closely related problems:

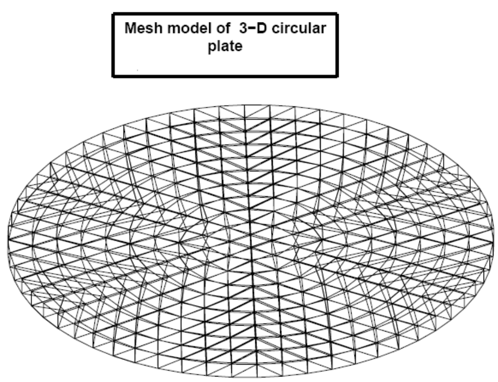
- Focusing of waves in order to bring about a local increase in the wave energy.
- Defocusing of waves in order to mitigate the wave field in a certain area.

The most obvious application in the first case would be to increase the output from some nearby wave energy device. The climate issue and the world's craving for renewable energy is making wave energy increasingly attractive, although, admittedly, there is also a cost side to this that cannot be easily overlooked.

In the second case there may also be important applications. Marine operations frequently rely on benign conditions: If waves can be directed away from the critical area, the threshold for the operation might be raised. In addition, permanent installations will also usually benefit from less severe waves.

The main driver behind the present investigation comes from optics: A light wave passing through a transparent medium (e.g. from air to glass) will be refracted, the underlying reason being the change in wave length that takes place at the interface. A similar situation may arise for water waves, as they propagate over a given submerged obstacle: A step like change in water depth is causing a change in wave length, and possibly a subsequent change in wave direction.

Of course, in general terms, wave reflection will also be part of the story, both in optics and hydrodynamics, and reflection will cause an overall loss of energy in the main direction of the waves. However, locally this can be more than compensated for by wave enhancement produced by refraction.



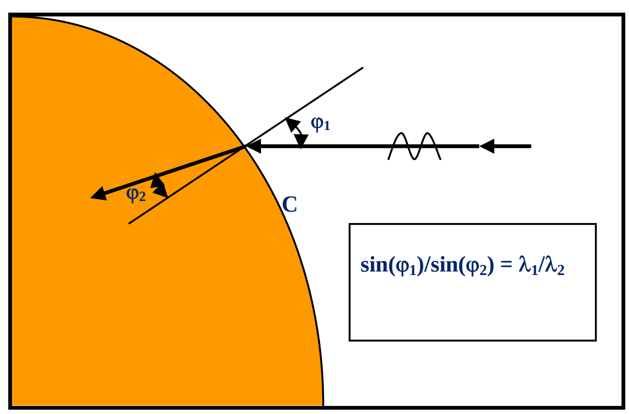


Fig. 1. Mesh model of a 3-D circular plate (left) and optical refraction at interface between two transparent media (right).

The main difference between between optics and hydrodynamics is that in the latter case there is no sharp, well defined boundaries and no strict adherence to a "Snell-type" law of refraction:

$$\frac{\sin(\phi_1)}{\sin(\phi_2)} = \frac{\lambda_1}{\lambda_2} = \eta$$

where η is the refraction index, φ_1 and φ_2 the direction of the incident and refracted wave, and with corresponding interperetation for the λ's. In hydrodynamics the interface is more "blurred", and the problem has to be investigated numerically. However, the important fact here is that a change in water depth will cause a change in wave length, although the scales over which this takes place are different.

Numerical examples

The base case for the investigation is an ambient water depth of 100.0 m, and one or more horizontal, flat plates residing at a water depth of 20m. The thickness of the plates is 2m (the precise figure is of minor importance) and the incoming waves are unidirectional (this is important). Four simple shapes will be considered:

1. Circular, disc shaped plate (with variable radius)
2. Hyperbolic plate
3. Rectangular plate
4. Twin rectangular plates

A more extensive number of geometric shapes have been investigated previously, but it turns out that, in most cases, the precise shape is of little consequence. For the present purpose the computations have been carried out with captive plates (i.e. fixed in space).

Also, the computations have been carried out with the linear version of the Wamit program. As a consequence nonlinear effects are not included or considered. The results should therefore be viewed with this in mind.

Lense data			
Shape	Target application	Maximum width (m)	Maximum length (m)
Rectangular	Wave amplification	75.0	150.0
Twin rectangular – 80 m slit	Mitigation	75.0	2 x 150.0
Hyperbolic	Mitigation	100.0	400.0
Circular disc	Amplification	Diameter D: 50.0 to 200.0	

Table 1. Overview of basic shapes used to create wave enhancement or wave supression.

In the examples given below, the sensitivity of the results to wave period, shape and magnitude is highlighted for a few simple cases:

- In Figs. 2 and 3 the sensitivity to the wave length is illustrated for the single, rectangular lense and for two given wave periods, 15.0 and 6.4 sec, corresponding to a radian frequency ω of about 0.42 and 0.98 rad/sec. It is seen that the wave amplification is much subdued, from nearly 100% increase to a far more moderate 15% increase when the wave period is reduced. This is related to corresponding changes in the "apparent" refraction index as defined above: This causes the focusing effect to be reduced, but also more "sustainable".

- In Figs. 4-5 the effect on wave amplification resulting from a change in lense magnitude is highlighted for two sets of circular lenses of diameter 50 and 200m. The basic features seen for the rectangular lense is retained for the 15.0 sec wave period, but it is amply demonstrated that the magnitude of the lense plays an important role. The local wave amplification is broadened and increase significantly for the larger lense, compared to the smaller one.
- In Figs. 6-7 a slighly different situation arises: In the case of Fig. 6 a distinctly hyperbolic (concave) lense has been applied, and the result is a defocusing of the waves from the center region, due to which a significant wave depression occurs

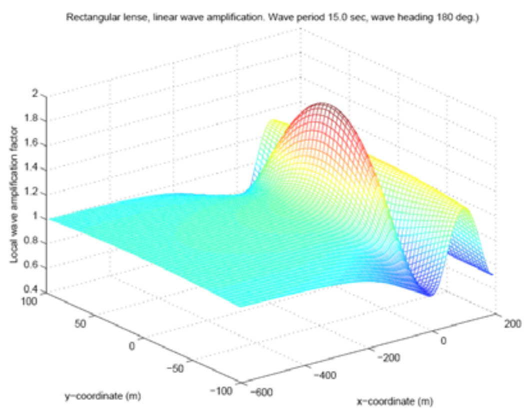


Fig. 2 Wave amplification, rectangular lense. Wave period T = 15.0 sec.

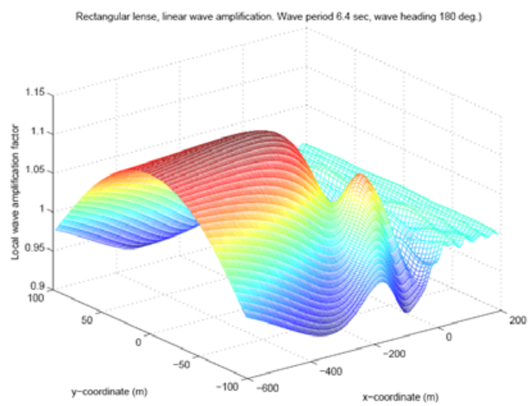


Fig.3. Rectangular lense. Wave period 6.4 s

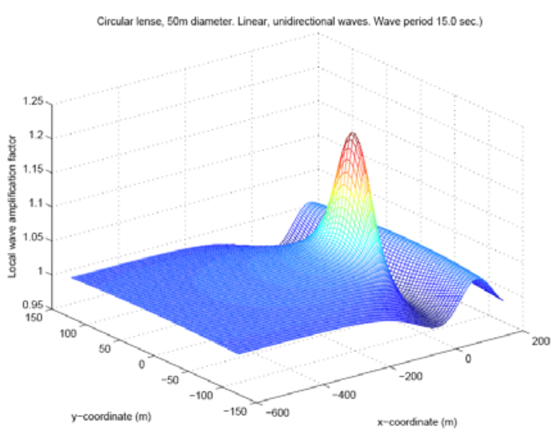


Fig.4. Circular lense. D(diameter)=50m. T=15.0 s

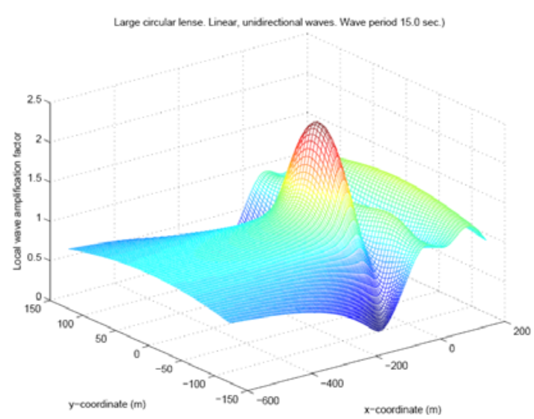
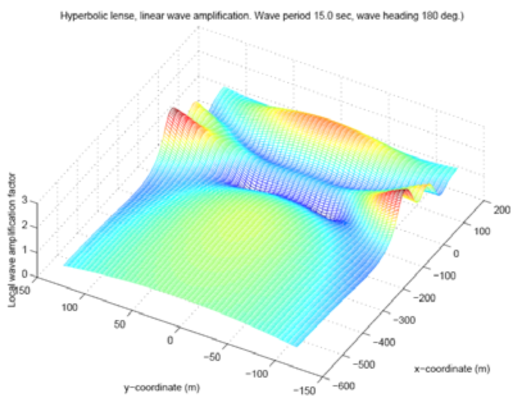


Fig. 5 Circular lense. D=200m. T=15.0 s.

locally downstream of the center region. At the same time waves are reflected back to an upstream focal region. The analogy with optics is quite apparent. Note that much of the same effect is accomplished by using two rectangular lenses with a slit between them (Fig. 7).



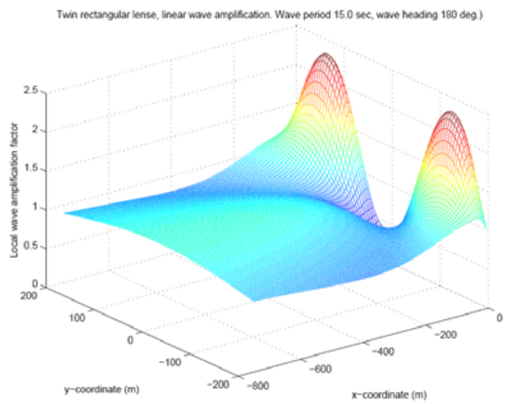
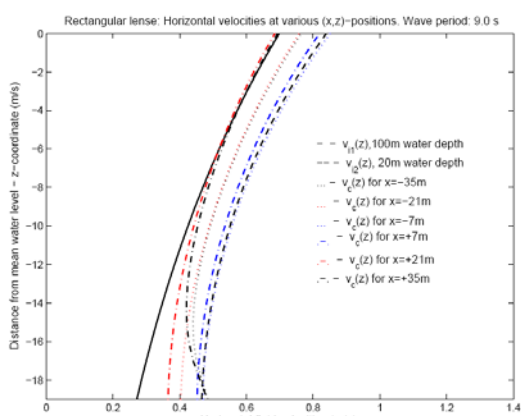


Fig. 6 Hyperbolic lense. Wave period 15.0 s. Fig. 7 Twin rectangular lense. T = 15.0 s

Fluid velocity profile over a single lense

The extent to which a change in wave length takes place as the waves pass over a submerge threshold (i.e. lense) can to some degree be tested. This is illustrated in Figs. 8 and 9 below where fluid velocities over the centerline of a single rectangular lense are plotted, for wave periods of 9.0 (left) and 15.0 sec (right).

At first, as the waves are progressing over the edge of the lense, the fluid velocities change only very slowly, except very close to the lense itself. In the case of the 9 sec wave, the flowfield has adjusted itself almost fully to the 20m constant depth solution around the mid-region of the lense. For the long wave case (T=15 sec), the presence of the lense is felt much further upstream, and no stable solution is reached over the lense. Tentatively, from these very simple examples it seems that it takes around a quarter of a wave length for the waves to readjust to the constant depth solution. For the longest waves, the lense quite simply is not wide enough.



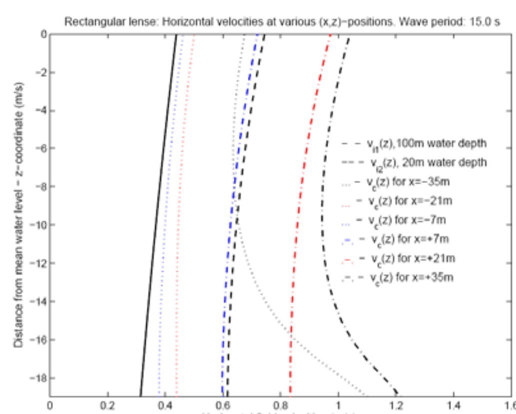


Fig. 8. Fluid velocities - rectangular plate. T=9s. Fig. 9. Velocities over the plate for T=15s.

Acknowledgements and references

P. Teigen, “On focusing and defocusing of waves in finite water depth”, ISOPE’2007, Lisbon, Proceedings.

The author would also like to acknowledge E. Mehlum for his work in this field around 1980.